FEUP FACULDADE DE ENGENHARIA
UNIVERSIDADE DO PORTO
DEPARTAMENTO DE ENGENHARIA E GESTÃO INDUSTRIAL

Operations Research Exercises

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Chapter 1

Decision Theory

1.1 Xpt0 Textile

XptO Ltd. is a textile company which is currently preparing its Winter collection, to be launched next year. Leopold, the manager of the company, is in doubt concerning the amount of investment to be allocated to this collection. He can do a big investment, a medium investment or a small investment. In recent years, the weather, which influences the success of the collection, has proved unreliable. Sometimes Autumn and Winter may be very similar but the Winter can also have several days of sun and heat.

Leopold knows that if the next Winter has many periods of sun and heat, the winter collection will be a failure. However, if the Winter is rigorous, the collection will bring huge profits to XptO. There is also the possibility that the Winter conditions are intermediate, corresponding to a less successful collection.

Leopold prepared the decision matrix that follows, with the profits in thousands of euros:

(K€)	Rigorous Winter	Mild Winter	Hot Winter
Big Investiment	5.000	2.000	-2.000
Medium investment	4.600	1.000	-50
Small investment	800	200	0

(a) Define who is the decision maker, what are the decisions and which are the events

(b) Suppose that the instability of recent years makes it very difficult to define the probabilities of occurrence of the events. Determine what type of investment should make XptO Ltd. based on the following criteria:

(i) MaxiMax;

(ii) MaxiMin;

(iii) Laplace;

(iv) Hurwicz (α=0.8);

(v) Savage.

(c) Consider now that, despite the instability, the following probabilities of occurrence of the events could estimated:

p(Rigorous Winter)=50%, p(Mild Winter)=30%, p(Hot Winter)=20%.

Determine what type of investment the company should make based on the following criteria:

(i) Maximization of the expected value;

(ii) Minimization of the expected opportunity loss.

1.2 A New Automotive Piece

An automotive company intends to invest in the development of a new piece.

For the development of the new piece the company has two alternatives. Either they make research and development (R&D) on their own or, alternatively, they can form a consortium with an engineering consulting firm.

Table 1 represents the expected profits for the next 5 years, depending on the success and the chosen alternative:

Profits (M€)	Big success (p_1)	Moderate success (p_2)	No success (p_3)
D (own R&D)	300	40	-60
C (Form a consortium)	200	30	-20

Table 1: Expected profits for the next 5 years

Based on feasibility studies and after several consultations to marketing and development companies, the estimated probabilities for the events were considered to be: $p_1 = 0.2$, $p_2 = 0.4$ e $p_3 = 0.4$.

(a) Which is the decision with the *Maximum Expected Value*? Draw the decision tree.

(b) Which decision should be taken if the decision criterion would be *Maximin* (pessimist)?

(c) Determine the value of perfect information and explain what it means.

1.3 Machine Acquisition

Gulas & Gulas, a company which operates in the food segment, considers the possibility of acquiring special machines to label bottles, as depicted in the figure.

The company must decide whether to purchase one or two labeling machines. However the cost of each machine will be lower if they purchase two at the same time.

If only one machine is purchased and the demand for the bottles is high, the second machine can still be purchased later.

The estimated probabilities for the demand of the product are:

- low demand 0.30;
- high demand 0.70.

The current value for the results if the 2 machines are purchased earlier is 750M€ if the demand for bottles is low, and is 1300M€ if the demand for bottles is high. The current value of the results if they buy one machine is 900M€ if the demand is low.

If the demand is high there are three hypotheses:

- doing nothing results in a current value of 900M€;
- subcontracting results in a current value of 1 100M€;
- if they buy the second machine then they will obtain a current value of 1 000M€.

(a) Build the decision tree for the problem.

(b) Determine, using the *Maximum Expected Value* criterion, how many machines they should purchase now.

1.4 Exploring Natural Gas

A large multinational company belonging to the energy sector intends to pay *Latifundios and Company* 60 000€ for the royalties to exploit natural gas in one of their properties. The offer of the multinational company includes the option for a future development. If they opt for it and if natural gas is discovered during the exploration phase, *Latifundios and Company* will receive 600 000€ extra.

Latifundios and Company believes that the interest shown by the multinational is a good indicator that the gas exists. They want therefore to evaluate the possibility of exploring the gas themselves. In order to do it they must hire a team of specialists in natural gas exploration.

The initial cost of the operation is 100 000€, which will be lost if no gas is found. However, if gas is discovered at an early stage, the estimate profit will be of 2 000 000€.

The decision alternatives of *Latifundios and Company* will then be:

- D_1 – accept the offer of the multinational company;
- D_2 – proceed with natural gas exploration.

The events are:

- S_1 – there is no natural gas in the property;
- S_2 – there is natural gas in the property.

Latifundios and Company estimates that there is a 60% chance that natural gas is found in the property.

The expected value (profit) for each pair (action, event) are in table 2

	S_1	S_2
D_1	60	660
D_2	-100	2000

Table 2: Expected value (in M€)

Determine the decisions recommended by the following criteria:

(a) *Maximin* (pessimist)

(b) *Laplace*

(c) *Maximum Expected Value*

(d) *Savage*

(e) *Hurwicz* (α=0,4)

(f) Consider now that the values represented in the decision matrix are costs and not profits. Determine all the previous decisions again.

1.5 Polido Guapo and his cLEAN Laundry

Polido Guapo is the manager of *cLEAN*

Recently the business had a big development and reaching capacity limits in both laundries that belong to the *cLEAN* company. There is no possibility to increase the size of the laundries.

Polido Guapo, however, does not want to open a new laundry because he is convinced that the success of his business is due to the good location in the city. For *Polido Guapo* the location of the laundry is the highest value of the business.

Polido Guapo recognizes that the operation of the company is experiencing difficulties motivated by the high occupation of space and, of course, he wants to improve the movement of the pieces during all the stages of the process (separation, cleaning, ironing, . . .). One hypothesis that would allow him to free some space would be through the installation of an air transport system (carrier) that would free part of the floor that is currently occupied by bins, which are used to keep the pieces together and move them between stages.

A company from another city that sells and installs this type of equipment proposed to install the carrier, but *Polido Guapo* is not capable to take decisions. The proposal of the company includes the installation of a carrier in a facility (the carrier would replace the bins) for 25 000€. If *Polido Guapo* decides to buy carriers for both laundries while the installation team is in town, the total cost will be only 45 000€. If he does not decide to install the carriers, then he should spend 1000€ to replace the bins by more modern ones.

It is clear that *Polido Guapo* knows that the carriers will reduce the time spend by the employees, improving service efficiency. The estimate earnings if a carrier is installed will be 16 000€ per installation. In addition, the potential gain will be due to the increase in the number of customers that will follow the increase in the available space.

Table 3 has estimates for the present value of the profit (for the sake of simplification only three values were considered for the increase in business):

Clearly *Polido Guapo* can order the installation of a carrier for now and install another one later on (at the price of 25 000€) if he considers that the business evolution is favorable (see table 3).

(a) Build a decision tree for this problem and use it to analyze the situation of *cLEAN*.

(b) Considering the *Maximum expected value* what should be the decision of *Polido Guapo*?

Business increase (%)	Profit (per installation)	Probability
0	16 000€	0.30
3	30 000€	0.50
6	50 000€	0.20

Table 3: Business increase (profit and probability

(c) What is the expected value for perfect information? Explain what that value means for *cLEAN*.

1.6 $\mathcal{LESTO}$, a new product

The commercialization of $\mathcal{LESTO}$, a new product of *Expedita* begins to take shape. However, as often happens in the launches of new brands, there is a considerable risk – will the product hold up well?

In a conservative posture, the *Expedita* finds it convenient to market $\mathcal{LESTO}$ only regionally before the national rollout, just to see how the market reacts. So, the first decision to take concerns this regional market study.

The company estimates the cost of 50 000€ for the market study. If they decide to do the market test, they should await the results of that test and then decide on the commercialization of $\mathcal{LESTO}$ throughout the country. On the other hand, if the initial choice is not to proceed with the market study, then the final decision, to sell or not to sell the product nationally, can be taken.

Expedita evaluates the success of the product in the domestic market, in 1.2M€, and failure would derive a cost of 500 000€.

The probabilities to be associated with the events should reflect the current scientific knowledge and the company's experience with similar products. Thus *Expedita* considers that there is a probability of 50% for the success (or failure) of $\mathcal{LESTO}$ at the national level (without any information from the market tests). However, if the test is run and it points to success, then the company believes the likelihood of national success of the product to be 70%, whereas in the opposite case (test points to failure), the probability of success in the domestic market will only be of 20%. Finally, it is assumed that the probability that the result of the test will point to a success is 60%.

(a) What is the appropriate strategy to adopt (use the *Maximum Expected Value*)?

(b) The objective of the market test is to obtain more accurate information about the national market, in the form of probabilities for future events to occur. Based on the available data, what should be the maximum amount that *Expedita* should pay for this market test?

(c) An increase in the value given to the success of $\mathcal{LESTO}$ (1.2M€) will have any consequence for the answer in (a)?

1.7 To be or not to be

[1]

Ser ou não ser... Eis a questão.
Que é mais nobre para a alma
Suportar os dardos e arremessos do fado sempre adverso,
Ou armar-se contra um mar de desventuras
E dar-lhes fim tentando resistir-lhes? Morrer--dormir--
Mais nada; imaginar que um sono põe remate
Aos sofrimentos do coração e aos golpes infinitos
Que são a natural herança da carne. Essa é a solução
Para se almejar. Morrer--dormir.
Dormir--talvez sonhar: É esse o dilema!
O não sabermos que sonhos poderá trazer o sono da morte
Quando alfim desenrolarmos toda a meada mortal,
Nos põe suspensos. É isso
Que torna verdadeira calamidade a vida assim tão longa.
Pois quem suportaria o escárnio e os golpes do mundo,
As injustiças dos opressores, a insolência dos orgulhosos,
A agonia do amor não retribuído, a demora da justiça,
A soberba dos chefes e o desprezo
Da inépcia contra o mérito perseverante,
Se estivesse em suas mãos obter sossego
Com um punhal? Que fardos suportaria,
Gemendo e suando nesta penosa vida,
Se não por temer algo após a morte--
A terra desconhecida de cujos limites
Nenhum viajante regressou--é um enigma para a vontade,
Fazendo-nos aceitar os males que já temos,
Do que correr para outros que desconhecemos?
William Shakespeare
Ser ou não ser (Hamlet acto 3 cena 1)

To be, or not to be: that is the question:
Whether 'tis nobler in the mind to suffer
The slings and arrows of outrageous fortune,
Or to take arms against a sea of troubles,
And by opposing end them? To die: to sleep;
No more; and by a sleep to say we end
The heart-ache and the thousand natural shocks
That flesh is heir to, 'tis a consummation
Devoutly to be wish'd. To die, to sleep;
To sleep: perchance to dream: ay, there's the rub;
For in that sleep of death what dreams may come
When we have shuffled off this mortal coil,
Must give us pause: there's the respect
That makes calamity of so long life;
For who would bear the whips and scorns of time,
The oppressor's wrong, the proud man's contumely,
The pangs of despised love, the law's delay,
The insolence of office and the spurns
That patient merit of the unworthy takes,
When he himself might his quietus make
With a bare bodkin? who would fardels bear,
To grunt and sweat under a weary life,
But that the dread of something after death,
The undiscover'd country from whose bourn
No traveller returns, puzzles the will
And makes us rather bear those ills we have
Than fly to others that we know not of?
William Shakespeare
To be, or not to be (Hamlet act 3, scene 1)

(a) Describe Hamlets' decisions. What are his alternative actions? What are his risks?

(b) Build a decision tree for Hamlet.

[1] Clemen, Robert T., Making Hard Decisions, An Introduction to Decision Analysis, Duxbury Press, 1996

Chapter 2

Modeling

2.1 Product Mixture

A company wants to schedule the production of three new products. The production requires two resources: labor and one type of raw material. The requirements in labor and raw material for each product are in table 1.

Product	A	B	C
Labor (hours per unit)	7	3	6
Raw material (kg per unit)	4	4	5
Profit (€per unit)	4	2	3

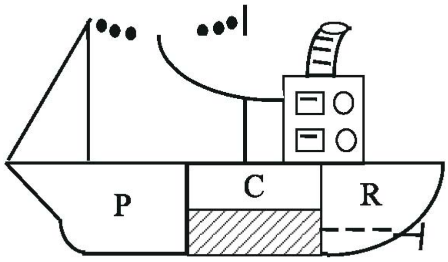
Table 1: Data provided by the engineering department

The raw material is limited to an amount of 200 kg/day. There is also a limit of 150 hours per day of labor.

The objective is to maximize the total profit.

Write the linear programming model for this problem.

2.2 Ship cargo



A shipping company has a ship with 3 cargo holds (front, center and rear) with the capacities presented in table 2:

Hold	Weight (tons)	Volume (m^3)
Front	2000	100000
Center	3200	14000
Rear	1800	80000

Table 2: Capacity (weight and volume) of each hold

This company has customers willing to ship the cargo described in table 3, and each cargo may be partially or fully accepted:

Cargo	Weight (tons)	Volume per ton ($\frac{m^3}{ton}$)	Profit ($\frac{Euros}{ton}$)
A	7000	60	20
B	6500	50	24
C	4000	25	16

Table 3: Weight, volume and profit associated to each cargo

To keep the ship balanced, the proportion between the weight and the volume, for each hold, has to be the same as the hold's capacities (weight and volume) proportion. Assume that each hold may carry different cargos together. The objective is to maximize the company's profit, in what concerns this ship usage.

Build a Linear Programming model for this problem.

2.3 Production and distribution (2 machines)

Two factories, **A** and **B**, that are located in different regions produce both products P_1 and P_2. The factories have two machines each and the machines produce both P_1 and P_2.

The products may be transported from one factory to the other in order to satisfy the demand. The number of units that may be produced daily, the production and transportation costs, the demand of each one of the products and the total number of days that each machine is available during the next month are represented in tables 4 and 5.

Write a linear programming model for this problem.

Factory	**A**				**B**			
Machine	$\mathbf{AA_1}$		$\mathbf{AA_2}$		$\mathbf{BB_1}$		$\mathbf{BB_2}$	
Availability (days)	30		28		26		28	
Product	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
Production per day	40	35	42	50	41	37	42	45
Cost per day	150	120	200	320	100	120	150	170

Table 4: Production capacities of the factories

Product	P_1		P_2	
Factory	**A**	**B**	**A**	**B**
Demand	1000	800	1700	1100
Transportation cost per unit	**A**→ **B** = 4	**B**→ **A** = 4	**A**→ **B** = 3	**B**→ **A** = 4

Table 5: Demand and transportation costs of the products

2.4 Oil Refinery

One oil refinery can mix three types of crude to produce normal and super gasoline.

The oil refinery has two mixing units, an older unit and a more recent one.

The older unit uses for each production cycle 5 barrels of crude A, 7 barrels of crude B and 2 barrels of crude C to produce 9 tanks of normal gasoline and 7 tanks of super. The more recent unit uses for each production cycle 3 barrels of crude A, 9 barrels of crude B and 4 barrels of crude C to produce 5 tanks of normal gasoline and 9 tanks of super.

The refinery has already signed contracts with some costumers that impose a production of at least 500 tanks of normal gasoline and 300 tanks of super.

For the total production they have in stock 1500 barrels of crude A, 1900 of crude B ande 1000 of crude C.

The profit of the oil refinery is 6 monetary units for each tank of normal gasoline and 9 monetary units for each tank of super.

Write the linear programming model for that problem.

2.5 Renting Space in a Warehouse (three months)

A company needs to rent space in a warehouse. The needs for the next 3 months are represented in table 6. The renting price per square meter is dependent on the number of months rented and is represented in table 7.

Month	Space need (m^2)
1	1500
2	500
3	5000

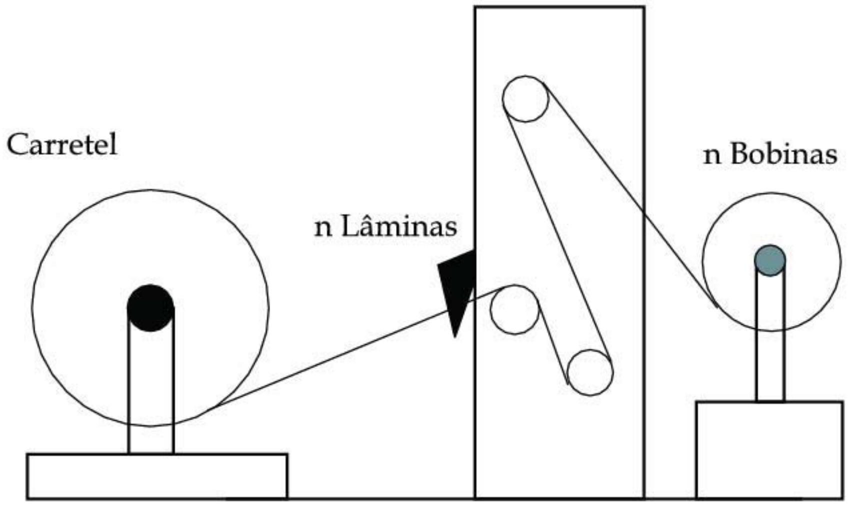
Table 6: Space needs for the next 3 months

Renting period (months)	Price per m^2 (€)
1	2800
2	4000
3	5000

Table 7: Renting prices

Write a linear programming model for this problem.

2.6 Production Planning in a Paper Mill



The paper is produced in jumbo rolls (characterized by a width and a diameter), that are divided in smaller rolls that can be sent directly to clients or that can be cut.

Consider the following example:

The paper is produced in rolls with a width of 6 meter. Out of these rolls the paper mill needs to produce 30 rolls of width 280cm, 60 rolls of width 200cm and 48 rolls of width 150cm.

One roll of 6 meter might be divided in 2 two rolls of 280 with a waste of 40cm.

Assume that they have enough rolls in stock to produce the quantities that are needed. Write the linear programming model for this example.

2.7 Scheduling of Human Resources

2.7.1 Part 1

One post office needs a different number of full-time employees during each day of the week.

	Minimum number
Monday	17
Tuesday	13
Wednesday	16
Thursday	19
Friday	14
Saturday	16
Sunday	11

The labor rules impose that each employee must have 2 days off after 5 days of work.

Write a linear programming model for this problem where the objective is to minimize the total number of full-time employees.

2.7.2 Part 2

Suppose now that the human resources needs can be met either by full-time employees or by part-time employees.

A full-time employee works 8 hours per day and a part-time employee works 4 hours a day. All the other labor rules apply also to the part-time employee.

The maximum number of part-time employees is 25% of the total number of emplyees.

The cost per hour of a full-time employee is 15 €and of a part-time employee is 10 €.

Write a linear programming model for this problem in order to minimize the total cost.

2.7.3 Part 3

Consider now that each employee can work an extra day each week. For instance an employee whose shift begins on Monday and ends on Friday can be asked to work on Saturday. Each hour of this extra day will be paid at 150%.

2.7.4 Part 4

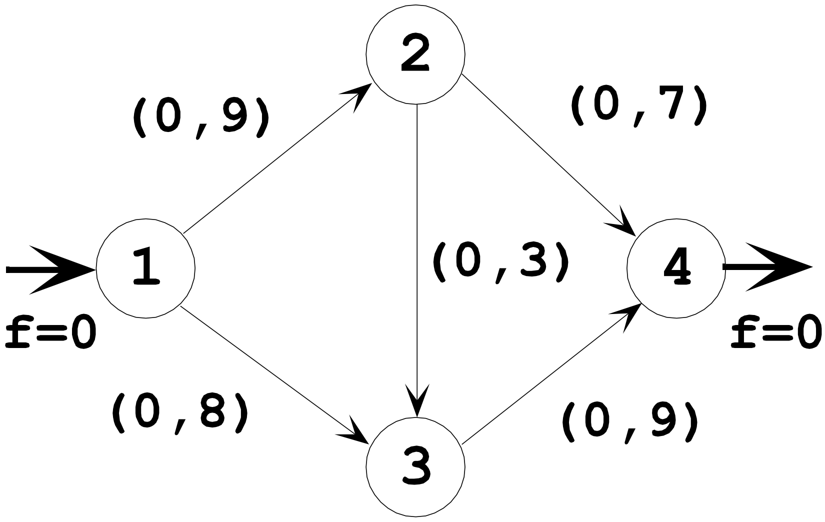
Consider once again that there are only full-time employees. Consider that the post office has 25 employees.

Write the scheduling model that maximizes the number of days-off during weekends (Saturday or Sunday).

2.8 Maximum Flow Problem

The maximum flow problem is the problem o finding a maximum feasible flow through a capacitated directed graph from a single-source to a single-sink.

Consider the graph represented in the following figure:

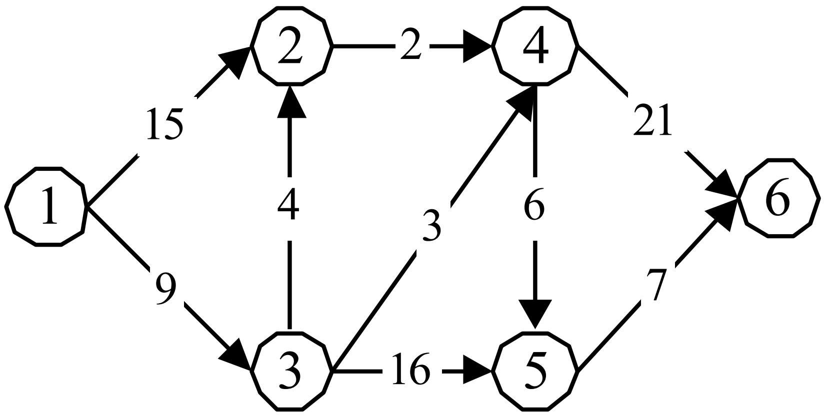


Write the linear programming model for the maximum flow problem represented in the figure.

2.9 Shortest Path Problem

The shortest path problem is the problem of finding a path between two vertices (or nodes) of a graph such that the sum of the weights of its constituent edges is minimized.

Consider the graph represented in the following figure:



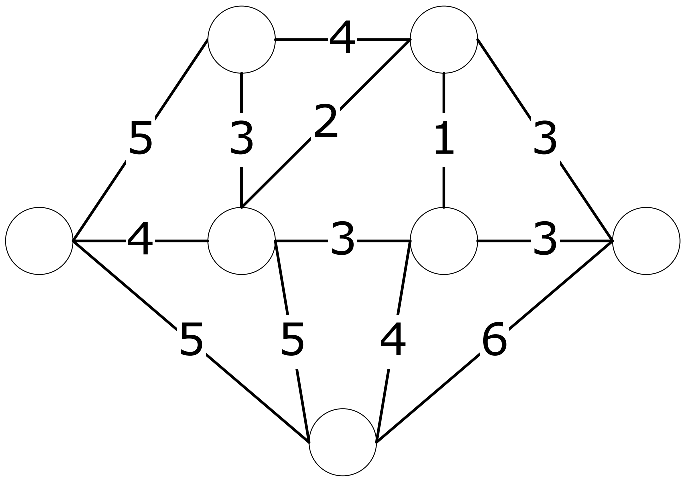
Write the linear programming model for the shortest path problem represented in the figure. The source node is node 1 and the sink node is node 6.

2.10 Minimal Spanning Tree

A tree is a connected acyclic graph. A graph is connected if there is a sequence of edges that connects all the nodes.

In the Minimum Spanning Tree Problem we want to find the tree of minimum total length that connects all the nodes of the graph.

Write the linear programming model for the Minimum Spanning Tree problem represented in the next figure.



2.11 Events in U.Porto

2.11.1 Part 1

Under the new cooperation agreement with Santander Totta Bank it was decided to invest 200000 €in cultural events and Sports, that will be organized in the next 5 years, with the aim of strengthening the bonds within University, and increase the awareness of UPorto brand in the city and in the country. To the team that prepared the data to provide ro the Rector do that a decision can be made, it was asked, for each proposed event, to estimate the budget and the number of people who would participate.

On schedule the results of team work were presented to the Rector.

Event	Estimated budget	Participants
"In U.Porto everything is drama"	80 000€	100 000
"Science in U.Porto"	40 000€	10 000
"Running in U.Porto"	30 000€	10 000
"Dawn with books in U.Porto"	15 000€	5 000
"Inside U.Porto"	10 000€	20 000
"Folk festival in U.Porto"	30 000€	10 000
"After six, U.Porto is Jazz"	40 000€	30 000
"U.Porto is only choirs"	20 000€	25 000
"On U.Porto paths"	25 000€	50 000
"To be U.Porto or not to be"	20 000€	30 000

Build the linear programming model that would allow to find the best set of events to organize, considering that the Rector of U.Porto wants to maximize the total number of participants.

2.11.2 Part 2

After analyzing investment solution for the next 5 years, the Rector asked the same team to classify the various events in the following categories: Performing Arts, Music, Philosophy, Architecture, Literature, Science and Sports, as he wished to add to the previous model a set restrictions to enable it to ensure that at least one event of each type would be organized.

Among the events classified as Sport and Science it is only needed to ensure that one was organized.

Event	Perf. Arts	Music	Philosophy	Arq.	Lit.	Sports	Science
In U.Porto everything is dram	X						
Science in U.Porto							X
Running in U.Porto						X	
Dawn with books in U.Porto					X		
Inside U.Porto				X			
Folk festival in U.Porto	X	X					
After six, U.Porto is Jazz	X	X					
U.Porto is only choirs	X	X					
On U.Porto paths				X			
To be U.Porto or not to be			X				

2.12 ALETROP Airport

Aletrop airport is the aircraft base of the airline PAT. It is a modern airport, and an airline in expansion, which wants to remain competitive in a highly competitive industry. Increasing competitiveness supposes, in particular, the achievement of two objectives: to improve service quality and to reduce operating costs. Furthermore, the safety of an airline is an aspect of prime importance, being intimately connected to maintenance. To keep a plane in good technical condition, preventive maintenance has to be performed through minor inspections between landing and subsequent takeoff. The company management is also considering the hypothesis of offering these maintenance services to other airlines, even if has to increase the maintenance teams. The crucial element in these teams is the head of maintenance, a highly qualified technician, who needs to take specific training for each airplane type and thus obtain the respective license. Each license relates to a category of aircraft and there 4 different license types:

License types	Aircrafts
1	Boeing 717 (100 seats)
2	Boeing 777 (300 a 500 seats)
3	Airbus A319 (124 seats)
4	Airbus A340 (350 seats)

Each technician can have a maximum of 2 licenses. The first license takes several years to obtain, and therefore more is expensive for the company, while the second license takes less years to obtain, naturally getting cheaper. The cost of the second license also depends on the previous license that that technician has. Currently there are 9 maintenance teams, each headed by a licensed technician, who work in 3 shifts.

Cost (M$)				
Previous license	License to take			
	1	2	3	4
0	2	4	2	4
1	-	1	2	3
2	1	-	2	3
3	1	3	-	2
4	1	2	1	-

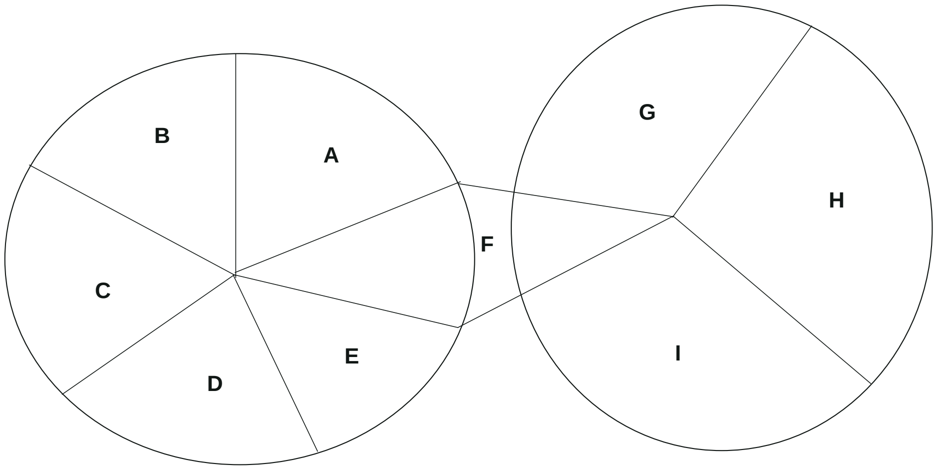
Shift	Head of maintenance team	License type
1	1	1, 2
	2	1
	3	2
2	4	3, 4
	5	2
	6	3
3	7	4
	8	3, 4
	9	3

In order to provide services to other airlines, the company intends to have 4 licenses of each type, when considering all heads of maintenance. This can be achieved by sending to training the current heads of team (hence technicians who already have 1 license) or other people who still have no license. However, form each shift at most one head of team may leave for training. Write a mathematical programming model in order to establish license training policy that minimizes costs for Aletrop.

2.13 ShopShopping construction site

2.13.1 Part 1

Before starting the construction of ShopShopping, the first offshore shopping center of the world, Feimiro needs to set the location of the several construction sites. To do that, he began by dividing the building area into sectors, as depicted in the following figure:



It is known that it is possible to build at most one construction site in each sector. Feimiro's company , the only one licensed to to build construction sites in offshore platforms, mounts two types of construction sites:

Construction site type α Small Construction site, with a capacity of handling and storage of 1000 tons of materials throughout the complete building process. These construction sites type α can only serve the sector where they are installed.

Construction site type β Large site with a capacity of handling and storage of 5000 tons of materials throughout the complete building process. These construction sites type β can serve any sector.

By imposition of the construction companies involved, if mounted a shipyard α in sector **B**, then it is necessary to set up a construction site β in sector **A** or a construction site α in sector **C**.

Because of regulatory constraints, the minimum number of construction sites to build in such a construction work as this one is 7.

The costs of mounting a construction sites type α or *eta* in a given sector are shown in the following table (in k€).

Sector	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
Site α	100	100	100	100	100	200	100	50	100
Site β	300	300	300	400	200	300	200	100	300

The running costs of the construction sites were also determined and are independent of the type of construction site. For every ton of material that is moved from one construction sites to the sector where the shipyard is located, the cost is 100€. The movement of materials from one construction sites located in one sector to another sector costs 200€ t.

The estimates for the total demand of construction materials in each sector are shown in the following table (in t).

Sector	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
Materials demand	1000	2000	5000	5000	1000	1500	2000	3000	4000

Build the linear programming model for this problem.

2.13.2 Part 2

20 minutes after Feimiro has presented the problem, you are already handing back the requested model, thinking that you can go home to rest. However, Feimiro has been thinking and wants to add a new constraint to the problem.

“ I thought it would cost less money if we demand that each sector is served by one and just one construction site. It should be easy to change the model, and since you have done much of the work in so little time, you will spend with this little change no mote that 2 minutes.”

(a) Will Feimiro spend less money in this case? Why?

(b) This new Feimiro’s idea will require a really small change in the model?

(c) Build the new model.

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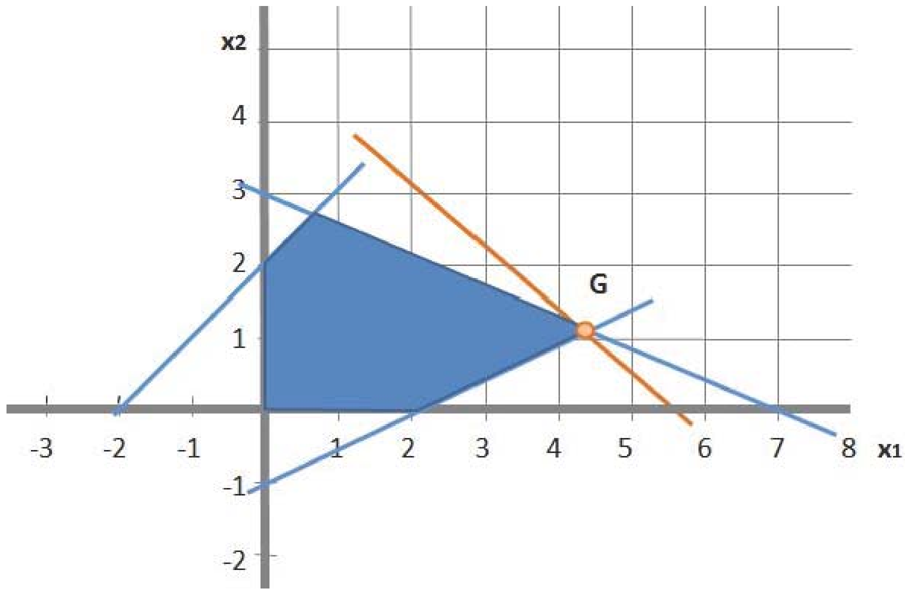
Chapter 3

Linear Programming

3.1 Problem PL I

Consider the Linear Programming problem presented in the following figure, where $F = x_1 + x_2$ is the objective function. The shadow zone represents the feasible solution region and the optimal solution is on point **G**.



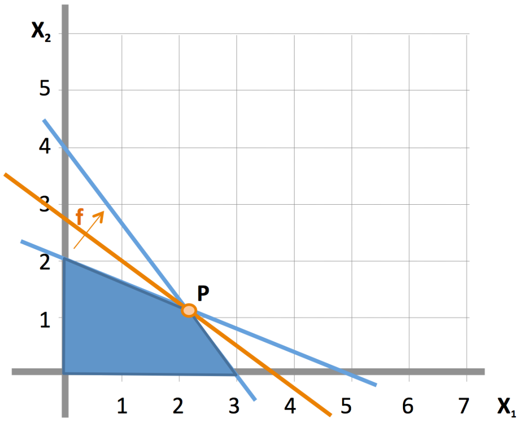
Suppose that the coefficient of x_1 in the objective function is kept constant and equal to 1.

(a) What is the maximum value that the objective function coefficient x_2 can take, so that the solution **G** remains optimum?

(b) And what is the minimum value?

3.2 Problem PL II

Consider the Linear Programming problem presented in the following figure, where $F = 3x_1 + 4x_2$ is the objective function. The shadow zone represents the feasible solution region and the optimal solution is on point **P**.



Suppose that the coefficient of x_2 in the objective function is kept constant and equal to 4.

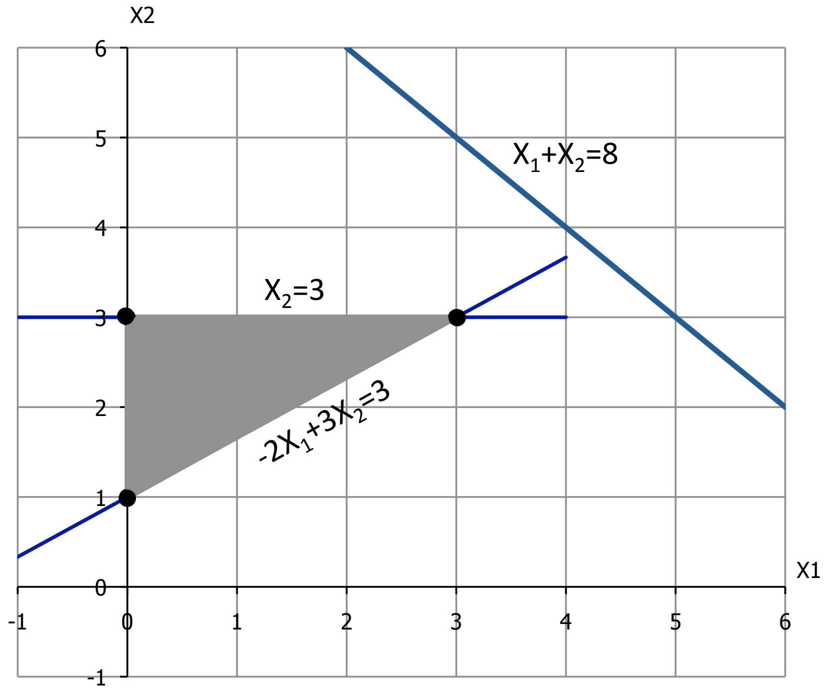
(a) What is the maximum value that the objective function coefficient x_1 can take, so that the solution **P** remains optimum?

(b) And what is the minimum value?

3.3 Problem PL1

Consider the Linear Programming (LP) problem presented in the following figure, where the shadow zone represents the feasible solution region and the objective is:

$$\min X_1 + X_2$$



(a) What is the optimal solution of this problem?

(b) Which of the following constraints are active on the optimal solution?

(i) $X_2 \leq 3$;

(ii) $-2X_1 + 3X_2 \geq 3$;

(iii) $-2X_1 + 3X_2 \leq 3$;

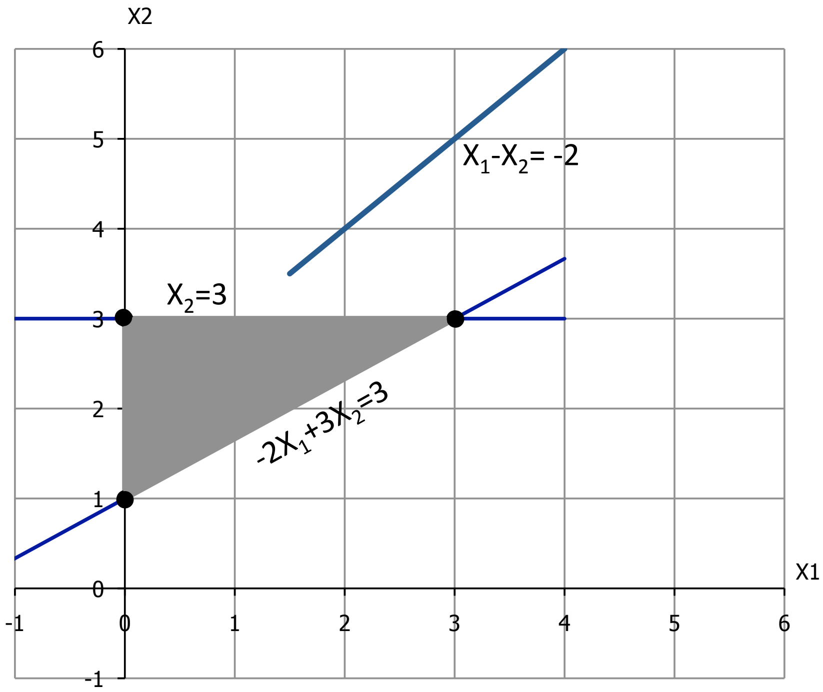
(iv) $X_1 \geq 0$;

(v) $X_2 \geq 0$.

3.4 Problem PL2

Consider the Linear Programming (LP) problem presented in the following figure, where the shadow zone represents the feasible solution region and the objective is:

$$\max -X_1 + X_2$$



(a) What is the optimal solution of this problem?

(b) Which of the following constraints are active on the optimal solution?

(i) $X_2 \leq 3$;

(ii) $-2X_1 + 3X_2 \geq 3$;

(iii) $-2X_1 + 3X_2 \leq 3$;

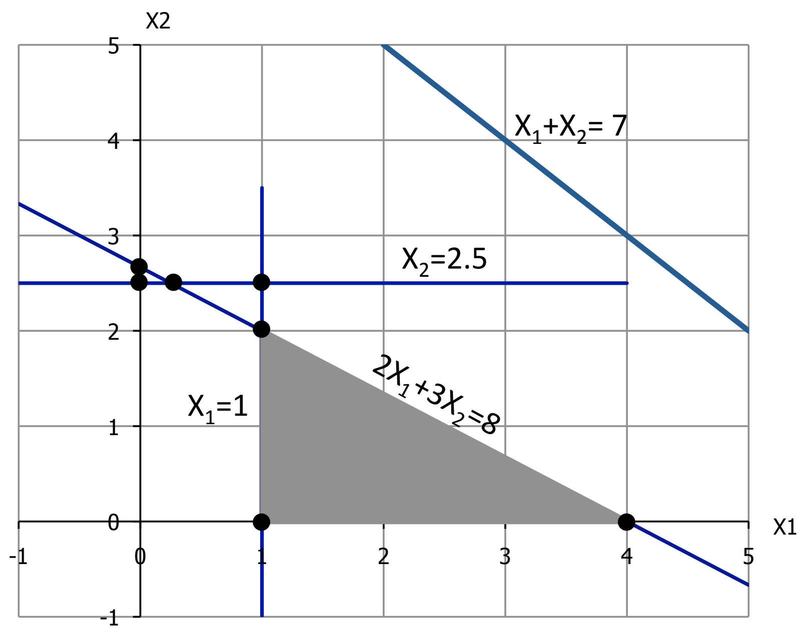
(iv) $X_1 \geq 0$;

(v) $X_2 \geq 0$.

3.5 Problem PL3

Consider the Linear Programming (LP) problem presented in the following figure, where the shadow zone represents the feasible solution region and the objective is:

$$\max X_1 + X_2$$



(a) What is the optimal solution of this problem?

(b) Which of the following constraints are active on the optimal solution?

(i) $X_1 \geq 0$;

(ii) $X_2 \geq 0$;

(iii) $X_1 \geq 1$;

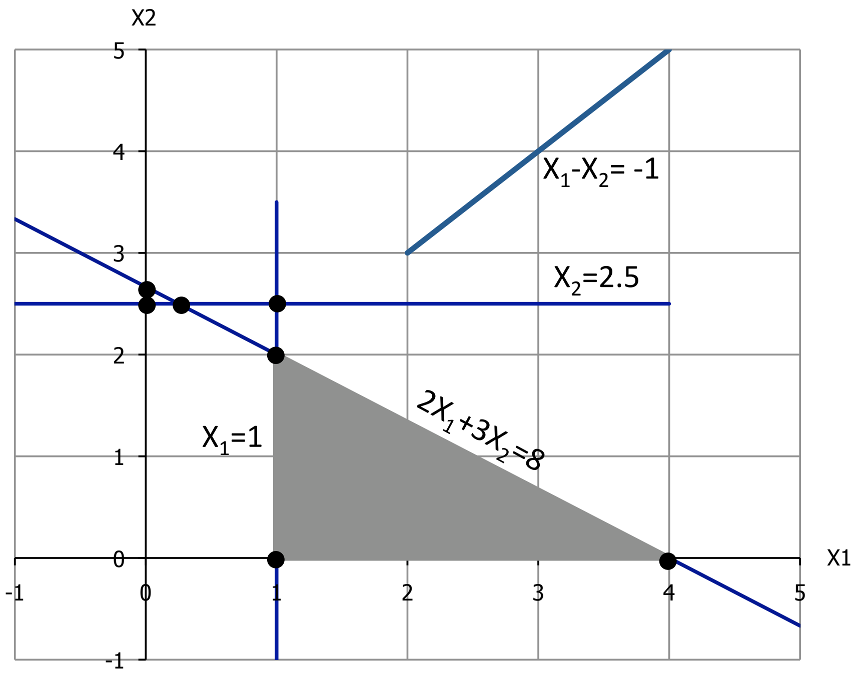
(iv) $X_2 \leq 2.5$;

(v) $2X_1 + 3X_2 \leq 8$.

3.6 Problem PL4

Consider the Linear Programming (LP) problem presented in the following figure, where the shadow zone represents the feasible solution region and the objective is:

$$\max X_1 - X_2$$



(a) What is the optimal solution of this problem?

(b) Which of the following constraints are active on the optimal solution?

(i) $X_1 \geq 0$;

(ii) $X_2 \geq 0$;

(iii) $X_1 \geq 1$;

(iv) $X_2 \leq 2.5$;

(v) $2X_1 + 3X_2 \leq 8$.

Chapter 4

Using Excel solver

	A	B	C	D	E	F	G	H	I
1		Scirocco	Eos	Sharan	Alhambra				
2									
3									
4		4000	3500	4500	5000	0			
5									
6		1	1	1	1	0	<=	200000	
7		0	0	1	1	0	<=	100000	
8		1	1	0	0	0	<=	65000	
9		1	0	0	0	0	<=	100000	
10		0	1	0	0	0	<=	50000	
11		0	0	1	0	0	<=	50000	
12		0	0	0	1	0	<=	20000	
13		22	-2	-4	2	0	<=	0	
14		-1	-1	2/3	2/3	0	>=	0	
15									

Figure 1: Excel model

4.1 Production Planning at VW Autoeuropa

Volkswagen Autoeuropa produces 4 types of vehicles: Scirocco, Eos, Sharan and Alhambra. The capacity of the Palmela plant limits the total output to a maximum of 200,000 units per year. Due to constraints related with the existing production equipment, the total units of both Sharan and Alhambra can not exceed 100,000 units, and the total units of both Scirocco and Eos can not exceed 65,000 units. The marketing strategy of Volkswagen requires the production of minivans (Sharan and Alhambra) to be at least 1.5 times the production Scirocco and Eos models. Volkswagen's environmental protection policy Volkswagen requires that in each of their factories the production mix respects an average CO^2 emission of 150 g/Km. The profits, the selling potential and the CO^2 are in the following table:

Model	Profit (€)	Selling potential	CO^2 emissions (g/Km)
Sharan	4500	50000	146
Alhambra	5000	20000	152
Eos	3500	50000	148
Scirocco	4000	100000	172

Figures 1, 2 and 3 reproduce the Excel model and the corresponding solver reports obtained after finding the optimal solution.

(a) Explain each one of the lines of the Excel.

(b) Use the Answer Report to find out which is the optimal solution and its value. Wich are the binding constraints for this optimal solution?

Célula de Objectivo (Máximo)

Célula	Nome	Valor Original	Valor Final
$F$4		0	493611111,1

Células de Variável

Célula	Nome	Valor Original	Valor Final	Número inteiro
$B$2	Scirocco	0	10555,55556	Contin
$C$2	Eos	0	36111,11111	Contin
$D$2	Sharan	0	50000	Contin
$E$2	Alhambra	0	20000	Contin

Restrições

Célula	Nome	Valor da Célula	Fórmula	Estado	Margem
$F$14		-3,63798E-12	$F$14>=$H$14	Enlace	0
$F$6		116666,6667	$F$6<=$H$6	Sem Enlace	83333,33333
$F$7		70000	$F$7<=$H$7	Sem Enlace	30000
$F$8		46666,66667	$F$8<=$H$8	Sem Enlace	18333,33333
$F$9		10555,55556	$F$9<=$H$9	Sem Enlace	89444,44444
$F$10		36111,11111	$F$10<=$H$10	Sem Enlace	13888,88889
$F$11		50000	$F$11<=$H$11	Enlace	0
$F$12		20000	$F$12<=$H$12	Enlace	0
$F$13		-2,91038E-11	$F$13<=$H$13	Enlace	0

Figure 2: Answer Report of the Solver

Células de Variável

Célula	Nome	Final Valor	Reduzido Custo	Objectivo Coeficiente	Permissível Aumentar	Permissível Diminuir
$B$2	Scirocco	10555,55556	0	4000	263500	500
$C$2	Eos	36111,11111	0	3500	500	3863,636364
$D$2	Sharan	50000	0	4500	1E+30	6944,444444
$E$2	Alhambra	20000	0	5000	1E+30	7319,444444

Restrições

Célula	Nome	Final Valor	Sombra Preço	Restrição Lado Direito	Permissível Aumentar	Permissível Diminuir
$F$14		-3,63798E-12	-3541,666667	0	39393,93939	15151,51515
$F$6		116666,6667	0	200000	1E+30	83333,33333
$F$7		70000	0	100000	1E+30	30000
$F$8		46666,66667	0	65000	1E+30	18333,33333
$F$9		10555,55556	0	100000	1E+30	89444,44444
$F$10		36111,11111	0	50000	1E+30	13888,88889
$F$11		50000	6944,444444	50000	27500	47500
$F$12		20000	7319,444444	20000	20000	20000
$F$13		-2,91038E-11	20,83333333	0	866666,6667	253333,3333

Figure 3: Sensitivity Report of the Solver.

(c) Use the sensitivity report to find out:

(i) If the unit profit of the Eos model would change from 3500 to 2500 €what would be the optimal solution? And what would be the answer if the profit of the Scirocco model would be reduced by the same amount? Detail your answers.

(ii) Through an aggressive discount policy it is possible to increase the selling potential of each one of the models. For which model would the impact be highest? Why? Which could be the maximum price reduction? Detail your answers.

(iii) Interpret the shadow price of the constraint in line 14.

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Chapter 5

Integer Programming

5.1 PIB

The "Branch-and-Bound" algorithm has been used to solve an integer programming problem (minimization) and the following problems have been generated and solved:

	A	B	C	D	E	F	G	H	I
x_1	5	$5\frac{13}{18}$	–	3	$3\frac{1}{2}$	–	5	3	–
x_2	2	$2\frac{4}{9}$	–	3	3	–	$2\frac{5}{8}$	$3\frac{1}{8}$	–
Z	-20	$-23\frac{2}{3}$	WAS	-21	-22	WAS	$-23\frac{1}{8}$	$-21\frac{5}{8}$	WAS

(a) Draw the tree with the problems that corresponds to this resolution. Include in each one of the branches the constraints that has been added.[1]

(b) Which is the optimal solution?

[1] WAS means without admissible solutions

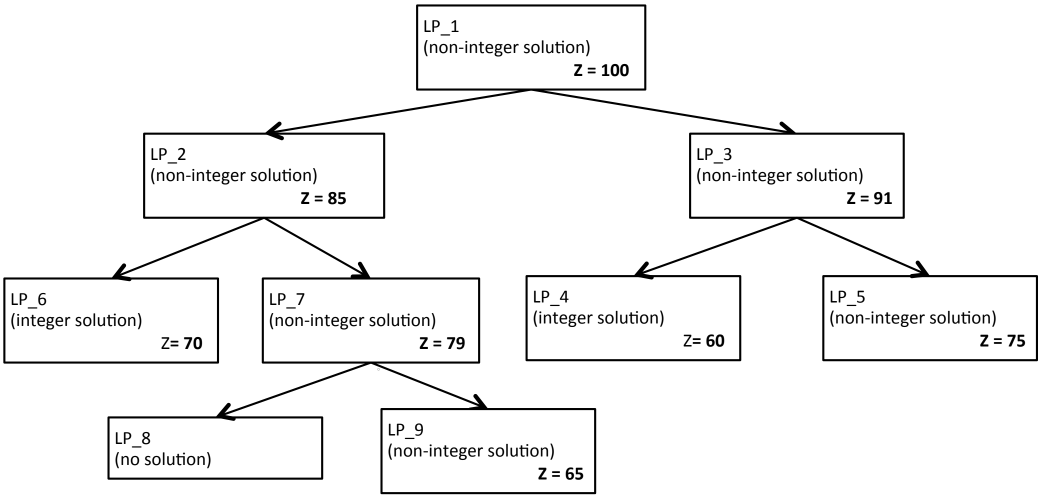

Figure 1: “Branch-and-Bound” tree.

5.2 PIC

Consider a maximization problem where all the variables are integer. The tree in figure 1 is obtained along the resolution process.

(a) Which is now the best **upper bound** on the optimal (integer) solution?

(b) Which is now the best **lower bound** on the optimal (integer) solution?

(c) Which nodes have already been explored? Explain why.

(d) Which nodes have not been explored yet? Explain why.

(e) Do we already know the optimal solution? Explain why.

(f) Which is the maximum gap if the algorithm finishes at this point?

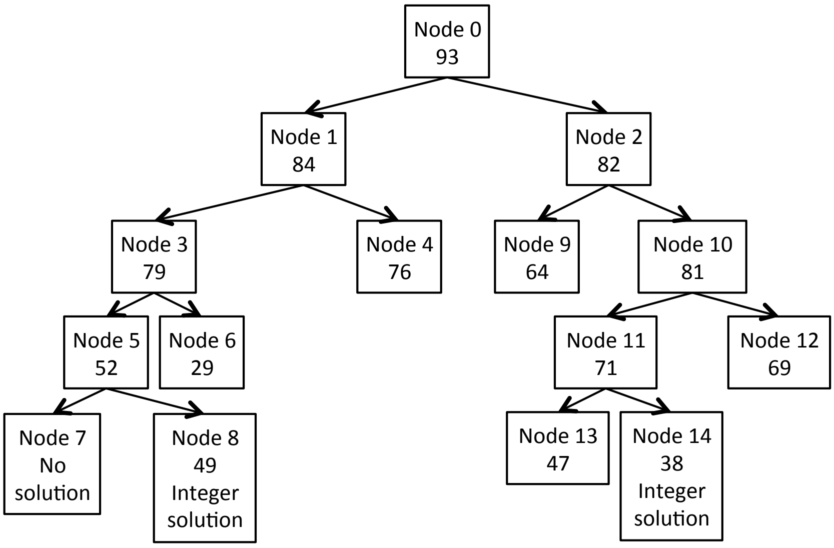

Figure 2: Branch-and-Bound tree.

5.3 PID

The “Branch-and-Bound” in figure 2 has been built for a maximization problem..

The node sequence is represented in the tree, as well as the value of the objective function in each node.

(a) Which information can be extracted from the tree?

(b) Which nodes have to be explored?

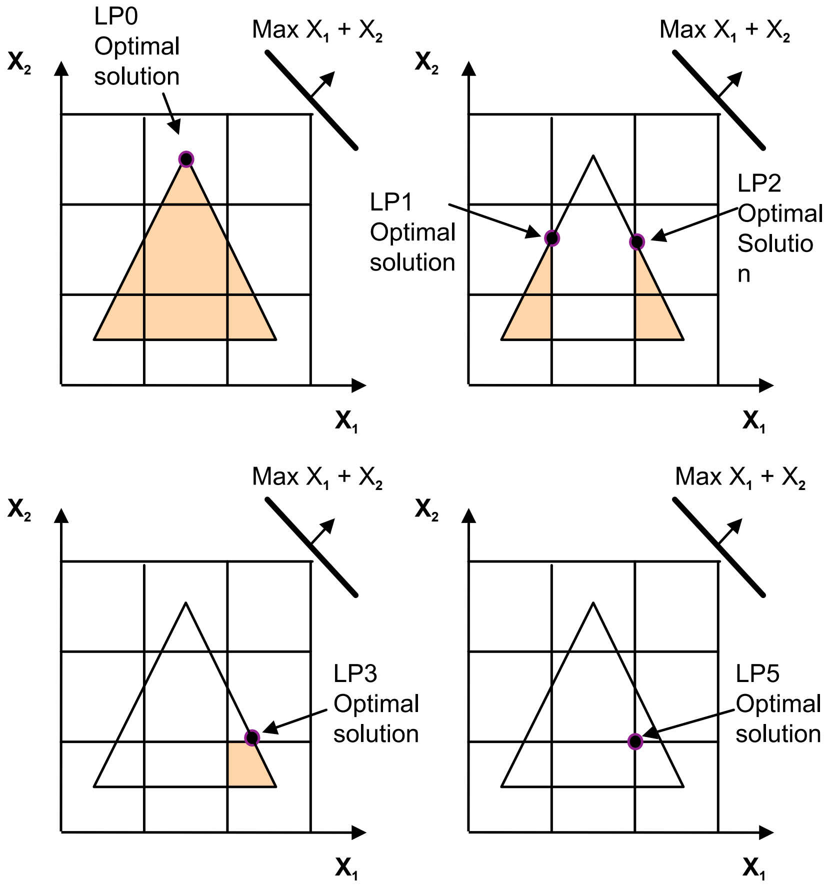

Figure 3: Steps followed in the resolution of the "Branch-and-Bound" of an Integer Programming Problem.

5.4 Triangle

Consider the Integer Programming Problem described in figures 3 and 4.

We want to maximize x_1+x_2, such that x_1 and x_2 belong to the admissible solutions area defined by the $\triangle$. x_1 and x_2 are integer.

Describe the steps that have been followed in the resolution of the problem by interpreting the figures 3 and 4.

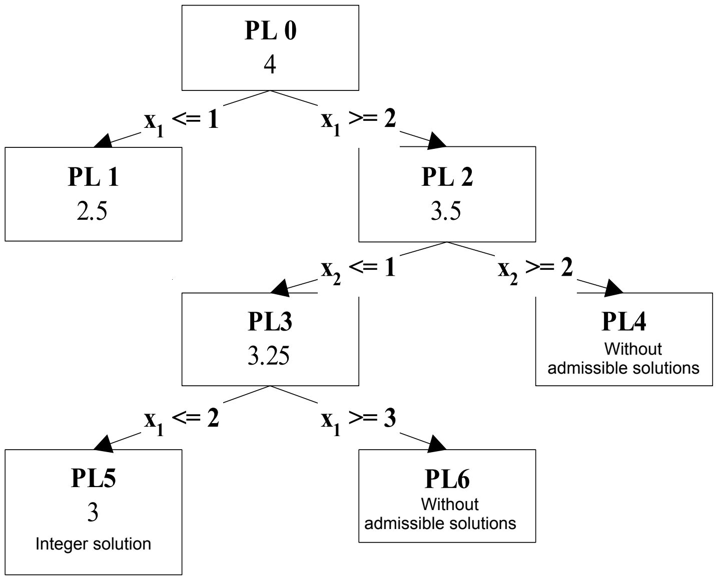

Figure 4: Branch and Bound tree

5.5 PIE

The "Branch-and-Bound" algorithm has been used to solve an integer programming problem and the following subproblems have been generated:

	A	B	C	D	E	F	G	H	I
X_1	0.3	0	1	0	0	0		1.4	1
X_2	3	3	2	2.7	3	3.6		0.8	1.6
X_3	0.45	0.75	0	1	0	0.3		1	0.8
Z	25.8	24	24	22.7	21	26.4	WAS	23.6	24.4

(a) Is this a maximisation or a minimization problem?

(b) Draw the "Branch-and-Bound" tree of this resolution, including the constraints that have been added in each branching.