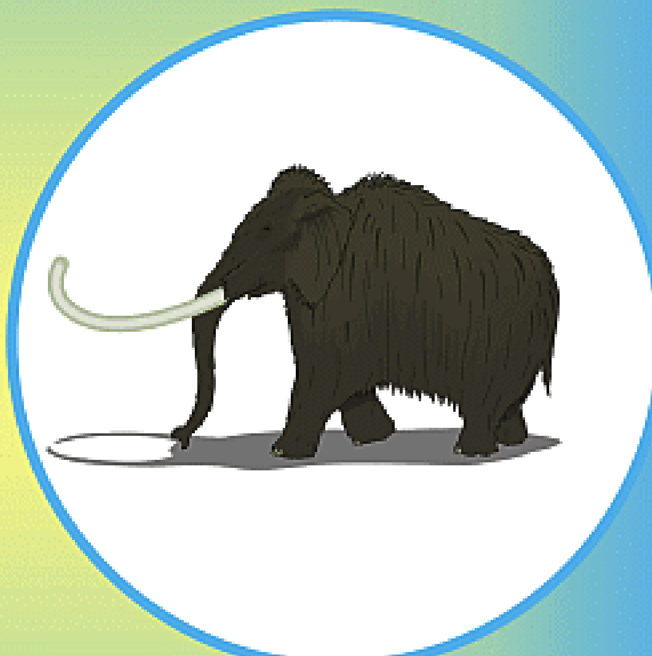
MATH MAMMOTH

Grade 4-B

Complete Worktext

- Division, Long Division and Problem Solving
- Geometry
- Fractions
- Decimals



By Maria Miller

www.MathMammoth.com

Chapter 5: Division
Introduction

The fifth chapter of *Math Mammoth Grade 4-A Complete Worktext* includes lessons on division, long division, remainder, part problems, average, and problem solving. It is a long chapter, as division and long division are "in focus" in fourth grade.

We start out reviewing basic division by single-digit numbers. Then students study some basic division topics such as division terms, division by 1 and 0, and dividing by whole tens and hundreds.

The lesson Finding Parts with Division is very important. It shows an important relationship between fractions and division. For example, we can find 3/4 of a number by first finding 1/4 (divide by 4), then multiplying that result by 3.

The lesson on remainder is just before the first lesson on long division, because that is where the student needs to understand this concept very well.

Long division is taught in several small steps over many lessons. We start with the situation where each of the thousands, hundreds, tens, and ones can be divided evenly by the divisor. Then is introduced remainder in the ones. Next comes the situation where we have a remainder in the tens. Finally, when we have a remainder in the hundreds, and so on.

All along the long division lessons the process is explained so that the student can understand what it is based on. After the many lessons that practice and explain long division, we see a comparison between repeated subtraction and long division. The purpose of this lesson is to shed light in the basic idea of long division, and not to practice a new calculation method.

After long division is mastered, we study the concept of average. Next comes *Long Division with Remainder*, which also contains a section on packing problems

The following two lessons contain plenty of part-related problems to solve. These problems deal with fractional parts of a total, and include both dividing and multipying. I have included many diagrams and pictorial representations of these problems to help the student. Encourage him to draw a picture for those problems that don't have any.

The last two topics in this section are divisibility and a two-digit divisor in long division. These topics are introductory only, and we continue them in the fifth grade.

The Lessons in Chapter 5

Review of Division

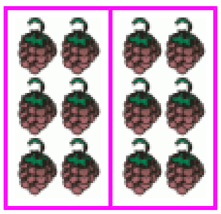
Multiplication has to do with equal-size groups.

Division *also* has to do with equal-size groups.
Division is the opposite operation of multiplication.

Division problems "start" with the total, whereas multiplication problems "end" with the total.

Division has two "meanings":

- dividing into so many groups: "All 50 kids were divided into 10 tables."
- dividing into certain-size groups. "The 30 students were divided into groups of 3."



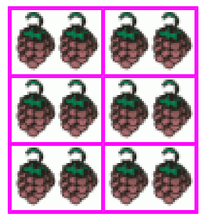
$2 \times 6 = 12$

"12 divided into 2 groups; how many in each group?"

$12 \div 2 = 6$

"How many sixes are in 12?

$12 \div 6 = 2$



$6 \times 2 = 12$

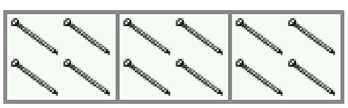
"12 divided into 6 groups; how many in each group?"

$12 \div 6 = 2$

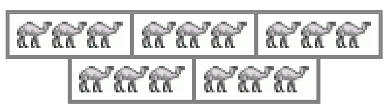
"How many twos are in 12?

$12 \div 2 = 6$

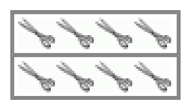
1. Write a multiplication sentence and two division sentences.



a.



b.



c.

2. Fact families: write two division and two multiplication sentences.

a. 21 7 and 3	**b. 24** 4 and ___	**c. 36** 4 and ___	**d. 55** 5 and ___

3. Practice a little. Continue the patterns.

a. 16 ÷ 2 = 18 ÷ 2 = 20 ÷ 2 =	**b.** 45 ÷ 5 = 40 ÷ 5 = 35 ÷ 5 =	**c.** 90 ÷ 10 = 100 ÷ 10 = 110 ÷ 10 =	**d.** 56 ÷ 7 = 49 ÷ 7 = 42 ÷ 7 =

4. Fill in the tables.

Eggs	6	12	24	36		54		78
Omelets	1				7		11	

Thumbtacks	8	24	32	48				
Pictures	1				8	10	12	13

5. Write a number sentence for each situation. (It's not always division!)
Tell what you can find out from your calculation.

a. Three children shared equally 18 marbles. 18 ÷ 3 = 6. Each kid got 6 marbles.	**b.** Jim has $34 and he wants a $45 book .
c. A fruit store received a shipment of 400 apples in four boxes.	**d.** Mrs. Davis shared 24 chocolate pieces equally between 6 persons.
e. Five boxes arrived at the bookstore, each containing 50 books.	**f.** Mom bought two books that cost $13 each.
g. A herd of cows had a total of 20 legs.	**h.** 60 books were placed on three shelves.

6. Divide.

a. 36 ÷ 4 = 50 ÷ 5 = 60 ÷ 12 =	**b.** 54 ÷ 9 = 42 ÷ 7 = 48 ÷ 6 =	**c.** 32 ÷ 8 = 64 ÷ 8 = 72 ÷ 9 =	**d.** 24 ÷ 3 = 27 ÷ 9 = 35 ÷ 7 =

7. Find what the number x stands for.

a. 64 ÷ x = 8
x = 8

b. 35 ÷ x = 7

c. 45 ÷ x = 9

d. 54 ÷ x = 6

8. For each division, write one multiplication sentence. Find the value of the unknown.

a. N ÷ 3 = 10

b. x ÷ 4 = 9

c. 60 ÷ T = 20

d. 81 ÷ y = 9

9. **a.** How many fives are in 45? In 60?

b. How many times can you subtract two from twenty? From forty?

c. How many $3-bargain books can you buy with $21? With $36? With $42?

10. Write a number sentence for each situation. (It's not always division!)
Tell what you can find out from your calculation.

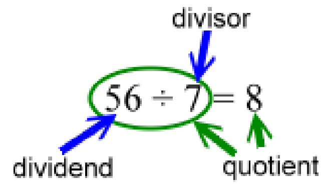
a. A pie that costs $12 is divided to 6 pieces.	**b.** 100 apples were boxed into 5 boxes.
c. Five boxes of nails cost $30.	**d.** A chocolate bar has 8 rows and 5 columns of squares.
e. A 48-inch piece of metal is cut into two equal pieces.	**f.** Each of the five boxes weighs 12 pounds.

Division Terms, Zero and One

Division terminology

Both the expression 56 ÷ 7 and its answer are called "the quotient"!

You can call "56 ÷ 7" the quotient written, and the 8 as the quotient solved.



1. What is missing from these divisions; the dividend, the divisor, or the quotient? Also fill in the missing number.

a. 80 ÷ ____ = 40 ____________________ is missing.

b. ____÷ 7 = 5 ____________________ is missing.

c. 120 ÷ 10 = ____ ____________________ is missing.

2. Write the division problem. Then solve for x.

a. The divisor is 7, the dividend is x, and the quotient is 3. __ ÷ __ = __. x = __

b. The dividend is 140, the divisor is x, and the quotient is 7. __ ÷ __ = __. x = __

c. The quotient is x, the divisor is 5, and the dividend is 150. __ ÷ __ = __. x = __

3. Write:

a. three division problems with a quotient of 6;

b. three division problems with a dividend of 24;

c. three division problems with a divisor of 3.

4. Fill in the tables.

Numbers	Product (written)	Product (solved)	Quotient (written)	Quotient (solved)
12 and 3				
10 and 5				
20 and 4				
100 and 10				

5. Estimate each division result by rounding.

a. Round the divisor:	**b.** Round the dividend:	**c.** Round both:
80 ÷ 21 =	46 ÷ 5 =	121 ÷ 19 =
90 ÷ 28 =	162 ÷ 40 =	159 ÷ 41 =
120 ÷ 59 =	98 ÷ 11 =	402 ÷ 98 =
200 ÷ 52 =	297 ÷ 30 =	203 ÷ 52 =

Division with zero

We check a division problem by multiplication.

Does 0 ÷ 3 = 0? Check if 0 × 3 = 0. Yes, it is.
Does 0 ÷ 11 = 0? Check if 0 × 11 = 0. Yes, it is.

Does 3 ÷ 0 = 0? Check if 0 × 0 = 3. It is **not**.

Does 3 ÷ 0 perhaps 3? Check if 0 × 3 = 3. It is **not**.

In fact, dividing by zero is a real problem. No matter what number you suggest as an answer to the problem 3 ÷ 0, the multiplication check won't work because you'll end up multiplying by zero, and can *never* get the dividend as an answer!

That is why division by zero is said to be *undeterminate* - you cannot determine the answer.

Remember though that you *can* divide zero by any number (except zero). The answer to that is always zero. For example, 0 ÷ 7 = 0. The check works: 0 × 7 = 0.

What about 0 ÷ 0?

We cannot really determine any single answer, because all of these could work:

If 0 ÷ 0 = 1, then check 0 × 1 = 0 works.
If 0 ÷ 0 = 7, then check 0 × 7 = 0 works.
If 0 ÷ 0 = 0, then check 0 × 0 = 0 works.

So 0 ÷ 0 is usually said to be *undeterminate* as well.

6. Divide. Mark off the problem if it is impossible to do.

a. 64 ÷ 8 =	**b.** 55 ÷ 5 =	**c.** 50 ÷ 10 =	**d.** 5 ÷ 1 =
48 ÷ 8 =	6 ÷ 0 =	0 ÷ 10 =	1 ÷ 1 =
32 ÷ 32 =	7 ÷ 7 =	0 ÷ 0 =	9 ÷ 0 =

7. Solve for x (if you can!)

a. 64 ÷ x = 1 **b.** 35 ÷ x = 35 **c.** 7 ÷ x = 0 **d.** x ÷ 18 = 1

8. **a.** Write two division problems with a quotient of 1.

b. Write two division problems with a dividend of 0.

Dividing Whole Hundreds and Thousands

Solving division problems always involves thinking backwards of multiplication, or "how many times".	
500 ÷ 5 = ? Think backwards of multiplication: Because 5 × 100 = 500, then 500 ÷ 5 = 100.	Or ask, "How many times does (the divisor) go into (the dividend)?" **7,000 ÷ 1000 = ?** How many times does 1,000 go into 7,000?

1. Practice a little.

a. 400 ÷ 4 = 400 ÷ 40 = 400 ÷ 400 =	**b.** 5,000 ÷ 5 = 5,000 ÷ 50 = 5,000 ÷ 500 =	**c.** 7,000 ÷ 7,000 = 7,000 ÷ 700 = 7,000 ÷ 70 =

2. Practice some more.

a. 320 ÷ 8 = 320 ÷ 80 = 3,200 ÷ 8 =	**b.** 540 ÷ 60 = 540 ÷ 6 = 5,400 ÷ 60 =	**c.** 360 ÷ 6 = 3,600 ÷ 60 = 36,000 ÷ 6 =

3. Write two division problems with a quotient of 200 and a divisor greater than 8.

4. Fact families: write two division and two multiplication sentences using the two given factors.

a. 1,800 60 and 30	**b. 25,000** 500 and ______	**c. 100,000** 10 and ______

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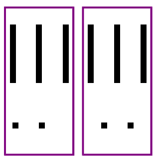
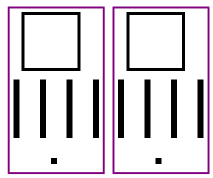
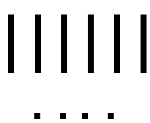
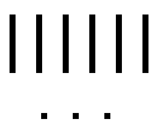
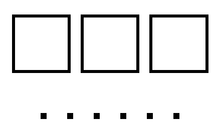
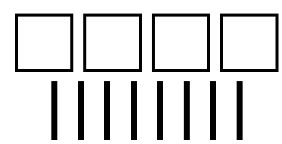
Long Division 1

<u>Divide hundreds, tens, and ones separately.</u> Write the dividend inside the long division "corner", and the quotient on top.	
<u>64 ÷ 2 = ?</u> Divide tens and ones separately: 6 tens ÷ 2 = 3 tens (t) 4 ones ÷ 2 = 2 ones (o)  t o 3 2 2) 6 4	**<u>282 ÷ 2 = ?</u>** 2 hundreds ÷ 2 = 1 hundred (h) 8 tens ÷ 2 = 4 tens (t) 2 ÷ 2 = 1. (o)  h t o 1 4 1 2) 2 8 2

1. Make groups. Divide. Write the dividend inside the "corner" if it is missing.

a. Make 2 groups	**b.** Make 3 groups	**c.** Make 3 groups	**d.** Make 4 groups
			
2) 6 2	3)	3)	4)

2. Divide thousands, hundreds, tens, and ones separately.

a. 4) 8 4 **b.** 3) 3 9 3 **c.** 3) 6 6 0 **d.** 4) 8 0 4 0

e. 3) 6 6 **f.** 6) 6 0 3 6 **g.** 3) 3 3 0 **h.** 4) 4 8 0 4

h t o 0 $4\overline{)\,248}$ h t o 0 6 2 $4\overline{)\,248}$ 4 does not go into 2. You can put zero in the quotient in the hundreds place or omit it. But 4 does go into 24, six times. Put 6 in the quotient.	th h t o 0 $5\overline{)\,3505}$ th h t o 0 7 0 1 $5\overline{)\,3505}$ 5 does not go into 3. You can put zero in the quotient. But 5 does go into 35, seven times.
Explanation: The 2 of 248 is of course 200 in reality. If you divided 200 by 4, the result would be less than 100, so that is why the quotient won't have any whole hundreds. But then you combine the 2 hundreds with the 4 tens. That makes 24 tens, and you CAN divide 24 tens by 4. The result 6 tens goes to the quotient. Check the final answer: 4 × 62 = 248.	**Explanation:** 3,000 ÷ 5 will not give any whole thousands to the quotient because the answer is less than 1,000. But 3 thousands and 5 hundreds make 35 hundreds together. You can divide 3,500 ÷ 5 = 700, and place 7 to the quotient in the hundreds place. Check the final answer: 5 × 701 = 3,505.

If the divisor does not "go into" the first digit of the dividend, look at the first two digits of the dividend.

3. Divide. Check your answer by multiplying the quotient and the divisor.

a. $3\overline{)\,123}$

b. $4\overline{)\,284}$

c. $6\overline{)\,360}$

d. $8\overline{)\,248}$

e. $2\overline{)\,184}$

f. $7\overline{)\,427}$

g. 0 6
$3\overline{)\,1833}$

h. $4\overline{)\,2404}$

i. $7\overline{)\,4970}$

j. $5\overline{)\,4505}$

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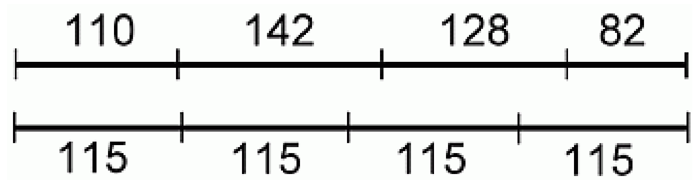
Average

The Millers went on a trip. The first day, they drove 110 miles, the second day, 142 miles, the third day, 128 miles, and the last day, 82 miles. The Millers drove a total of 460 miles.

In the diagram, we have put those distances as sticks one after another, though of course in reality they did not drive just straight stretches of roads.

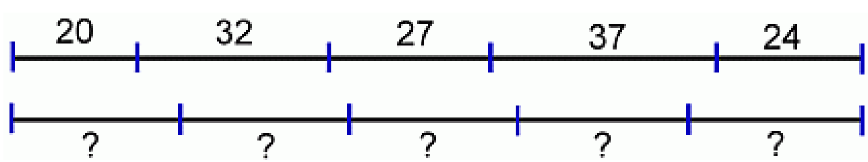


IF they had driven 115 miles each day, it would have totaled the same 460 miles.

On average, the Millers drove 115 miles a day, or their *average* daily mileage was 115 miles.

What is the average of 20, 32, 27, 37, and 24?

First find the total by adding. Then, divide that into equal parts.



20 + 32 + 27 + 37 + 24 = 140.   140 ÷ 5 = 28.

The average of 20, 32, 27, 37, and 24 is 28.

If these number were, for example, the ages of club members, we can say the average age of the members is 28 years. However, they could also be distances, or weights, or volumes, or just plain numbers.

1. Judith's test scores were 78, 87, 69, and 86.
   Find her average score.

2. John measured the temperature five times during a day.
   These are his measuring results:
   18°C, 22°C, 26°C, 23°C, and 16°C.
   Find the average temperature for the day.

3. Dad drove a 414 km stretch in six hours.
   How many kilometers did he drive,
   on the average, in one hour?

You can also use the average "backwards":

During a 20-hour drive from Denver to Dallas, Dad's average speed was 40 miles per hour. How far is Denver from Dallas?

You can multiply 20 hours × 40 miles/hour = 800 miles.

Note that in reality, he did not drive with a totally even speed all of the time because he had to stop at crossings, slow down on curves, stop for a snack and so on. We do not know how his speed varied on the trip. All we are given is that his *average* speed was 40 miles per hour. (And, of course the average speed was calculated by dividing the length of the trip by the total number of hours the trip took.)

4. The average pay of a translator is $42 per hour.
   How much would it cost to hire a translator for 11 hours?

5. The package of eggs says that an egg's average weight is 55 grams.
   How much would a dozen eggs weigh?

6. Mom's weekly grocery bills in June were $234, $178, $250, and $198.
   How much did Mom spend on groceries in June?
   What was her average weekly grocery bill?

7. For her hospital stay, mom was charged an average of $76 daily.
   What was the total cost of her one-week stay?

8. The kids ran a race. These are the resulting times:

Ann	12 min
Judy	15 min
Rose	14 min
Elizabeth	19 min
Grace	12 min
Nancy	18 min

Michael	12 min
Greg	10 min
James	11 min
Caleb	15 min
Hans	17 min

Find the girls' average running time and the boys' average running time separately.

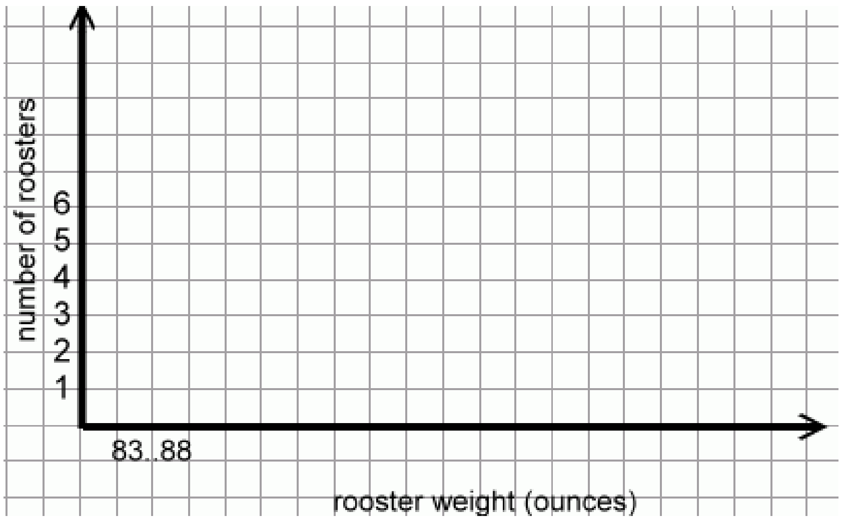
Are boys or girls quicker on the average?

What is the difference of the two averages?

9. Maria was studying how much 1-year old roosters usually weigh. She went to a farm and weighed 20 roosters. The numbers below are their weights, in *ounces*.

96, 94, 90, 101, 84, 102, 101, 95, 108, 113, 87, 95, 97, 84, 90, 99, 89, 93, 92, 100.

a. Make a bar graph of the data.

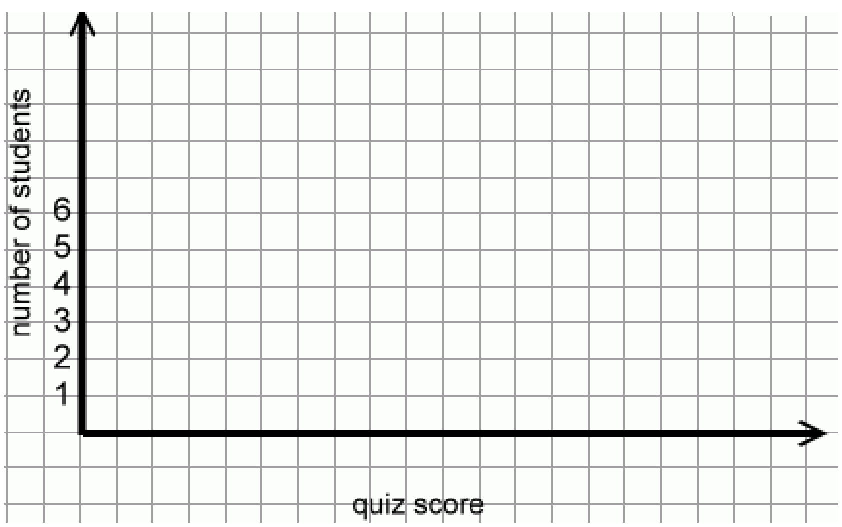


Weight (ounces)	Frequency
83..88	
89..94	
95..100	
101..106	
107..112	
113..118	

b. Maria calculated the average several times, and got different results from her calculator. She must have made errors in punching the buttons! Use the graph and the data to figure out which one is the *correct answer:* 89 ounces, 95 1/2 ounces, or 100 1/2 ounces?

10. Here you see ten students' quiz scores. 24 20 24 16 28 30 14 22 23 19

a. Make a frequency table and a bar graph.



Test score	Frequency
13..15	
16..18	
19..21	
22..24	
25..27	
28..30	

b. Calculate the average score.

c. Both the bar graph and the average tell us what the "middle" or "typical" result in the test was. Explain how you can guess what the average is approximately, just using the graph.

Remainder and Long Division

When using long division, the division is not always exact either.

At this point there are no more digits to drop down from the dividend. The last subtraction yields 6, which is the remainder.
So 125 ÷ 7 = 17, R6.

Note that the remainder 6 is LESS THAN 7, the divisor.

$$\begin{array}{r} 1\ 7 \\ 7\overline{)1\ 2\ 5} \\ -\ 7 \\ \hline 5\ 5 \\ -\ 4\ 9 \\ \hline \boxed{6} \end{array} \qquad \begin{array}{r} {}^{4} \\ 17 \\ \times\ \ 7 \\ \hline 119 \\ +\ \ \ \ 6 \\ \hline 125 \end{array}$$

To check, multiply the answer (17) by the divisor (7), and then add the remainder (6). You get the original dividend (125).

1. Divide. Check each result by multiplying and adding.

a. 514 ÷ 3

b. 673 ÷ 8

c. 1,905 ÷ 6

d. 8,205 ÷ 4

e. 7,805 ÷ 7

f. 2,575 ÷ 2

2. Find the divisions that are right or not. Fix those that are wrong.

a.
$$\begin{array}{r} 7\,7 \\ 6\,\overline{)\,4\,6\,3} \\ \underline{-\,4\,2} \\ 4\,9 \\ \underline{-\,4\,2} \\ 7 \end{array}$$

b.
$$\begin{array}{r} 3\,5\,3 \\ 7\,\overline{)\,2\,4\,7\,3} \\ \underline{-\,2\,1} \\ 3\,7 \\ \underline{-\,3\,5} \\ 2\,3 \\ \underline{-\,2\,1} \\ 2 \end{array}$$

c.
$$\begin{array}{r} 3\,5\,1 \\ 9\,\overline{)\,4\,0\,5\,9} \\ \underline{-\,3\,6} \\ 4\,5 \\ \underline{-\,4\,5} \\ 0\,9 \end{array}$$

d. How can you spot the error in (a) above just by looking at the remainder 463 ÷ 6 = **77 R 7** ?

3. Can you put 412 balls into bags evenly so that each bag has

a. 4 balls? **b.** 5 balls? **c.** 6 balls?

4. Write a division sentence for each problem, and solve it. What does the answer mean?

a. Arrange 112 chairs into rows of 9.	**b.** Arrange 800 erasers into piles of 3.

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Chapter 6: Geometry
Introduction

The sixth chapter of *Math Mammoth Grade 4-A Complete Worktext* is about geometry.

Students are now introduced to the concept of an angle, and learn about acute, right, obtuse, and straight angles. Students learn how to measure angles with a protractor, and estimate some common angles.

After angles, we study rectangles and parallelograms in more detail, and students learn to draw them, given either some side lengths or angle measures. The lesson on polygons concentrates on learning their names.

Next we study triangles, and classify them according to the angles. Classifying triangles according to their sides (equilateral vs. isosceles triangles) is left for the 5th grade. In the lesson about circles, we learn the terms circle, radius, and diameter. Students draw circles and circle designs using a compass.

The last topics in the geometry section are perimeter, area, and volume. These are topics that mostly involve calculations, and in many math books that is all you will find, but I have also included problems of drawing figures with a given area or perimeter.

The concepts of area and perimeter are very important, and sometimes students confuse them. For that reason, I included a lesson where we compare both concepts in detail.

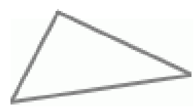
The study of geometry is full of strange-sounding words to learn. I encourage you to get the student(s) started with a *geometry notebook*, where they will write every new concept or term, and draw a picture or pictures and text to explain the term. This notebook will then be their own creation, and while working with it, the terms also will stick better in their memory. The students could also do the drawing exercises in this book, or just keep it as a terminology notebook, either way.

The Lessons in Chapter 6

Lines, Rays, and Angles

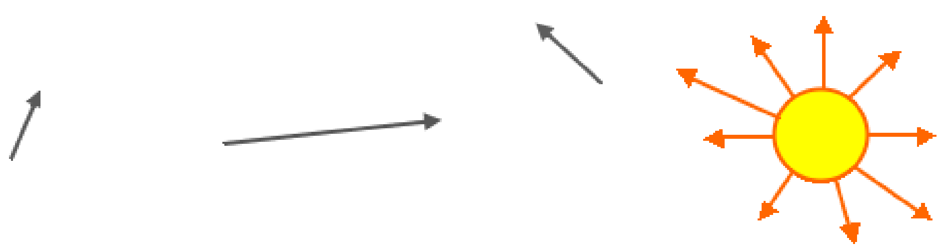
A line has no beginning point or end point. Imagine it continuing indefinitely in both directions. We can illustrate that by little arrows on both ends.

A line segment has a beginning point and an end point.

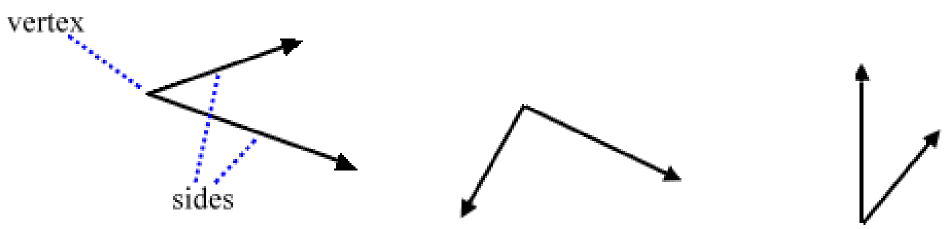


All the sides of this triangle are line segments.

A ray has a beginning point but no end point.
Think of sun's rays: they start at sun and go on forever...

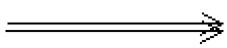


What is an angle? Many people think that an angle is some kind of slanted line, but in mathematics, **an angle** is made up from two rays that have the same beginning point. That point is called the vertex and the two rays are called the sides of the angle.

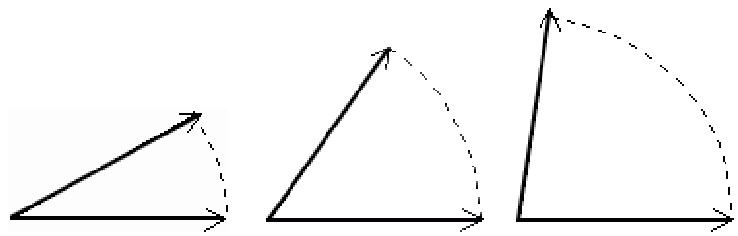


Imagine that the two sides of the angle started side by side, and then opened up to a certain point. When the two sides "open up", they draw an imaginary arc of a circle (see below).

Illustrate this with two pencils as the two sides of an angle. Turn the one pencil while keeping the other stationary, and imagine a circle is drawn in the air while you rotate the other pencil.

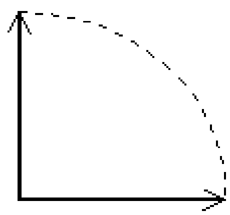


This angle is called the **zero angle**.

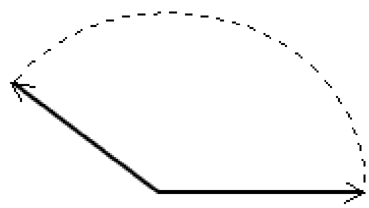


In each picture the angle is opened more and more and keeps getting bigger. The arc of the circle is larger.

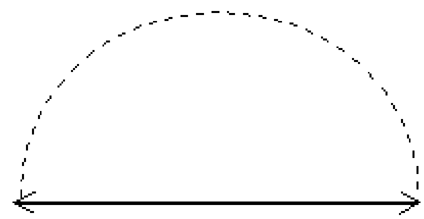
These angles are **acute angles**, which means they are less than a right angle. Think of the acute angles as *sharp* angles. If someone stabbed you with the vertex of an acute angle, it would feel sharp.



This is **the right angle.** For example, table corners are right angles.

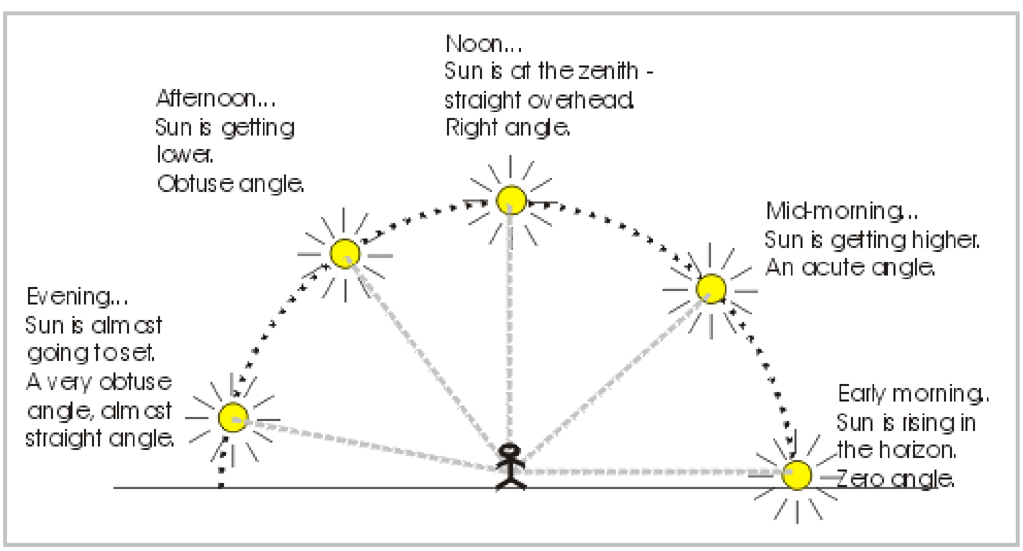


The angle is opened even more and is bigger than the right angle. It is an **obtuse angle.** Obtuse angles are *dull* angles.



This angle is called **the straight angle**. It is as if the pencils are lying down flat or *straight* on the floor.

You can also think of a sun rising in the morning in the horizon, and gradually getting higher, and traveling through the sky along an arc of a circle.

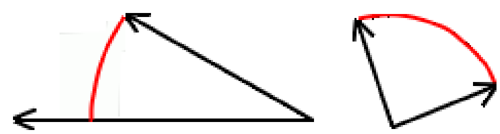


<u>How big is the angle?</u>

It does not matter how long the sides of the angle are. Remember, they are rays, and rays don't have an endpoint. When we draw them on paper, we have to make them end somewhere.

The sides of the angle might even seem to have different lengths. That doesn't matter either. The size of the angle is ONLY determined by how much it has "opened". Think how big an arc of a circle the sides have drawn, as compared to a whole circle.

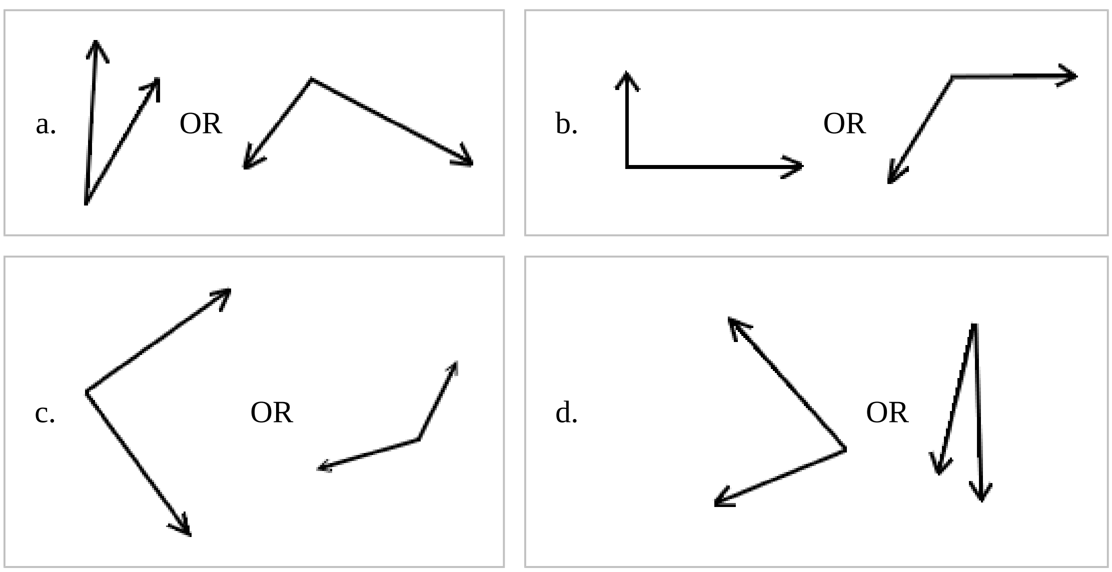
Which of these two angles is bigger? Remember to look at how much the angle has opened, or how much of a part of a circle the sides have drawn.



Many times the arrows are omitted from the rays, and the arc of the circle is drawn as a very tiny arc near the vertex.

Even the little arc is not necessary.

1. Which angle is bigger, the first or second picture?



2. Draw three different acute angles, three different obtuse angles, a right angle, and a straight angle. Then also draw the imaginary arc of a circle in your angles.

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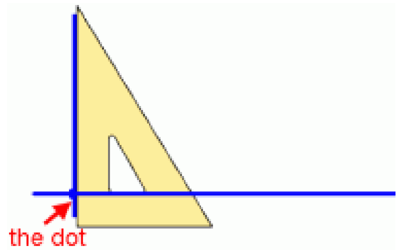
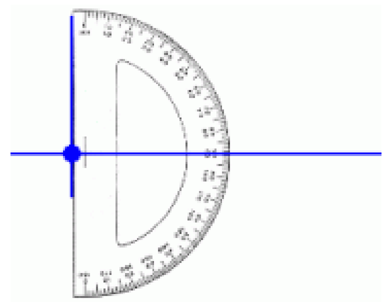
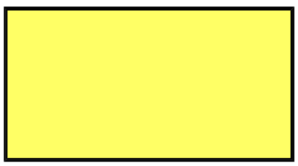
Rectangles

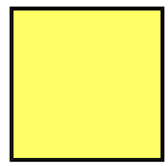
Remember? The pictures show how to draw a line that is perpendicular to a given line, using either a triangular ruler or a protractor.







A *rectangle* has four sides, and all of its angles are right angles.

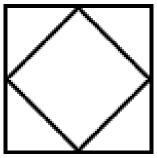


A *square* is a special rectangle: all of its sides are of equal length. We say the sides are *congruent*.

1. Draw a square with each side 3 1/2 inches long.

2. Draw a rectangle so its longer sides are 6 cm 2 mm and its shorter sides are 4 cm 7 mm.

3. Draw a large square with each side 6 inches long. After you're done, find the midpoint of each side. Join the midpoints as in the figure below.

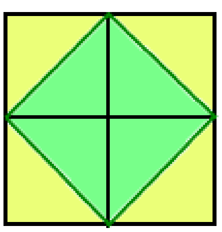


What figure is formed in the middle?
How long are its sides?

4. Do the same as above, but instead of a square, start out with a rectangle with 10 cm and 6 cm sides. What figure will be formed in the middle now?

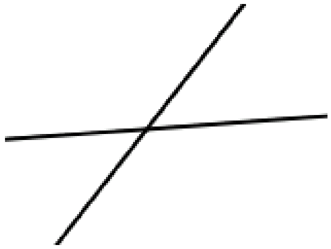
   How long are its sides now?

5. Draw the figure below so that the sides of the outside square measure 12 cm.

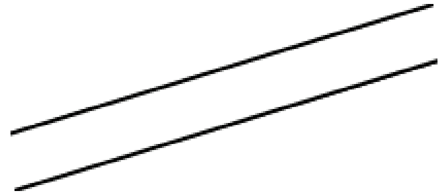


Parallelograms

Two lines or line segments can either **intersect** (cross) each other or be **parallel.**



These lines intersect.



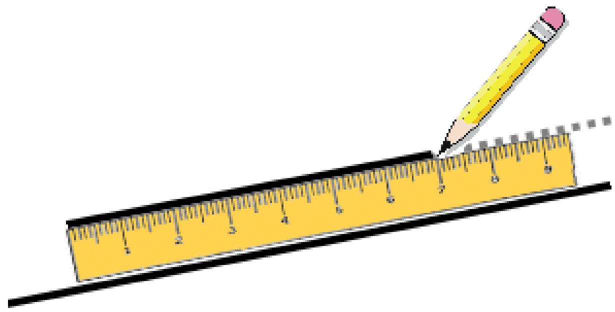
These lines are parallel.

Parallel lines will never meet each other, no matter how far you would continue them in both directions.

You can draw parallel lines with a ruler.

Align the bottom side of the ruler with an existing line. Then draw a line above the top of the ruler.

You can also carefully slide the ruler up or down if you want the parallel line to be a little further or a little closer to the existing line.



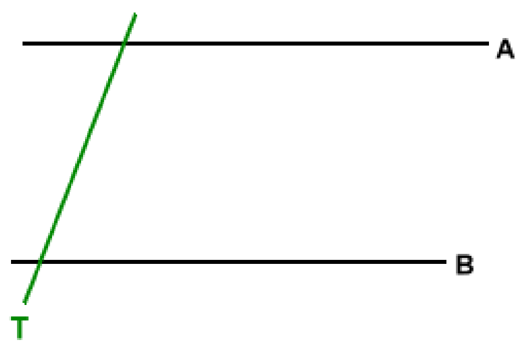
This method is not totally accurate, but it is good enough for now.

In this picture you see two parallel lines, A and B. The third line is marked with "T", and it intersects the other two.

Draw a *fourth line* for the picture so it is parallel to the line T, and crosses the other two. Call it S.

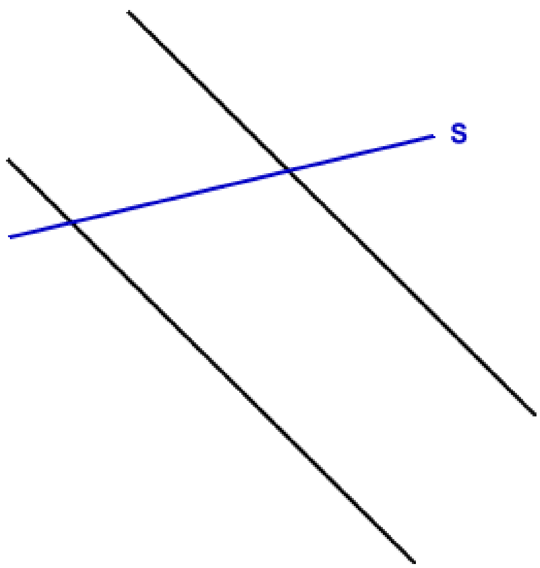
Color the figure formed in the middle. It is called a **parallelogram**, because two of its sides are parallel to each other, and so are the other two.

In other words, a parallelogram has two *pairs* of parallel sides.



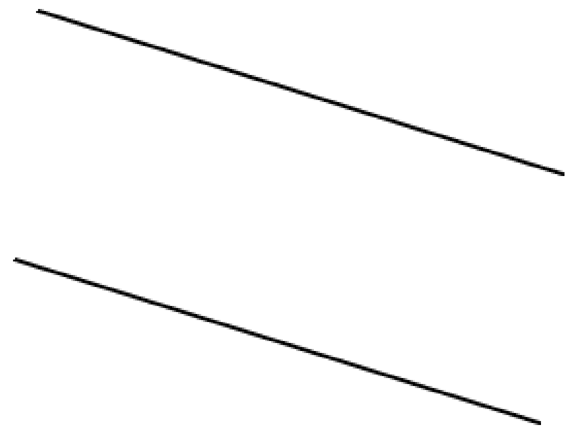
1. **a.** Draw a line that is parallel to the line S so that you get a parallelogram.

b. Measure all the sides of the parallelogram and write the side lengths in the figure, next to the sides.

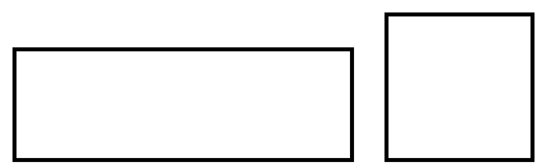


c. Make another parallelogram! Draw two lines that are parallel to each other and that cross the two existing lines.

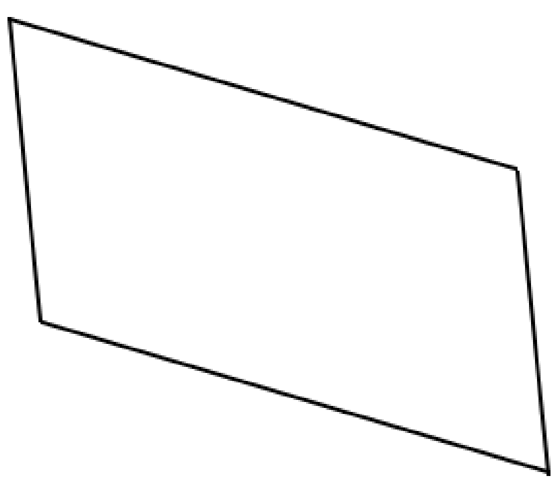
d. Measure all the sides of the parallelogram and write the side lengths in the figure, next to the sides. What can you notice?



2. Is a rectangle a parallelogram? To answer this, think if it has two pairs of parallel sides. How about a square?



3. Measure all the angles of this parallelogram. What do you notice?



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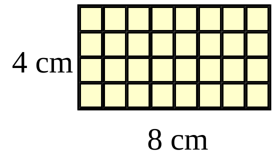
Area Versus Perimeter

Sometimes it's easy to confuse perimeter and area. Just remember that • AREA has to do with covering; • PERIMETER is about "going around".	**Area:** 4 cm × 8 cm = 32 cm^2.  **Perimeter:** 4 cm + 8 cm + 4 cm + 8 cm = 24 cm

1. **a.** Draw a square with a perimeter of 4 inches. What is its area?

b. Draw a square with an area of 9 cm^2. What is its perimeter?

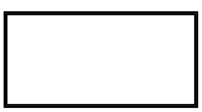
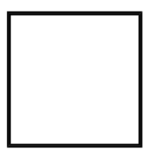
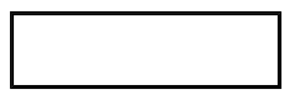
2. Which fits each situation - perimeter, area, or volume, if you need to find out...

a. How much fence is needed to go around a yard?

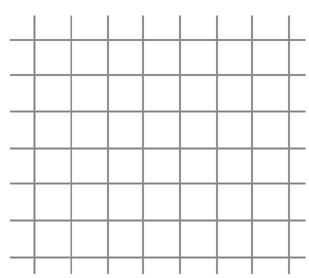
b. How much water fits into a bottle?

c. How much carpet will cover the floor?

3. Draw the square inches inside the figures, and find the area and perimeter.

 **a.** A rectangle 4 inches wide, 2 inches tall	 **b.** A square with 4-in sides	 **c.** 1 ft 2 inches wide, 4 inches tall

4. The sides of an L-shape are: 4,1, 3, 3, 1, and 4. What are its area and perimeter?



5. If a rectangle's *area* is 24, what can its perimeter be? Fill out the table. You can sketch rectangles in the grid.

Rectangle's sides	Area	Perimeter
	24	
	24	
	24	
	24	

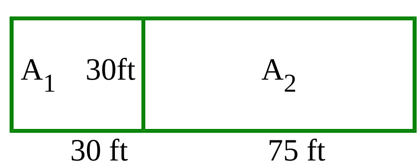
6. If a rectangle's *area* is 36, what can its perimeter be?

Rectangle's sides	Area	Perimeter
	36	
	36	
	36	
	36	
	36	

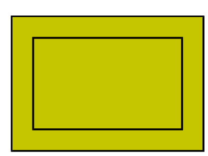
7. If a rectangle's *perimeter* is 20, what can its area be?

Rectangle's sides	Area	Perimeter
		20
		20
		20
		20
		20

8. **a.** Find the areas of the two parts A_1 and A_2.
b. Find the total area.
c. Find the perimeter.



9. The outer dimensions of a picture frame are 19 cm and 14 cm. The frame is 2 cm wide. What is the area of the inside?



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Chapter 7: Fractions
Introduction

In the third grade, children studied the concept of a fraction, added and subtracted like fractions (with the same denominator), and compared some easy fractions. In fourth grade, it is time to slightly expand the fraction topics. We study

- mixed numbers
- addition and subtraction with like fractions and mixed numbers with like fractional parts
- comparing fractions
- equivalent fractions
- finding the fractional part of a quantity again. This topic has already been studied in the division chapter.

Then in fifth grade, students tackle *all* of the four operations of fractions. Our studies here are still laying groundwork for that, emphasizing the concepts and using visual models a lot.

These lessons are also important because they are the basis for understanding decimal numbers, the topic of the next chapter. Remember, all decimals are just another way of writing fractions with denominators 10, 100, 1,000 etc.

The topics in this chapter are first studied with the help of pictures in order to help to cement the concepts. Avoid presenting fraction math as a list of computational rules. Children easily confuse the various fraction rules, because there are so many, such as:

- a rule for converting a mixed number to a fraction, and vice versa
- a rule for adding like fractions
- a rule for finding a common denominator
- a rule for changing fractions to like fractions
- a rule for adding unlike fractions
- a rule for simplifying fractions
- a rule for finding equivalent fractions
- a rule for multiplying fractions
- a rule for dividing fractions
- a few rules for doing the four operations with mixed numbers

There is a place for the rules, as shortcuts for ideas that are already understood, but do not start with them. In fourth grade, there is no hurry to study all of these rules. Let the big ideas sink in conceptually first. Then, if a child understands the concept, notices a shortcut (a rule), and wants to use one, let him go ahead with the rule.

The Lessons in Chapter 7

Helpful Resources and Games on the Internet

Visual Fractions
Great site for studying all aspects of fractions: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by either a number line or a circle with a Java applet. Also couple of games, for example: make cookies for Grampy.
http://www.visualfractions.com/

Who Wants pizza?
Explains the concept of fraction, addition and multiplication with a pizza example, then has some interactive exercises.
http://math.rice.edu/~lanius/fractions/index.html

Fraction Model
Adjust the the numerator and the denominator, and the applet shows the fraction as a pie/rectangle/set model, as a decimal and as a percent.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=44

Clara Fraction's Ice Cream Shop
Convert improper fractions to mixed numbers and scoop the right amount of ice cream flavors on the cone.
http://www.mrnussbaum.com/icecream/index.html

MathSplat
Click on the right answer to addition problems or the bug splats on your windshield!
http://fen.com/studentactivities/MathSplat/mathsplat.htm

Fraction Worksheets: Addition and Subtraction
Create custom-made worksheets for fraction addition and subtraction. Choose "Like Fractions" for this level.
http://www.homeschoolmath.net/worksheets/fraction.php

Equivalent Fractions from National Library of Virtual Manipulatives (NLVM)
See the equivalency of two fractions as the applet divides the whole into more pieces.
http://nlvm.usu.edu/en/nav/frames_asid_105_g_2_t_1.html

Equivalent Fractions
Draw two other, equivalent fractions to the given fraction. Choose either square or circle for the shape.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=80

Fraction Frenzy
Click on pairs of equivalent fractions, as fast as you can. See how many levels you can get!
http://www.learningplanet.com/sam/ff/index.asp

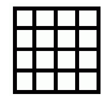
Fresh Baked Fractions
Practice equivalent fractions by clicking on a fraction that is not equal to others.
http://www.funbrain.com/fract/index.html

Fractioncity
Make "fraction streets" and help kids with comparing fractions, equivalent fractions, addition of fractions of like and unlike denominators while they drive toy cars on the streets. This is not an online activity but has instructions of how to do it at home or at school.
http://www.teachnet.com/lesson/math/fractioncity.html

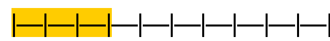
One Whole and Its Fractional Parts

When we use fractions, they always relate to some kind of <u>*one whole*</u>.

Maybe the one whole is this square. If it is divided into 16 parts, each part is $\frac{1}{16}$ of the whole.



Maybe the one whole is this line. $\frac{3}{10}$ of it is colored.



Maybe the one whole is Daddy's salary. If we need to find 5/6 of it,
we imagine dividing the salary into 6 parts, and taking five of those parts.

Now you write down two more examples. You can draw a picture, too.

Maybe the one whole is ______________________________,

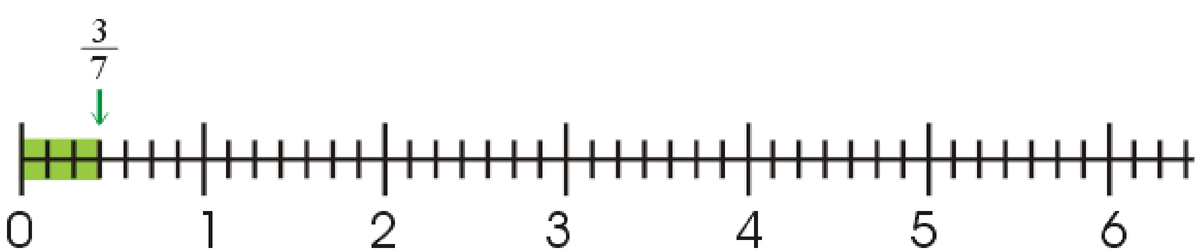
and it is divided into ___________________________________.

Maybe the one whole is ______________________________,

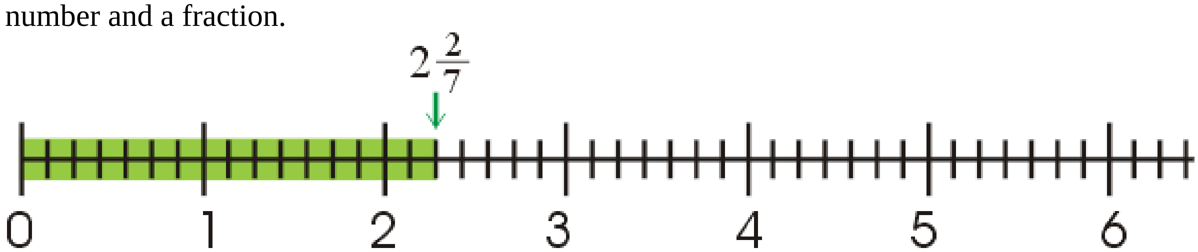
and ____

$\frac{3}{11}$ The top number is the **<u>numerator</u>**. It *numerates* or counts *how many pieces* there are.
The bottom number is the **<u>denominator</u>**. It *denominates* or *names* what kind of parts they are.

If you have a fraction alone as a number, such as $\frac{3}{7}$, then the one whole you are comparing to is the number 1. We can illustrate 3/7 on a number line where each whole-number interval from 0 to 1, from 1 to 2, from 2 to 3, and so on, is divided into seven parts.

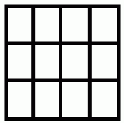


A number line is great for illustrating *mixed numbers*, too. In mixed numbers, you have both a whole number and a fraction.



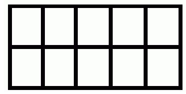
1. Color parts. Write the colored part and the white part as fractions.

a. Color 1 part.



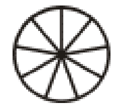
$\frac{1}{12}$ and —

b. Color 5 parts.



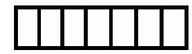
and

c. Color 8 parts.



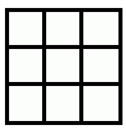
and

d. Color 3 parts.

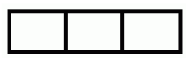


and

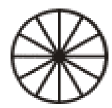
2. Color and write one whole as a fraction.



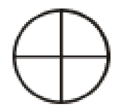
a. 1 =



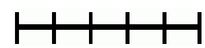
b. 1 =



c. 1 =



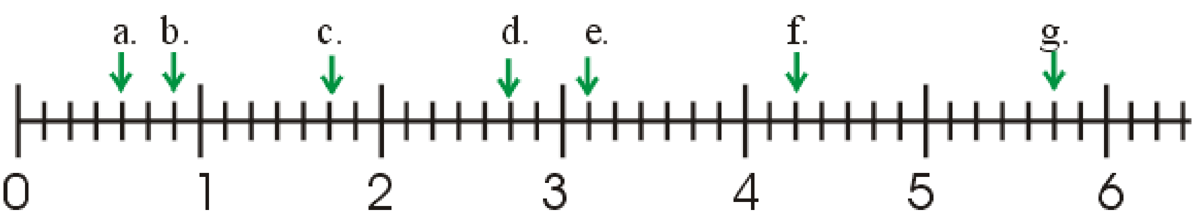
d. 1 =



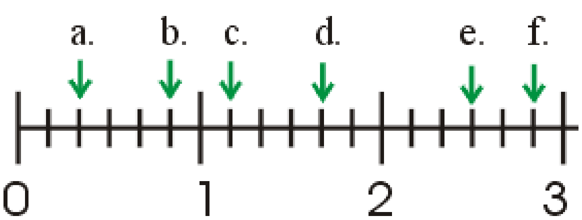
e. 1 =

In problems 3 - 5, write the fractions and mixed numbers that the arrows mark on the number line.

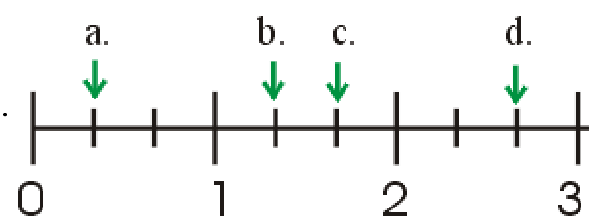
3.



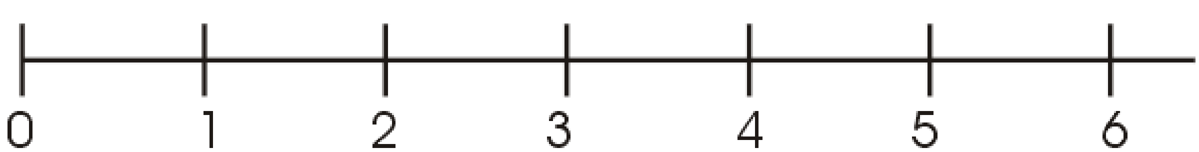
4.



5.



6. Divide each interval from a whole number to the next into four parts, and then mark these mixed numbers on the number line: **a.** 1 2/4 **b.** 3/4 **c.** 4 1/4 **d.** 5 1/2 **e.** 3 1/4 **f.** 2 3/4



7. Answer.

a. Jackson's ate $\frac{3}{4}$ of the pie.
How much is left?

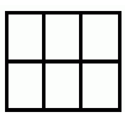
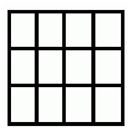
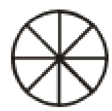
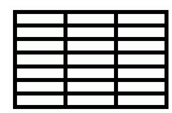
b. Jerry ate $\frac{1}{6}$ of the pizza.
How much is left?

c. Five boys shared the chocolate bar equally. Each one got $\frac{}{}$ of the bar.

8. Write 1 whole as a fraction.

a. $1 = \frac{}{8}$ **b.** $1 = \frac{}{20}$ **c.** $1 = \frac{}{50}$ **d.** $1 = \frac{}{156}$

9. Color. Write an addition sentence, adding the colored and white parts. What is the sum?

a. Color 1 part.  $\frac{1}{6} + \frac{}{} =$	**b.** Color 10 parts. 	**c.** Color 3 parts. 	**d.** Color 15 parts. 

10. What is missing from 1 whole?

a. $\frac{3}{4} + \frac{}{} = 1$ **b.** $\frac{6}{7} + \frac{}{} = 1$ **c.** $\frac{1}{8} + \frac{}{} = 1$ **d.** $\frac{11}{15} + \frac{}{} = 1$

11. Solve.

a. Mary drank $\frac{1}{4}$ liter of juice from a 1-liter pitcher, and her brother drank another $\frac{1}{4}$.
How much juice is left in the pitcher?

b. A loaf of bread was cut into 25 slices, and Jack and John ate three slices each.
What fractional part of the bread is left?

c. Remember division? Find 1/5 of 940. Then find 4/5 of 940.

d. A restaurant bill is $45.50. It is divided so that
Cindy pays 2/5 of it and Sandy pays 3/5 of it.
Find each one's share of the bill in dollars.

e. Dad used 2/9 of his $2,700 paycheck.
What fractional part is left of his paycheck?
How many dollars are left?

Subtracting Fractions and Mixed Numbers

Again, we only subtract like fractions and mixed numbres with like fractional parts.

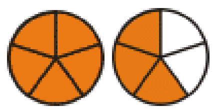
Subtracting like fractions is just as easy as adding them. Just think about "so many of this kind of pieces" and taking away some. You can illustrate subtraction of fractions by crossing out pieces from a picture.

Example 1. $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$

Here are 5 sixths. If you take away two of them, how many sixths will you have left? Three sixths, of course!

Example 2. $1\frac{3}{5} - \frac{4}{5}$

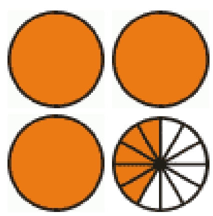
$= \frac{8}{5} - \frac{4}{5} = \frac{4}{5}$



Here it helps to first change the mixed number into a fraction. One whole and three fifths is the same as 8 fifths. Then subtract 4 fifths.

Example 3. $3\frac{5}{12} - 1\frac{9}{12} = ?$

Try to cross out $1\frac{9}{12}$ from the picture!



First subtract 1: $3\frac{5}{12} - 1 = 2\frac{5}{12}$.

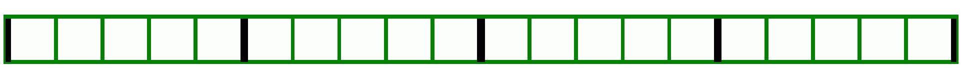
Then subtract 5 twelfths: $2\frac{5}{12} - \frac{5}{12} = 2$.

Lastly, subtract 4 twelfths: $2 - \frac{4}{12} = 1\frac{8}{12}$.

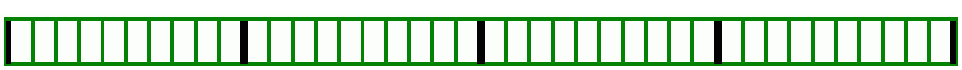
1. Subtract.

a. $\frac{11}{12} - \frac{7}{12} =$ **b.** $\frac{7}{10} - \frac{5}{10} =$ **c.** $1 - \frac{2}{3} =$ **d.** $1 - \frac{7}{8} =$

2. Subtract. You can shade parts and then cross or erase some out to help you.

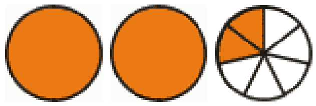
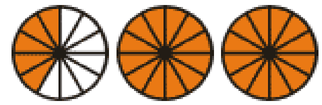
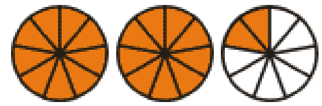


a. $3 - 1\frac{4}{5} =$ **b.** $2\frac{3}{5} - 1\frac{1}{5} =$ **c.** $1\frac{1}{5} - \frac{4}{5} =$



d. $2\frac{2}{10} - \frac{7}{10} =$ **e.** $3\frac{8}{10} - 2\frac{4}{10} =$ **f.** $4 - 2\frac{2}{10} =$

3. Subtract. You can cross out parts to help.

a. $2\frac{2}{7} - 1\frac{2}{7} =$ 	**b.** $2\frac{5}{12} - \frac{7}{12} =$ 
c. $2\frac{1}{8} - 1\frac{6}{8} =$ 	**d.** $2\frac{2}{9} - \frac{5}{9} =$ 

4. Try your skills with these subtractions. Try to imagine the fractions in your mind.

a. $\frac{5}{4} - \frac{2}{4} =$

b. $1\frac{1}{4} - \frac{1}{4} - \frac{2}{4} =$

c. $6\frac{5}{6} - 3\frac{3}{6} =$

d. $1\frac{2}{5} - \frac{3}{5} =$

e. $7\frac{1}{5} - 2 - \frac{4}{5} =$

f. $3\frac{1}{3} - \frac{2}{3} =$

5. Subtract from whole numbers.

a. $7 - \frac{2}{4} =$

b. $5 - \frac{5}{7} =$

c. $6 - \frac{8}{9} =$

d. $10 - 2\frac{3}{5} =$

e. $5 - 1\frac{1}{2} =$

f. $8 - 2\frac{2}{3} =$

6. Food problems!

a. Three pizzas were ordered for a get-together. Each was divided into 12 pieces. Edward ate three pieces, Abigail ate two pieces, Mom and Dad together ate a whole pizza, and Jack and John ate four pieces each. How much pizza was left? Give your answer as a fraction.

b. A bag of flour contains 6 cups of flour. Mom used 2 2/3 cups for a bread recipe. How much flour is left after she makes one batch of bread? How many batches more can she make with the remaining flour?

Puzzle corner

Solve what number x represents.
The top and bottom problems are related!

a. $4 - x = 3\frac{1}{4}$

b. $7 - x = 3\frac{1}{4}$

c. $x - \frac{4}{9} = 5$

d. $x - \frac{4}{9} = 5\frac{1}{9}$

e. $5\frac{7}{10} - x = 2\frac{3}{10}$

f. $5\frac{1}{10} - x = 2\frac{3}{10}$

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Chapter 8: Decimals
Introduction

In fourth grade, we study the concept of decimal numbers with one or two decimal digits, adding and subtracting them, and multiplying decimals by a whole number. It is important that the student grasps these simple topics well, because we are laying a groudwork towards fifth and sixth grade, when decimal operations and using decimals take more of a "center stage".

Right now, the focus is first of all grasping that decimals are nothing more than fractions with a denominator 10 or 100. I have only included decimals with one or two decimal digits to keep from confusion, to keep it simple enough so that the concepts can be well understood.

With that in mind (decimals are fractions), we study adding and subtracting them. The important ideas to grasp are:

- In problems of the type 0.5 + 0.9, we get 14 tenths, which is *more* than one whole. The answer is NOT 0.14, but 1.4. If the student has problems, have him compare it with fraction addition.

- In a problem such as 0.5 + 0.11, the answer is NOT 0.16. We cannot add the decimal parts as if they were "whole numbers". Instead, we rewrite 0.5 as 0.50, and the problem becomes 0.50 + 0.11 = 0.61.

Then we multiply decimals by whole numbers. This is essentially just repeated addition. For example, $3 \times 0.8 = 0.8 + 0.8 + 0.8 = 2.4$. Note how the answer has one decimal digit, because we kept adding 0.8, which also has one decimal digit (tenths).

After this idea is understood, the student can just multiply 3×8 using the knowledge of multiplication tables, and remember to place the decimal point in the answer.

In the lesson Using Decimal Numbers, we do some conversions between metric measuring units. This topic will be studied further in 5th and 6th grades.

In general, decimal numbers will be studied in a lot more detail in grades 5 and 6.

The Lessons in Chapter 8

Decimal Numbers - Tenths

You might have seen this picture before. The number line between 0 and 1 is divided into ten parts. Each of these ten parts is $\frac{1}{10}$, a **tenth**.

Under the tick marks you see the *distances* as *decimal numbers* such as 0.1, 0.2, 0.3, and so on. These are the same numbers as the fractions $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, and so on.

We can write any fraction with tenth parts (denominator 10) using the decimal point.
Simply write after the decimal point *how many tenths* the number has.

0.6 means six tenths or $\frac{6}{10}$.

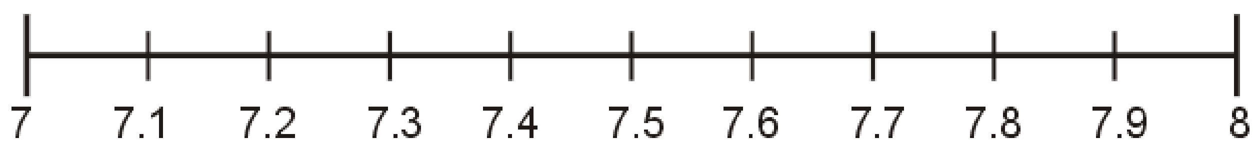
1.5 means 1 whole 5 tenths or $1\frac{5}{10}$.

Note: $\frac{1}{8}$ is ***not*** 0.8. Instead, 0.8 is eight tenths, or $\frac{8}{10}$.
The denominator is always 10!

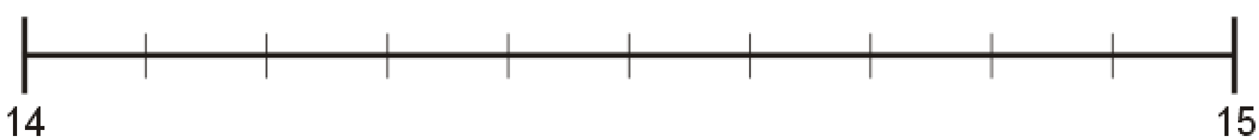
"**Decimal**" comes from the Latin root *decem*, which simply means ten. The number system we use is called the **decimal number system**, because the place value units are in tens: you have ones, tens, hundreds, thousands, and so on, each unit being 10 times the previous one.

In common language, the word "decimal number" has come to mean numbers which have digits after the decimal point, such as 5.8 or 9.302. In reality, any number within the decimal number system could be termed a decimal number, including whole numbers such as 12 or 381.

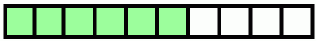
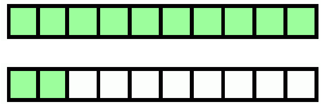
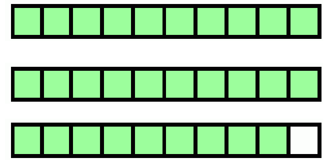
1. Write the mixed number under each decimal number.



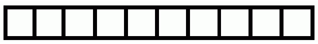
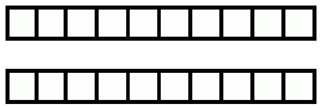
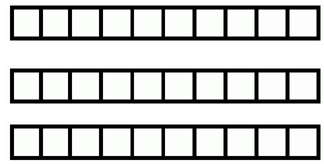
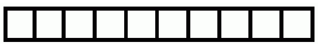
2. Write the decimal numbers under the tick marks.



3. Write the decimal and the fraction that the picture shows.

a. 	**b.** 	**c.** 

4. Shade parts to show the decimals.

a.  0.4	**c.**  1.6	**d.**  2.8
b.  0.1		

5. Write the fractions as decimals and vice versa.

a. $\frac{7}{10}$	**b.** $2\frac{4}{10}$	**c.** $10\frac{9}{10}$	**d.** 0.9	**e.** 29.3

6. Make a number line from 2 to 3.5 with tick marks at every tenth. Label them with decimal numbers.

7. Compare. Write <, >, or = in between the numbers.

a. 0.5 $\square$ 0.9 **b.** 1.3 $\square$ 0.3 **c.** 5.1 $\square$ 4.9 **d.** 0.4 $\square$ $\frac{1}{2}$ **e.** 16.0 $\square$ 16

8. Put in order from smallest to largest:

1.2 0.9 2.6 0.1 $2\frac{1}{2}$ 2.3 3.0 $\frac{1}{2}$

9. Mark these temperatures with dots on the thermometer: 37.4°C, 36.2°C, 38.7°C, 41.8°C, 40.5°C