1 **a** $(x-3)^2 + (y+2)^2 = 25$

b sub. $(x-3)^2 + [(2x-3)+2]^2 = 25$

$(x-3)^2 + (2x-1)^2 = 25$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1, 3$

$\therefore$ $(-1, -5)$ and $(3, 3)$

$AB^2 = 4^2 + 8^2 = 80$

$AB = \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$

2 **a** $= (\frac{-5+3}{2}, \frac{6+8}{2}) = (-1, 7)$

b radius $= \sqrt{16+1} = \sqrt{17}$

$\therefore$ $(x+1)^2 + (y-7)^2 = 17$

c grad of radius $= \dfrac{7-6}{-1-(-5)} = \frac{1}{4}$

$\therefore$ grad of tangent $= -4$

$\therefore$ $y - 6 = -4(x+5)$

$[\ y = -4x - 14\ ]$

3 **a** $(x+4)^2 - 16 + (y-8)^2 - 64 + 62 = 0$

$(x+4)^2 + (y-8)^2 = 18$

$\therefore$ centre $(-4, 8)$ radius $3\sqrt{2}$

b grad of $l = 2$ $\therefore$ grad of perp. $= -\frac{1}{2}$

eqn. of line perp to l through centre:

$y - 8 = -\frac{1}{2}(x+4)$

$y = 6 - \frac{1}{2}x$

intersects l when:

$2x + 1 = 6 - \frac{1}{2}x$

$x = 2$ $\therefore$ $(2, 5)$ is closest point

dist. $(2, 5)$ to centre

$= \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

min. dist. $= 3\sqrt{5} - 3\sqrt{2} = 3(\sqrt{5} - \sqrt{2})$

4 **a** $PQ = \sqrt{1+9} = \sqrt{10}$

radius $= \frac{1}{2}PQ = \frac{1}{2}\sqrt{10}$

b = midpoint of PR

$= (\frac{0+7}{2}, \frac{4+3}{2}) = (\frac{7}{2}, \frac{7}{2})$

c midpoint of $PQ = (\frac{0+1}{2}, \frac{4+1}{2}) = (\frac{1}{2}, \frac{5}{2})$

centre of C_1 = midpoint of $(\frac{1}{2}, \frac{5}{2})$ and $(\frac{7}{2}, \frac{7}{2})$

$= (\frac{\frac{1}{2}+\frac{7}{2}}{2}, \frac{\frac{5}{2}+\frac{7}{2}}{2}) = (2, 3)$

$\therefore$ eqn. of C_1:

$(x-2)^2 + (y-3)^2 = (\frac{1}{2}\sqrt{10}\,)^2$

$x^2 - 4x + 4 + y^2 - 6y + 9 = \frac{5}{2}$

$2x^2 - 8x + 8 + 2y^2 - 12y + 18 = 5$

$2x^2 + 2y^2 - 8x - 12y + 21 = 0$

5 **a** midpoint $AB = (\frac{0+2}{2}, \frac{3+7}{2}) = (1, 5)$

grad $AB = \dfrac{7-3}{2-0} = 2$

$\therefore$ perp. grad $= -\frac{1}{2}$

$\therefore$ $y - 5 = -\frac{1}{2}(x-1)$

$[\ y = \frac{11}{2} - \frac{1}{2}x\ ]$

b circle touches y-axis at $(0, 3)$

$\therefore$ y-coord of centre $= 3$

sub. $3 = \frac{11}{2} - \frac{1}{2}x$

$x = 5$

$\therefore$ centre $(5, 3)$ radius 5

$\therefore$ $(x-5)^2 + (y-3)^2 = 25$

c grad of radius $= \dfrac{7-3}{2-5} = -\frac{4}{3}$

$\therefore$ grad of tangent $= \frac{3}{4}$

$\therefore$ $y - 7 = \frac{3}{4}(x-2)$

$4y - 28 = 3x - 6$

$3x - 4y + 22 = 0$

6 $AP^2 = (x+3)^2 + (y-4)^2$

$BP^2 = x^2 + (y+2)^2$

$AP = 2BP$ $\therefore$ $AP^2 = 4BP^2$

$\therefore$ $(x+3)^2 + (y-4)^2 = 4[x^2 + (y+2)^2]$

$x^2 + 6x + 9 + y^2 - 8y + 16 = 4x^2 + 4y^2 + 16y + 16$

$x^2 - 2x + y^2 + 8y - 3 = 0$

$(x-1)^2 - 1 + (y+4)^2 - 16 - 3 = 0$

$(x-1)^2 + (y+4)^2 = 20$

in form $(x-a)^2 + (y-b)^2 = r^2$ $\therefore$ circle

centre $(1, -4)$ radius $2\sqrt{5}$

7 **a** $= (\frac{-4+(-2)}{2}, \frac{9+(-5)}{2}) = (-3, 2)$

b radius $= \sqrt{1+49} = \sqrt{50}$

$\therefore\ (x+3)^2 + (y-2)^2 = 50$

c sub. (2, 7) into eqn of C:

$(2+3)^2 + (7-2)^2 = 50$

$25 + 25 = 50$

true $\therefore$ R lies on C

d 90°

PQ is a diameter

$\therefore\ \angle PRQ$ is the angle in a semicircle

8 **a** $x^2 + (y-2)^2 - 4 - 16 = 0$

$\therefore$ centre (0, 2)

b C_2: $(x-1)^2 - 1 + (y-4)^2 - 16 - 60 = 0$

$\therefore$ centre (1, 4)

grad $= \frac{4-2}{1-0} = 2$

$\therefore\ y = 2x + 2$

c sub. into eqn of C_1:

$x^2 + [(2x+2) - 2]^2 - 20 = 0$

$x^2 + (2x)^2 - 20 = 0$

$x^2 = 4$

$x = \pm 2$

from diagram, $x = -2$ at P

$\therefore$ P (−2, −2)

l perp to line through centres

$\therefore$ grad $= -\frac{1}{2}$

$\therefore\ y + 2 = -\frac{1}{2}(x+2)$

$[\ y = -\frac{1}{2}x - 3\ ]$

9 **a** $(x-4)^2 - 16 + (y+2)^2 - 4 + 12 = 0$

$(x-4)^2 + (y+2)^2 = 8$

centre (4, −2) radius $2\sqrt{2}$

b dist. P to centre

$= \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$

$\therefore$ max. $PQ = 5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$

min. $PQ = 5\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$

c tangent perp. to radius

$PQ^2 = (5\sqrt{2})^2 - (2\sqrt{2})^2 = 50 - 8 = 42$

$PQ = \sqrt{42} = 6.48$

10 **a** radius $= b$

$\therefore\ (x-a)^2 + (y-b)^2 = b^2$

b sub. $y = x$ into eqn

$(x-a)^2 + (x-b)^2 = b^2$

$x^2 - 2ax + a^2 + x^2 - 2bx + b^2 = b^2$

$2x^2 - 2(a+b)x + a^2 = 0$

tangent $\therefore$ repeated root

$\therefore$ "$b^2 - 4ac$" = 0

$4(a+b)^2 - 8a^2 = 0$

$a^2 - 2ab - b^2 = 0$

$a = \frac{2b \pm \sqrt{4b^2 + 4b^2}}{2} = b \pm \sqrt{2}\,b$

$a > 0, b > 0\ \therefore\ a = (1 + \sqrt{2})b$