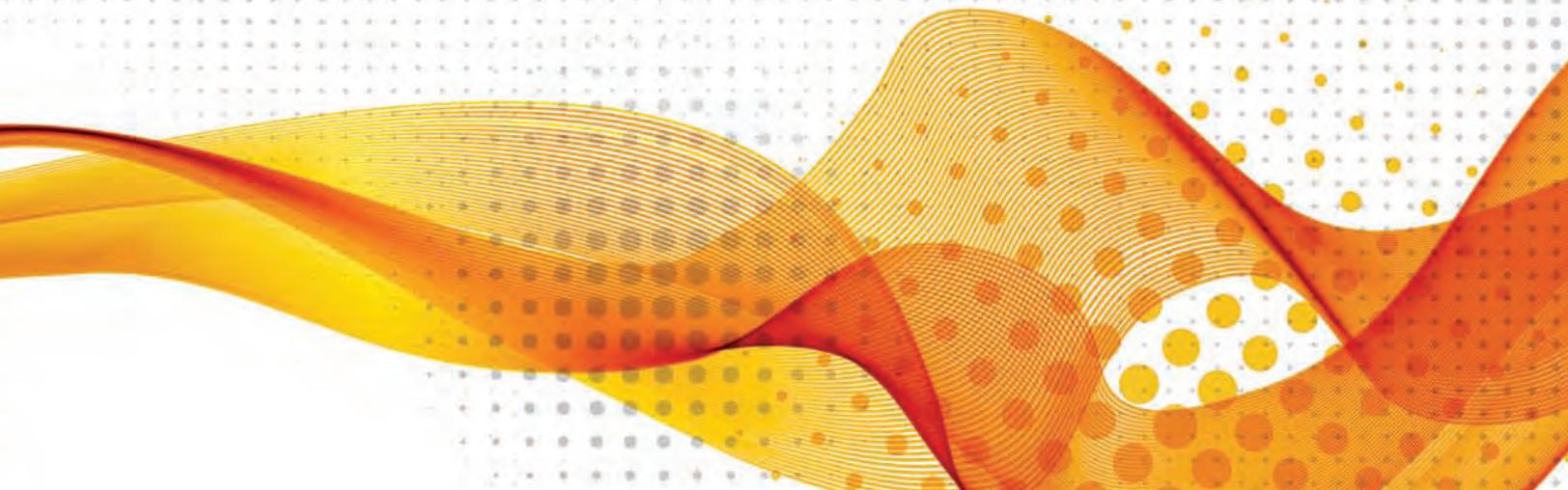


ESSENTIAL MATH SKILLS

Preparation for High School Equivalency Tests



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Senior Consultants
Bonnie Goonen
Susan Pittman-Shetler



Essential Education
Learning Made CERTAIN

INTERACTIVE PRACTICE WORKBOOK

Aligned to the Common Core State Standards and College and Career Readiness Standards



Essential Education
Learning Made CERTAIN

Essential Math Skills

Interactive Practice Workbook

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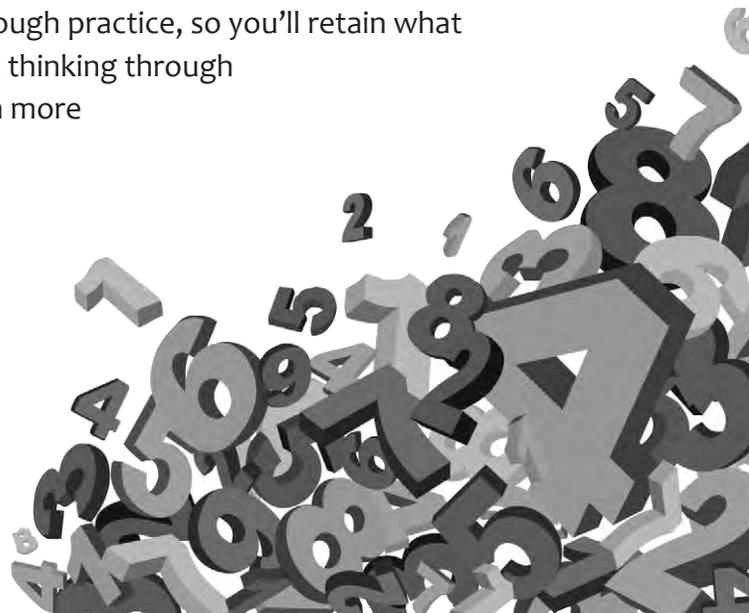
Introduction

The *Essential Math Skills* workbook will help you improve your mathematical reasoning, giving you new ways to think about and understand math. Math sometimes seems difficult because it appears abstract, but this book will help you recognize and work with applications of math, from personal finance to science. As you think through real-world math applications, you'll improve your grasp of math basics as well as learn problem solving, algebra, and geometry. You'll be successful in abstract thinking about math problems because your understanding of math will be deeper. You'll understand what math problems, expressions, and procedures really represent.

Understand
Plan
Attack
Check

Because reasoning is central to mathematics, this workbook focuses on developing strong quantitative, algebraic, and geometric reasoning skills, as well as problem-solving procedures to help you think through what you're practicing. This book introduces the **UnPAC** process, a simple procedure that you can use to think through the solution to math problems at every level. You'll learn through practice, so you'll retain what you've learned. When you're an active learner, thinking through problems and working out solutions, you learn more quickly and remember more.

You can use this workbook to get extra practice to supplement a class or online learning program, such as Essential Education's GED Academy™ and Essential Skills Online math programs. If you're in a class, your teacher can help you choose appropriate sections from this workbook to supplement your



learning. If you're studying on your own, you can identify the sections that are most relevant to what you're learning.

This workbook can also be used on its own as a stand-alone learning tool by working through each section. Using this workbook, you'll solidify the foundations of your math knowledge and learn to apply reasoning and problem-solving skills to high-school-level mathematics.

Each lesson has four parts:

- **Connections** introduces important ideas for the lesson. Connect what you'll learn to what you already know, and learn how the concepts apply to your life.
- **Learn It!** guides you through a strategy to achieve the goal for the lesson and understand the math involved.
- **Practice It!** provides practice for the strategy you've learned. You'll also think about how to apply the strategy to varying situations.
- **Check Your Skills** gives you a check-up to see what you've learned. You'll answer question types like those on the 2014 GED® test and test your mastery of the lesson.

This math book contains practice problems that involve different levels of knowledge and thinking:

- ★ If a problem has one star, it's a procedural problem. You need to apply the right math skills to solve it.
- ★★ If a problem has two stars, it will require more thinking through. You'll have to consider options for solving it, and it will show a deeper understanding of math.
- ★★★ If a problem has three stars, it will really get you thinking about how you approach math problems and about what the math skills you're learning really mean.

As you work through this book, keep track of math ideas that recur and help you address many different problems. These ideas will be the foundation of your math knowledge and will help you in reasoning about math in any situation.

Build Your Math Skills

Challenge yourself to think of two or more approaches to solving a problem. You can compare processes and improve your math reasoning skills.

The **UnPAC** Process

Connections

Have you ever ...

- Estimated whether you had enough money to buy the groceries in your cart?
- Figured out how much material you needed for a household project?
- Found a method to track your car's gas mileage?



Math is about solving problems, and when you run into problems like these, you think them through to find solutions. The better you understand the process you already use to solve problems, the better you'll be able to tackle new and more difficult problems.

No matter how far you go in math, you can use the same basic problem-solving strategy. You'll **UnPAC** the problem.

-
- | | | |
|-----------|-------------------|---|
| Un | Understand | Ask: <ul style="list-style-type: none">• What information do you have?• What information are you missing?• What final result do you want? |
| P | Plan | Make a plan to solve the problem. <ul style="list-style-type: none">• Use math tools to make better plans.• Use simple, logical thinking and common sense. |
| A | Attack | Solve the problem! Carry out your plan step-by-step to find a solution. |
| C | Check | After you find a solution, check it. <ul style="list-style-type: none">• Does it make sense?• Does it answer the question? |
-



UnPAC the Problem

You can tackle any problem with **UnPAC**. Imagine you're creating a Christmas budget. How would you come up with a solution that works for you?

You have 15 gifts to buy. Five of them are for close family members and friends, and you want to budget more for their gifts. The other 10 are more distant family and work acquaintances, so you don't want to spend as much on their gifts. With your year-end bonus, you have about \$900 to spend on gifts.

Un Understand

Your first step is to understand the problem.

- ? 1. What do you want to find?

In this case, you want a budget. You want a specific amount of money to spend on each gift, and it needs to add up to the total you budgeted to spend, \$900.

- ? 2. What information do you have, and what information do you need?

It helps to look at information in a summary, as notes, sketches, or a table, like this:

Information You Have	Information You Need
Total budget: \$900	Budget for each expensive gift
Expensive gifts: 5	Budget for each less expensive gift
Less expensive gifts: 10	

P Plan

Make a plan to solve the problem. What steps will you take? Do you know any math tools that will help?

- ? 3. Make a plan to create your budget.

Here's an example plan:

- Assign $\frac{2}{3}$ of your total budget to expensive gifts and $\frac{1}{3}$ to inexpensive gifts.
- Divide the budget for expensive gifts by the number of expensive gifts.
- Divide the budget for inexpensive gifts by the number of inexpensive gifts.

A Attack

Solve the problem! Carry out your plan to find a solution. Work one step at a time, especially if the problem is complicated. If there is a specific question, make sure you answer it.

- ? 4. Create a budget based on your plan.

You might attack the example plan like this:

Step 1

Expensive gifts: $\frac{2}{3} \times 900 = 600$ Inexpensive gifts: $\frac{1}{3} \times 900 = 300$

Step 2

Since you have \$600 for five expensive gifts, each expensive gift can be about \$120.

$$\text{budget} \div \text{number of gifts} = \text{budget per gift}$$

$$600 \div 5 = x$$

$$120 = x$$

Step 3

Since you have \$300 for 10 inexpensive gifts, each inexpensive gift can be about \$30.

$$\text{budget} \div \text{number of gifts} = \text{budget per gift}$$

$$300 \div 10 = x$$

$$30 = x$$

C Check

After you find a solution, always check your answer.

- ? 5. Check that your budget works.

You'll want to make sure you have enough money! For example, you might total the cost of all the presents in a spreadsheet and make sure the budget equals \$900.

Using

Un P A C

Try using estimation to **check** answers.

Understand

Plan

Attack

Check

Practice
It!

Using **UnPAC**, answer the following questions.



1. Online, you find two sellers who carry the sleeping supplement you want. One has bottles of 50 capsules for \$9.95. The other has bottles of 200 capsules for \$31.89. Which is the better value?

a. **Understand:** What information do you want to find? What information do you have? What information do you need?

b. **Plan:** How will you solve the problem?

c. **Attack:** Solve it! Remember to answer the question.

d. **Check:** Check your answer.



2. You're leading a fundraiser for a local after-school program. You have 236 cookies donated by parents, and your goal is to raise \$150. What are two possible cookie prices, and how many cookies would you have to sell at those prices to raise \$150?

a. **Understand:** What information do you want to find? What information do you have? What information do you need?

b. **Plan:** How will you solve the problem?

c. **Attack:** Solve it! Remember to answer the question.

d. **Check:** Check your answer.



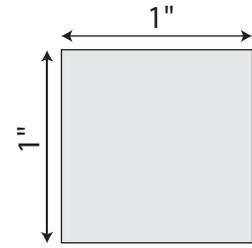
3. You're tiling the backsplash above part of your kitchen counter with multi-colored one-inch square tiles of blue, teal, green, and yellow. You need an equal amount of each color for your pattern. If the backsplash is 1.25 feet by 4.5 feet, how many blue tiles do you need?

a. **Understand:** What information do you want to find? What information do you have? What information do you need?

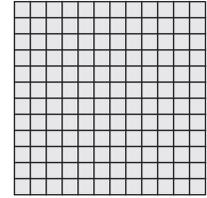
b. **Plan:** How will you solve the problem?

c. **Attack:** Solve it! Remember to answer the question.

d. **Check:** Check your answer.



Think: How many one-inch tiles fit in one square foot?



4. You're buying a hybrid car that gets 51 miles per gallon. Your current car gets 24 miles per gallon. Both cars have 10 gallon tanks. With gas at \$3.90 per gallon, how much money will you save on gas in the first 10,000 miles?

a. **Understand:** What information do you want to find? What information do you have? What information do you need?

b. **Plan:** How will you solve the problem?

c. **Attack:** Solve it! Remember to answer the question.

d. **Check:** Check your answer.

Using

Un P A C

To help **plan**, look for math keywords. *Angle* often means you need geometry. *Total* often means you need to add. *Per* often means to divide.

Understand
Plan
Attack
Check

5. Andy is planning a movie night with his friends. There will be 10 people total, Andy plus nine guests. He's serving mini pizzas, and he wants three pizzas per person, with three pepperonis on each pizza. He's bought three 20-slice pepperoni packages. Does Andy have enough pepperoni? What advice would you give Andy?

a. **Understand:** What information do you want to find? What information do you have? What information do you need?

b. **Plan:** How will you solve the problem?

c. **Attack:** Solve it! Remember to answer the question.

d. **Check:** Check your answer.



6. Ana is going to a craft fair where she'll sell her handmade scarfs. She has six scarf designs, and she's made 15 of each. She's paid \$128 for the booth, and each scarf costs \$9.80 to make. How much should she charge for each scarf if she wants to break even after selling a third of her scarves?

a. Use **UnPAC** to solve this problem. Explain your thinking in creating your plan.

b. What alternative plan could you use?

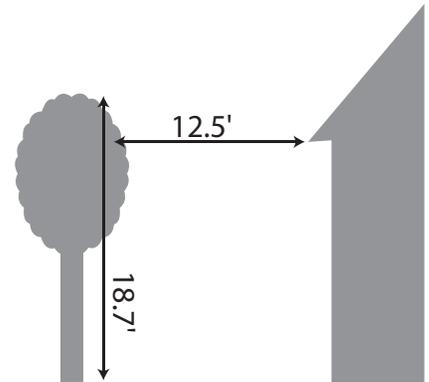


Check Your Skills

UnPAC these problems to answer the following questions.

1. A car rental agency charges \$25 per day for an economy car and \$30 per day for a mid-sized car. There is a weekly rental discount of 10% for a seven-day rental. How much will you save on a week's rental if you choose an economy car instead of a mid-sized car?
- \$22.75
 - \$31.50
 - \$33.20
 - \$35.00

2. You need to cut down a diseased tree 12.5 feet away from your house. If the tree is 18.7 feet tall, how many feet should be trimmed off the top of the tree to give it a two foot clearance when it falls, to make certain it doesn't hit your house?
- 4.8
 - 5.4
 - 6.9
 - 8.2



3. You have a choice of two moving companies to help you move into a new house. You estimate that you'll need two movers for two hours to load the truck, and the same time to unload the truck. You'll also need the movers to drive the truck 10 miles to the new house. How much more expensive will Manny's Reliable Movers be than BB's Moving Guys?

Service	Manny's Reliable Movers	BB's Moving Guys
Two Movers	\$79 per hour	\$61 per hour
Driving	\$1.98 per mile	\$4.95 per mile

- \$42.30
- \$57.25
- \$64.80
- \$85.93

4. You're planning on painting one of your living room walls dark red. The wall is 9.1 feet by 12.5 feet. You can buy one gallon of paint, which will cover approximately 350 square feet, for \$29.95, and you can buy paint in quarter-gallon buckets for \$12.50 each. What is the least you can pay to get enough paint to cover your wall? *Area of a rectangle = length \times width*
- a. \$12.50
 - b. \$25.00
 - c. \$29.95
 - d. \$42.45
5. Dwayne is trying to build up credit. He's decided to get a secured credit card, but he needs a \$450 security deposit. If he saves \$15 a week, how long will it take him to save up the money for the deposit?
- a. 15 weeks
 - b. 20 weeks
 - c. 25 weeks
 - d. 30 weeks
6. Your mp3 player has 319 megabytes of free space. If an average song is about two megabytes, approximately how many songs could you fit on your mp3 player?
- a. 159
 - b. 173
 - c. 191
 - d. 201
7. Out of 25 students in your son's second grade class, 15 are in soccer, and 12 are in karate. If there are six students in both soccer and karate, how many students are not in either activity?

It can be challenging to think through a plan to solve a complex problem. Go step by step.

*Remember
the Concept*

Understand—Find what you know and need to know.

Plan—Make a plan.

Attack—Solve it!

Check—Does the answer make sense?

Understand

Plan

Attack

Check

Concrete and Abstract Problem Solving

Connections

Have you ever ...

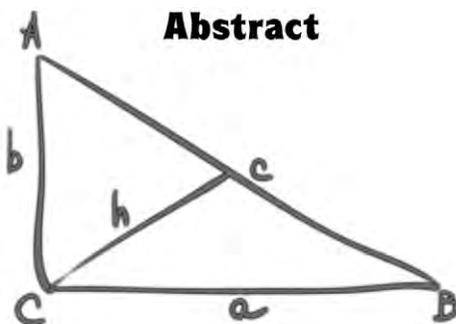
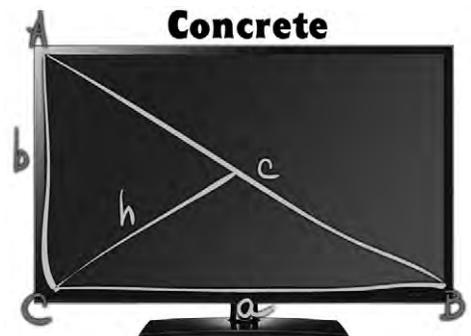
- Been stumped by what kind of math to use to solve a word problem?
- Found an equation confusing because you didn't know what it meant?

Math problems come in two types: concrete and abstract.

Concrete problems are down-to-earth examples. Any time you use math to balance your checkbook or figure out how much dog food to buy, you're doing concrete math.

Abstract problems aren't tied to examples. They make you think mathematically and logically about numbers. You'll need to master both concrete and abstract problem solving.

We all face **concrete** math problems in our day-to-day lives, whether it's putting together a shopping budget or calculating a bill for a client. In the math classroom, concrete problems are usually word problems or investigations of real-world problems. Concrete math problems can be easier because the numbers relate to familiar objects and events. The difficulty is usually figuring out what kind of math to use.



Abstract math problems can be more difficult. When you see $5x + 2 = 15$, those numbers don't relate to anything real that you can grasp. They're just numbers. Nonetheless, you can think about those numbers and understand ideas about numbers in general. When you solve an abstract problem, you show that you really know the foundations of math. To make it easier, you can relate the problem to real life.

Learn
It!

Solving a Concrete Problem

The biggest challenge in a concrete problem is understanding how the real-world problem relates to math.

Your water bill went up significantly last month. You used about 300 gallons more water than the same month last year. Looking around the house, you find that one of the bathroom sinks is dripping and won't stop. It doesn't seem to be dripping that much, though. Could the dripping sink be the cause of the high water bill?

Un Understand

Your first job is to understand the problem. Look for numbers you know and unknown values. Then, put it into **math terms**. What relationships are there between values? What math operations describe those relationships. Look for keywords, such as *per*.

- ? 1. Put the question into math terms.



For this question, you need to know whether the dripping sink could have caused the high water bill. In **math terms**, could the amount of water dripping away be as much as 300 gallons per month?

Is dripping water $>$, $<$ or $=$ 300 gallons/month?

You know that you used 300 gallons more water than expected. You need to know how much water is dripping from the faucet (the unknown value).

P Plan

Plan how to solve the problem, considering math tools you know. Which ones will be useful?

- ? 2. Make a plan to decide whether the dripping water is greater than, less than, or equal to 300 gallons per month.

Here's an example of a possible plan:

- Measure the amount of water that drips from the sink in an hour.
- If 300 gallons drips in 30 days, how much drips in one hour? You might use ratios.
- Compare the two amounts.

A Attack

Solve the problem. Now that you've thought through a plan, you'll need to carry out the math. Part of the plan involves measuring. Imagine that you put a measuring cup under the sink and checked it periodically. In one hour, you gathered a quart of water.

- ? 3. Calculate whether the dripping sink could be responsible for the extra water usage.

You might attack the example plan like this:

Step 1: The sink is dripping one quart of water per hour (a quarter gallon per hour).

Step 2: How much per hour is 300 gallons per 30 days? You can write 300 gallons per 30 days as a fraction and convert it to gallons per hour. The word *per* means division, which is the same as a fraction.

$$\frac{300 \text{ gallons}}{30 \text{ days}} = \frac{10 \text{ gallons}}{1 \text{ day}} = \frac{10 \text{ gallons}}{24 \text{ hours}} = \frac{5}{12} \text{ gallons per hour}$$

Step 3: Compare the two amounts.

$$\frac{5}{12} > \frac{1}{4}$$

The drip is only using a quarter gallon of water an hour. Five-twelfths is $\frac{1}{6}$ greater than $\frac{1}{4}$. That means a sixth of a gallon of extra water usage per hour is unexplained.

C Check

Don't forget to check your answer. Does it make sense? What's your next step?

- ? 4. Check your answer.

If the sink is dripping a quarter gallon an hour, that means in a day, it will drip six gallons. Over 30 days, that's 180 gallons. The result is the same. A lot of extra water usage remains unexplained. In terms of gallons, you're still 120 gallons over what you expected.

Review & Practice

Learn about ratios on pages 79–90. Get practice with fractions on pages 305–312 and with ratios on pages 325–326.

Practice
It!

Answer the following concrete math problems.

- ★★ 1. You have \$41.50 left in your day's vacation budget, and you want to rent rollerblades to skate around Venice Beach. If the rollerblades cost \$9.98 plus \$7.50 per hour, how many hours' rental can you afford?

a. Solve the problem. b. What math skills or equations did you use to solve it?

- ★★ 2. Karen is meeting Karla for lunch in half an hour. The restaurant is 3.1 miles away, and she'll average 28 miles per hour on the trip. How soon does Karen need to leave?

a. Solve the problem. b. What math skills or equations did you use to solve it?

- ★★ 3. You need to order supplies for a conference of 250 attendees. The budget is \$3,000, and you can choose from the following supplies your vendor offers.

Supplies	Price
Pens	\$4.50 each
Pencils	\$2.25 each
Notebooks	\$6.00 each
Coffee and Tea Bags (Assorted)	\$1.25 per serving
Name Badges	\$2.00 each
Folders	\$1.00 each
Bottled Water	\$1.00 each

How much can you spend per attendee? Propose a list of supplies per attendee that's within your budget.

a. Solve the problem. b. What math skills or equations did you use to solve it?

- ★★ 4. In a science experiment, a student finds that 81.746% of moths in the local area have primarily white wings. If the student observed 126 moths, how many had primarily white wings?



a. Solve the problem. b. What math skills or equations did you use to solve it?



Solving an Abstract Problem

An abstract problem tests your mathematical reasoning because it contains only numbers and symbols. By relating it to real-world problems, you can understand it better. Here is an example of an abstract problem:

$$\frac{70}{2} = \frac{x}{10}$$

Un Understand

To understand an abstract math problem, try to relate it to what you know.

- Do you recognize the structure? Is it a type of problem you've seen?
- Can you understand what it means and relate it to real-world problems?



1. What's a real-world problem this equation might represent?

You might recognize this as a ratio. This equation could describe 70 **some things** per two **something else**. For example, you might travel 70 miles in two hours. How many miles would you travel in 10 hours? If you have 70 widgets for every two thingamabobs, then how many widgets would you have for 10 thingamabobs?

P Plan

Think through how to solve this problem and make a plan.



2. Make a plan to solve this problem.

You could think about this in a couple of ways. You might know that it would help to cross-multiply, a common way to solve ratios. You might also know that you want to get x on its own. In both cases, you'd use a similar strategy.

- Multiply 70 times 10.
- Divide the result by two.

You could also do some simplification to make the problem easier.

- Simplify the fraction $\frac{70}{2}$.
- Multiply the result by 10.

Using

Un P A C

Try thinking of more than one **plan** to solve the problem. You can choose the easiest, or try both to **check** your result.

Un derstand
P lan
A ttack
C heck

A Attack

Solve the problem, using the plan you think is best. Especially in an abstract problem, work carefully, step by step.

- ? 3. Solve the problem to find the value of x .

Simplifying the fraction seems like it might make the problem easier, so you might attack the problem using that strategy.

Step 1

$$\frac{70}{2} = \frac{35}{1} = 35$$

Step 2

$$\begin{aligned} 35 &= \frac{x}{10} \\ 35 \times 10 &= \frac{x}{10} \times 10 \\ 350 &= x \end{aligned}$$

Review & Practice

Learn about ratios on pages 79–90, and get practice on pages 325–326.

C Check

Does your answer make logical sense? One way to check is to plug the answer into the original problem.

- ? 4. Check the answer.

If you change x to 350 in the original problem, it looks like this:

$$\frac{70}{2} = \frac{350}{10}$$

If you think about what the problem means, it's clear that the answer is correct. Since 10 is five times as many as two, you'd expect the answer to be five times as many as 70. That's 350. If you have 70 **something** for every two **something else**, then you'd have five times as many **some things** for every 10 **something else**.

Practice
It!

Solve the following abstract math problems. As you work through these problems, think of concrete examples to help you understand what each problem means, and use your knowledge of math to identify the type of problem. Consider more than one way to find the solution.



1. $52.5 + (21 \times 3.1) = x$

- Understand:** What type of problem is this? What would it mean in the real world? What is an example of a real-world problem this could represent?
- Plan:** How will you solve the problem?
- Attack:** Solve it!
- Check:** Check your answer.



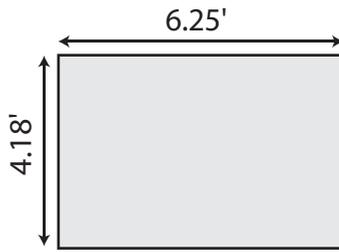
2. $\frac{4}{15} \times 2 = x$

- Understand:** What type of problem is this? What would it mean in the real world? What is an example of a real-world problem this could represent?
- Plan:** How will you solve the problem?
- Attack:** Solve it!
- Check:** Check your answer.

*Review &
Practice*

Get more review and practice with multiplying fractions on pages 305–308.

- ★★ 3. What is the area of this rectangle? *Area of a rectangle = base \times height ($A=bh$).*



- a. **Understand:** What type of problem is this? What would it mean in the real world? What is an example of a real-world problem this could represent?
- b. **Plan:** How will you solve the problem?
- c. **Attack:** Solve it!
- d. **Check:** Check your answer.

- ★★ 4. $5x + 3 = 13$

- a. **Understand:** What type of problem is this? What would it mean in the real world? What is an example of a real-world problem this could represent?
- b. **Plan:** How will you solve the problem?
- c. **Attack:** Solve it!
- d. **Check:** Check your answer.



Check Your Skills

Use your concrete and abstract problem-solving abilities to answer the following questions.

1. Zane took all the change in his change jar and put it in rolls to bring to the bank. He ended up with one \$10 roll of quarters, one \$2 roll of nickels, one \$5 roll of dimes, and 32 extra coins. How many coins did he have?
- 122
 - 142
 - 162
 - 182



2. Find the value of x :

$$10 \times \frac{1}{4} = x$$

3. Cheryl takes on a part-time job as a parts inspector at a factory. She has to write reports about her inspections. Cheryl inspects 357 parts and finds that 18 are flawed. Approximately what percentage of parts are flawed?
- 2.4%
 - 4.8%
 - 5.0%
 - 7.1%

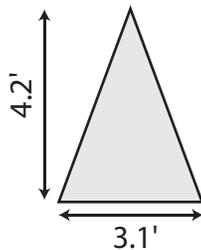
4. Find the value of x :

$$\frac{52}{13} = \frac{48}{x}$$

Review & Practice

Get some practice with worksheets for percents and ratios and proportions on pages 323–326.

5. You are planning a summer trip to the Grand Canyon. The trip is 824 miles each way, and gas will be approximately \$4 per gallon. Your car will get approximately 38 miles per gallon. How much should you budget for gas?
- \$135
 - \$149
 - \$162
 - \$175
6. What is the area of this triangle? *Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height}$ ($A = \frac{1}{2}bh$)*



7. Dennis wrote a science fiction novel. It's 876 pages long! He's trying to calculate the word count so that he can submit it to a publisher, and he estimates there are approximately 250 words per page. What is Dennis's estimated word count?
- 219,000
 - 242,000
 - 268,000
 - 291,000

*Remember
the Concept*

Concrete Problem

Understand the abstract math!

Abstract Problem

Relate it to concrete examples!

Estimation and Mental Math

Connections

Have you ever ...

- Quickly added up the prices of the clothes you wanted to buy to see if you had enough money?
- Questioned a bill because the total didn't sound right?
- Made an educated guess about how much money you had left for the rest of the weekend?

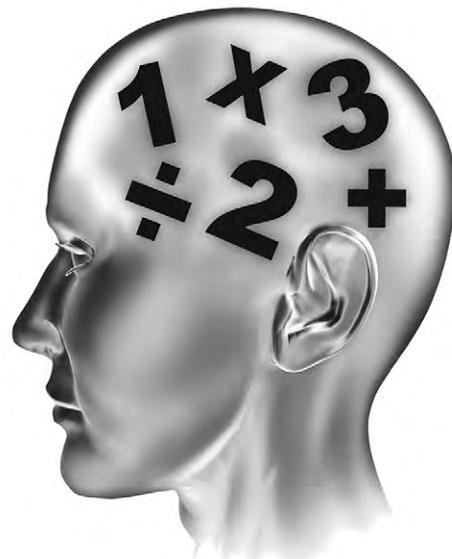
We do math all the time. If you've ever guessed an approximate solution or done a quick calculation in your head, you've used important math skills: **estimation** and **mental math**.

When you **estimate**, you use rounding and approximation to make a close guess about the solution to a math problem. You might estimate when you're solving a math problem with pen and paper or even a calculator, but it's most common to estimate when you make rough calculations in your head.

When you do **mental math**, you do math in your head. Often, you'll use estimation in mental math, but you can also do exact calculations. The key is practice. The better you know basic math and can manipulate numbers, the easier and more accurately you can do mental math.

Estimating and mental math work well together.

- Round and approximate to make the math easier.
- Figure out the solution quickly in your head.
- End up with a good guess that you can use.



Learn
It!

Using Estimation and Mental Math

Estimation helps you find an approximate answer quickly. Mental math helps you make calculations easily in your head.

Un Understand

Understanding the problem is always the first step.

Miguel and Clara started a charity to raise money for mosquito netting to prevent malaria in Africa. They've raised \$2,379, and every \$11 can buy a mosquito net. They've had \$622 in expenses. Make a quick estimation of approximately how many mosquito nets they can buy.



- ?
1. What are you trying to find? What do you know, and what do you need to know?

You want to know how many nets Miguel and Clara can buy. To figure that out, you need to know how much money they have and how much a mosquito net costs. You know:

How much money they have	How much mosquito nets cost
raised \$2379	\$11 each
expenses \$622	

P Plan

To create a plan, think about how you can use estimation and mental math. What numbers can you approximate to make the problem easier? How can you do the math quickly and easily in your head?

- ?
2. Make a plan to use estimation and mental math to solve the problems.

Here's an example of a possible plan:

- Round off the amount raised and the expenses to the hundreds, and subtract expenses from the amount raised.
- Round the cost to \$10, and divide the remaining money by \$10.

A Attack

Attack the problem!

- Go one step at a time to keep the numbers straight in your head.
- If you can, round less to make your result more accurate.
- As you practice, try more complex mental math with less (or no) estimation.

? 3. Solve the problem according to your plan.

You might attack the example plan like this:

Step 1

Round \$2,379 to \$2,400. Round \$622 to \$600. In your head, subtract.

$$24 - 6 = 18, \text{ so } 2400 - 600 = 1800$$

Step 2

In your head, divide. To divide quickly by 10, remove one zero from the numerator.

$$1800 \div 10 = 180$$

Miguel and Clara can buy approximately 180 mosquito nets.

C Check

Check your answer. Does it make sense? What's your next step?

? 4. Check your answer.

Try doing the problem backwards to check the answer.

- The approximate answer is 180 mosquito nets.
- If mosquito nets cost about \$10, that's \$1,800.
- \$1,800 plus the expenses rounded to \$600 is approximately \$2,400.
- \$2,400 is approximately how much they raised, so the answer is a good estimate.

Without rounding, the exact answer is 159, so the estimate is off by about 20.

Math Tip

When rounding, pay attention to how it changes your answer. How rounding affects your answer will depend on three things:

- Did you round up or down?
- How much did you round?
- What operations did you do with the numbers?

If you round one number up and another down by a similar amount and then add, you will get a close estimate. If you round two numbers up and multiply, the estimate might be far from the exact answer.

Practice
It!

Use estimation and mental math to solve the following problems.



1. Barbara has taken a part-time job in a restaurant, and one of her jobs is ordering supplies. The restaurant has an average of 112 customers per night, and half of them order chicken. Approximately how many portions of chicken should Barbara order for the week?

- Solve the problem using estimation and mental math.
- What estimation did you use? Why?
- What is the difference between your estimated solution and the exact solution?



2. A farmer plants 12 acres of corn. Each acre will yield approximately 180 bushels of corn, and the farmer can sell corn for \$5.08 per bushel. Approximately how much income can the farmer expect from his corn?

- Solve the problem using estimation and mental math.
- What estimation did you use? Why?
- What is the difference between your estimated solution and the exact solution?



- ★★ 3. Ethan is moving to a new apartment. With an online moving calculator, he figured out that his furniture will take up approximately 650 cubic feet on the truck. He also has 25 small one-cubic-foot boxes, 20 medium three-cubic-foot boxes, and 10 large 4.5-cubic-foot boxes. If he wants the smallest suitable truck, which truck should he get?

Truck Size	Capacity
14 feet	733 cubic feet
17 feet	865 cubic feet
20 feet	1,015 cubic feet

- a. Solve the problem using estimation and mental math.
- b. What estimation did you use? Why?
- c. What is the exact amount of cubic feet of space Ethan needs?

- ★★ 4. Isabella wants to take a vacation to New York. She needs 42,000 airline points to get a free flight, and she already has 39,420. She's signed up for a credit card to earn more points. She gets 1,500 points for signing up, and she'll earn one point for every dollar she spends. How much does she need to spend to earn the points for her vacation?

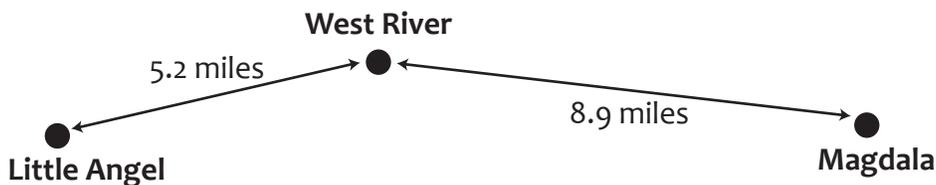
- a. Solve the problem using estimation and mental math.
- b. What estimation did you use? Why?
- c. What is the difference between your estimated solution and the exact solution?

Math Tip

Practice basic math facts to improve your mental math skills. Knowing math facts well will make all the math you do quicker and easier!

5. Madison is making LEGO® artwork for her son’s room. She wants the artwork to be 18 inches by 24 inches (432 square inches). A single brick is approximately 0.098 square inches. How many single bricks does Madison need?
- Solve the problem using estimation and mental math.
 - What estimation did you use? Why?
 - What is the difference between your estimated solution and the exact solution?

6. The distance from Little Angel to West River is 5.2 miles, and the distance from West River to Magdala is 8.9 miles. What is the total distance from Little Angel to Magdala through West River?



- Solve the problem using estimation and mental math.
 - What estimation did you use? Why?
 - What is the difference between your estimated solution and the exact solution?
7. How does the amount and direction of your estimate affect your solution? Why?



Check Your Skills

Use estimation to choose the best multiple choice answer.

Use estimation and mental math to solve the following problems. You can use estimation to choose the best multiple choice answer.

1. Liam and Roberto are discussing how many people attended a concert in a local park. The 30-foot by 30-foot area for the audience was packed with a standing crowd. If a single person takes up approximately four square feet, approximately how many people were there?
 - a. 125
 - b. 225
 - c. 325
 - d. 425

2. Mason’s favorite shop is having a sale. He wants to buy a video game that normally costs \$59.95 and is on sale for 35% off. How much is the sale price of the game?
 - a. \$20.29
 - b. \$31.18
 - c. \$38.97
 - d. \$46.11

3. Olivia is training for a marathon, 26.2 miles. She runs five miles every morning, and her average time is 50 minutes. If she keeps the same pace on the marathon, how long will it take her to finish?
 - a. 3 hours, 18 minutes
 - b. 4 hours, 22 minutes
 - c. 5 hours, 44 minutes
 - d. 6 hours, 28 minutes

4. If Latasha burns 105 calories for every mile she runs, how many calories will she burn during a 26.2 mile marathon?
 - a. 2,005
 - b. 2,238
 - c. 2,480
 - d. 2,751

Math Tip

The closer your estimate is to the original number, the more accurate your answer will be.

5. What is the volume of a cube where each side is six feet long?
Volume = area of the base \times height
- 128 cubic feet
 - 154 cubic feet
 - 182 cubic feet
 - 216 cubic feet
6. Brianna and Alejandro plan to have lunch and see a movie. They decide that Brianna will pay for lunch, and Alejandro will pay for the movie. Lunch costs \$23.50, and each matinee movie ticket costs \$7.25 each. How much more will Brianna pay than Alejandro?
- \$8.00
 - \$9.00
 - \$10.00
 - \$11.00
7. Nathan reads the elevator safety sign in his building's elevator. It says maximum capacity 2,500 pounds and 12 people. Nathan starts to worry that 12 very heavy people will get in the elevator and break it. What would the average weight per person be for 12 people to weigh a total of 2,500 pounds?
- $208\frac{1}{3}$
 - $220\frac{1}{2}$
 - $232\frac{1}{3}$
 - $241\frac{1}{2}$
8. Alaska's Mt. McKinley is the highest point in the U.S., and California's Death Valley is the lowest point. What is the variation in altitude from the highest point in the U.S. to the lowest point?

*Remember the
Concept*

**Estimation and
Mental Math**

Round numbers to make it easy.

Practice to improve your mental math.

State	Highest Point	Highest Altitude	Lowest Point	Lowest Altitude
Alaska	Mt. McKinley	20,320	Pacific Ocean	0
California	Mt. Whitney	14,494	Death Valley	-282

Take on the challenge to do the exact math in your head.