Selected Answers

This section contains answers for the odd-numbered problems in each set of Exercises. When a problem has many possible answers, you are given only one sample solution or a hint on how to begin.

CHAPTER 0

LESSON 0.1

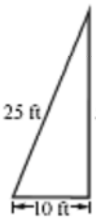
1a. Begin with a 10-liter bucket and a 7-liter bucket. Find a way to get exactly 4 liters in the 10-liter bucket.

1b. Begin with a 10-liter bucket and a 7-liter bucket. Find a way to get exactly 2 liters in the 10-liter bucket.

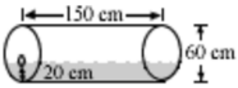
3. one possible answer: (14, 13)

5. *Hint:* Your strategy could include using objects to act out the problem and/or using pictures to show a sequence of steps leading to a solution.

7a.



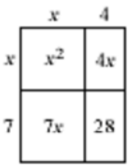
7b.



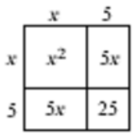
7c.

9. *Hint:* Try using a sequence of pictures similar to those on page 2.

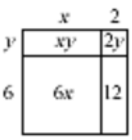
11a. $x^2 + 4x + 7x + 28$



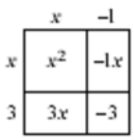
11b. $x^2 + 5x + 5x + 25$



11c. $xy + 2y + 6x + 12$



11d. $x^2 + 3x - x - 3$



13a. $n + 3$, where n represents the number

13b. $v = m + 24.3$, where v represents Venus's distance from the Sun in millions of miles and m represents Mercury's distance from the Sun in millions of miles.

13c. $s = 2e$, where s represents the number of CDs owned by Seth and e represents the number of CDs owned by Erin.

15a. $\frac{375}{1000} = \frac{3}{8}$ **15b.** $\frac{142}{100} = \frac{71}{50} = 1\frac{21}{50}$

15c. $\frac{2}{9}$ **15d.** $\frac{35}{99}$

LESSON 0.2

1a. Subtract 12 from both sides.
1b. Divide both sides by 5.
1c. Add 18 to both sides.
1d. Multiply both sides by – 15.

3a. $c = 27$ **3b.** $c = 5.8$ **3c.** $c = 9$
5a. $x = 72$ **5b.** $x = 24$ **5c.** $x = 36$

7a. $-12L - 40S = -540$
7b. $12L + 75S = 855$
7c. $35S = 315$
7d. $S = 9$. The small beads cost 9¢ each.
7e. $L = 15$. The large beads cost 15¢ each.
7f. $J = 264$. Jill will pay $2.64 for her beads.

9a. Solve Equation 1 for a. Substitute the result, $5b - 42$, for a in Equation 2 to get $b + 5 = 7((5b - 42) - 5)$.

9b. $b = \frac{167}{17}$ **9c.** $a = \frac{121}{17}$

9d. A has $\frac{121}{17}$ or about 7 denarii, and B has $\frac{167}{17}$ or about 10 denarii.

11a. Draw a 45° angle, then subtract a 30° angle.
11b. Draw a 45° angle, then add a 30° angle.
11c. Draw a 45° angle, then add a 60° angle.

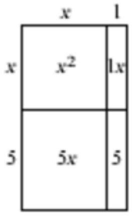
13a. 98 **13b.** –273

15. *Hint:* Try using a sequence of pictures similar to those on page 2. Also be sure to convert all measurements to cups.

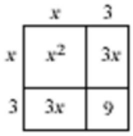
LESSON 0.3

1a. approximately 4.3 s
1b. 762 cm **1c.** 480 mi
3. 150 mi/h
5a. $a = 12.8$ **5b.** $b = \frac{4}{3} = 1.\overline{3}$
5c. $c = 10$ **5d.** $d = 8$

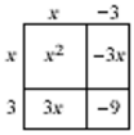
7a. 54 in.^{2} **7b.** 1.44 m^3
7c. 1.20 ft **7d.** 24 cm
9a. Equation iii. Explanations will vary.
9b. i. $t \approx 0.03$ **9b. ii.** $t \approx 0.03$ **9b. iii.** $t \approx 8.57$
9c. It would take approximately 9 minutes.
11a. r^{12} **11b.** $\frac{s^4}{3}$
11c. t^{-4} or $\frac{1}{t^4}$ **11d.** $48u^8$
13a. $x^2 + 1x + 5x + 5$



13b. $x^2 + 3x + 3x + 9$



13c. $x^2 + 3x - 3x - 9$



CHAPTER 0 REVIEW

1. *Hint:* Try using a sequence of pictures similar to those on page 2.
3a. $x = \sqrt{18}$ cm $= 3\sqrt{2}$ cm ≈ 4.2 cm
3b. $y = 5$ in.
5a. $x = 13$ **5b.** $y = -2.5$
7a. $c = 19.95 + 0.35m$
7b. possible answer: \$61.25
7c. \$8.40
9. 17 years old
11a. $h = 0$. Before the ball is hit, it is on the ground.
11b. $h = 32$. Two seconds after being hit, the ball is 32 feet above the ground.
11c. $h = 0$. After three seconds, the ball lands on the ground.
13a. $y = 1$ **13b.** $y = 8$
13c. $y = \frac{1}{4}$ **13d.** $x = 5$
15. *Hint:* Mr. Mendoza is meeting with Mr. Green in the conference room at 9:00 A.M.

LESSON 1.1

1a. 20, 26, 32, 38 **1b.** 47, 44, 41, 38
1c. 32, 48, 72, 108 **1d.** −18, −13.7, −9.4, −5.1
3. $u_1 = 40$ and $u_n = u_{n-1} - 3.45$ where $n \geq 2$; $u_5 = 26.2$; $u_9 = 12.4$
5a. $u_1 = 2$ and $u_n = u_{n-1} + 4$ where $n \geq 2$; $u_{15} = 58$
5b. $u_1 = 10$ and $u_n = u_{n-1} - 5$ where $n \geq 2$; $u_{12} = -45$
5c. $u_1 = 0.4$ and $u_n = 0.1 \cdot u_{n-1}$ where $n \geq 2$; $u_{10} = 0.0000000004$
5d. $u_1 = -2$ and $u_n = u_{n-1} - 6$ where $n \geq 2$; $u_{30} = -176$
5e. $u_1 = 1.56$ and $u_n = u_{n-1} + 3.29$ where $n \geq 2$; $u_{14} = 44.33$
5f. $u_1 = -6.24$ and $u_n = u_{n-1} + 2.21$ where $n \geq 2$; $u_{20} = 35.75$
7. $u_1 = 4$ and $u_n = u_{n-1} + 6$ where $n \geq 2$; $u_4 = 22$; $u_5 = 28$; $u_{12} = 70$; $u_{32} = 190$
9a. 399 km
9b. 10 hours after the first car starts, or 8 hours after the second car starts
11a. \$60 **11b.** \$33.75
11c. during the ninth week
13. *Hint:* Construct two intersecting lines, and then construct several lines that are perpendicular to one of the lines and equally spaced from each other starting from the point of intersection.
15a. $\frac{70}{100} = \frac{a}{65}$; $a = 45.5$
15b. $\frac{115}{100} = \frac{b}{37}$; $b = 42.55$
15c. $\frac{c}{100} = \frac{110}{90}$; $c \approx 122.2\%$
15d. $\frac{d}{100} = \frac{0.5}{18}$; $d \approx 2.78\%$
17. the 7% offer at \$417.30 per week

LESSON 1.2

1a. 1.5 **1b.** 0.4 **1c.** 1.03 **1d.** 0.92
3a. $u_1 = 100$ and $u_n = 1.5u_{n-1}$ where $n \geq 2$; $u_{10} = 3844.3$
3b. $u_1 = 73.4375$ and $u_n = 0.4u_{n-1}$ where $n \geq 2$; $u_{10} = 0.020$
3c. $u_1 = 80$ and $u_n = 1.03u_{n-1}$ where $n \geq 2$; $u_{10} = 104.38$

3d. $u_1 = 208$ and $u_n = 0.92u_{n-1}$ where $n \geq 2$; $u_{10} = 98.21$

5a. $(1 + 0.07)u_{n-1}$ or $1.07u_{n-1}$

5b. $(1 - 0.18)A$ or $0.82A$

5c. $(1 + 0.08125)x$ or $1.08125x$

5d. $(2 - 0.85)u_{n-1}$ or $1.15u_{n-1}$

7. 100 is the initial height, but the units are unknown. 0.20 is the percent loss, so the ball loses 20% of its height with each rebound.

9a. number of new hires for next five years: 2, 3, 3 (or 4), 4, and 5

9b. about 30 employees

11. $u_0 = 1$ and $u_n = 0.8855u_{n-1}$ where $n \geq 1$
$u_{25} = 0.048$, or 4.8%. It would take about 25,000 years to reduce to 5%.

13a. 0.542% **13b.** \$502.71

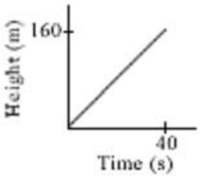
13c. \$533.49 **13d.** \$584.80

15a. 3 **15b.** 2, 6,..., 54,... , 486, 1458,..., 13122

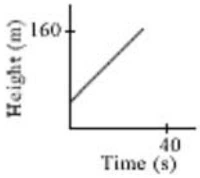
15c. 118,098

17a. 4 m/s **17b.** 10 s

17c.



17d.



17e.

19a. $x \approx 43.34$ **19b.** $x = -681.5$

19c. $x \approx 0.853$ **19d.** $x = 8$

LESSON 1.3

1a. 31.2, 45.64, 59.358; shifted geometric, increasing

1b. 776, 753.2, 731.54; shifted geometric, decreasing

1c. 45, 40.5, 36.45; geometric, decreasing

1d. 40, 40, 40; arithmetic or shifted geometric, neither increasing nor decreasing

3a. 320 **3b.** 320 **3c.** 0 **3d.** 40

5a. The first day, 300 grams of chlorine were added. Each day, 15% disappears, and 30 more grams are added.

5b. It levels off at 200 g.

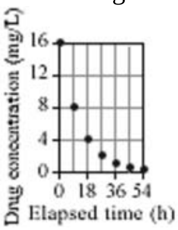
7a. The account balance will continue to decrease (slowly at first, but faster after a while). It does not level off, but it eventually reaches 0 and stops decreasing.

7b. \$68

9. $u_0 = 20$ and $u_n = (1 - 0.25)u_{n-1}$ where $n \geq 1$; 11 days

11a. Sample answer: After 9 hours there are only 8 mg, after 18 hours there are 4 mg, after 27 hours there are still 2 mg left.

11b.



11c. 8 mg

13a. $u_2 = -96$, $u_5 = 240$ **13b.** $u_2 = 2$, $u_5 = 1024$

15. 23 times

LESSON 1.4

1a. 0 to 9 for n and 0 to 16 for u_n

1b. 0 to 19 for n and 0 to 400 for u_n

1c. 0 to 29 for n and –178 to 25 for u_n

1d. 0 to 69 for n and 0 to 3037 for u_n

3a. geometric, nonlinear, decreasing

3b. arithmetic, linear, decreasing

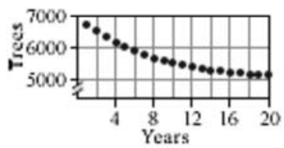
3c. geometric, nonlinear, increasing

3d. arithmetic, linear, increasing

5. i. C **5. ii.** B **5. iii.** A

7. The graph of an arithmetic sequence is always linear. The graph increases when the common difference is positive and decreases when the common difference is negative. The steepness of the graph relates to the common difference.

9a.



9b. The graph appears to have a long-run value of 5000 trees, which agrees with the long-run value found in Exercise 8b in Lesson 1.3.

11. possible answer: $u_{50} = 40$ and $u_n = u_{n-1} + 4$ where $n \geq 51$

13a. 547.5, 620.6, 675.5, 716.6, 747.5

13b. $\frac{547.5 - 210}{0.75} = 450$; subtract 210 and divide the difference by 0.75.

13c. $u_0 = 747.5$ and $u_n = \dfrac{u_{n-1} - 210}{0.75}$ where $n \geq 1$

15a. $33\frac{1}{3}$ **15b.** $66\frac{2}{3}$ **15c.** 100

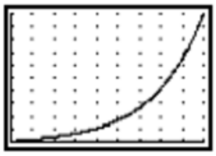
15d. The long-run value grows in proportion to the added constant. $7 \cdot 33\frac{1}{3} = 233\frac{1}{3}$

LESSON 1.5

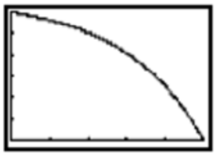
1a. investment, because a deposit is added
1b. \$450 **1c.** \$50 **1d.** 3.9%
1e. annually (once a year)
3a. \$130.67 **3b.** \$157.33 **3c.** \$184.00 **3d.** \$210.67
5. \$588.09
7a. \$1877.14 **7b.** \$1912.18 **7c.** \$1915.43

7d. The more frequently the interest is compounded, the more quickly the balance will grow.

9a. \$123.98
9b. for $u_0 = 5000$ and
$u_n = \left(1 + \frac{0.085}{12}\right)u_{n-1} + 123.98$ where $n \geq 1$



[0, 540, 60, 0, 900000, 100000]
11a. \$528.39
11b. for $u_0 = 60000$ and
$u_n = \left(1 + \frac{0.096}{12}\right)u_{n-1} - 528.39$ where $n \geq 1$

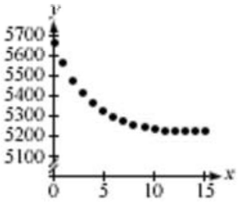


[0, 300, 60, 0, 60000, 10000]
13. something else
15a. 30.48 cm **15b.** 320 km **15c.** 129.64 m

CHAPTER 1 REVIEW

1a. geometric
1b. $u_1 = 256$ and $u_n = 0.75u_{n-1}$ where $n \geq 2$
1c. $u_8 \approx 34.2$ **1d.** $u_{10} \approx 19.2$ **1e.** $u_{17} \approx 2.57$
3a. –3, –1.5, 0, 1.5, 3; 0 to 6 for n and –4 to 4 for u_n
3b. 2, 4, 10, 28, 82; 0 to 6 for n and 0 to 100 for u_n

5. i. C **5. ii.** D **5. iii.** B **5. iv.** A
7. approximately 5300; approximately 5200; $u_0 = 5678$ and $u_n = (1 - 0.24)u_{n-1} + 1250$ where $n \geq 1$

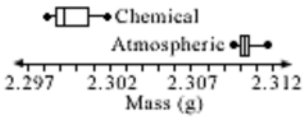


9. $u_{1970} = 34$ and $u_n = (1 + 0.075)u_{n-1}$ where $n \geq 1971$

CHAPTER 2

LESSON 2.1

1a. mean: 29.2 min; median: 28 min; mode: 26 min
1b. mean: 17.35 cm; median: 17.95 cm; mode: 17.4 cm
1c. mean: \$2.38; median: \$2.38; mode: none
1d. mean: 2; median: 2; modes: 1 and 3
3. minimum: 1.25 days; first quartile: 2.5 days; median: 3.25 days; third quartile: 4 days; maximum: 4.75 days
5. D
7. *Hint:* Consider the definitions of each of the values in the five-number summary.
9a. Connie: *range* = 4, *IQR* = 3; Oscar: *range* = 24, *IQR* = 18
9b. *range* = 47; *IQR* = 14
11. *Hint:* Choose three values above 65 and three values below 65.
13a. juniors: $\bar{x} \approx 12.3$ lb; seniors: $\bar{x} \approx 8.6$ lb
13b. juniors: *median* = 10 lb; seniors: *median* = 8 lb
13c. Each mean is greater than the corresponding median.
15a. chemical: 2.29816, 2.29869, 2.29915, 2.30074, 2.30182; atmospheric: 2.30956, 2.309935, 2.31010, 2.3026, 2.31163
15b.



15c. *Hint:* Compare the range, *IQR*, and how the data are skewed. If you conclude that the data are significantly different, then Rayleigh's conjecture is supported.
17a. $6\sqrt{2} \approx 8.5$ **17b.** $\sqrt{89} \approx 9.4$

17c. $\frac{\sqrt{367}}{2} \approx 9.6$

19a. $x = 7$ **19b.** $x = 5$

19c. $\bar{x} = \frac{7}{3} = 2.\bar{3}$

LESSON 2.2

1a. 47.0 **1b.** –6, 8, 1, –3 **1c.** 6.1

3a. 9, 10, 14, 17, 21 **3b.** *range* = 12; *IQR* = 7

3c. centimeters

5. *Hint:* The number in the middle is 84. Choose three numbers on either side that also have a mean of 84, and check that the other criteria are satisfied. Adjust data values as necessary.

7. 20.8 and 22.1. These are the same outliers found by the interquartile range.

9a. *Hint:* The two box plots must have the same endpoints and *IQR*. The data that is skewed left should have a median value to the right of the center.

9b. The skewed data set will have a greater standard deviation because the data to the left (below the median) will be spread farther from the mean.

9c. *Hint:* The highest and lowest values for each set must be equal, and the skewed data will have a higher median value.

9d. Answers will vary, depending on 9c, but should support 9b.

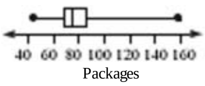
11a. First period appears to have pulse rates most alike because that class had the smallest standard deviation.

11b. Sixth period might have the fastest pulse rates because that class has both the highest mean and the greatest standard deviation.

13a. *median* = 75 packages; *IQR* = 19 packages

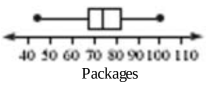
13b. $\bar{x} \approx 80.9$ packages; $s \approx 24.6$ packages

13c. Hot Chocolate Mix



five-number summary: 44, 67.5, 75, 86.5, 158; outliers: 147, 158

13d. Hot Chocolate Mix



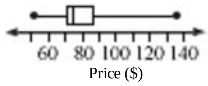
five-number summary: 44, 67, 74, 82, 100

13e. *median* = 74 packages; *IQR* = 15 packages; $\bar{x} \approx 74.7$ packages; $s \approx 12.4$ packages

13f. The mean and standard deviation are calculated from all data values, so outliers affect these statistics significantly. The median and *IQR*, in contrast, are defined by position and not greatly affected by outliers.

15a. mean: \$80.52; median: \$75.00; modes: \$71.00, \$74.00, \$76.00, \$102.50

15b. CD Players



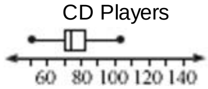
five-number summary: 51, 71, 75, 87, 135.5

The box plot is skewed right.

15c. *IQR* = \$16; outliers: \$112.50 and \$135.50

15d. The median will be less affected because the relative positions of the middle numbers will be changed less than the sum of the numbers.

15e. CD Players



five-number summary: 51, 71, 74, 82.87, 102.5 The median of the new data set is \$74.00 and is relatively unchanged.

15f. *Hint:* Consider whether your decision should be based on the data with or without outliers included. Decide upon a reasonable first bid, and the maximum you would pay.

17a. $x = 59$ **17b.** $y = 20$

LESSON 2.3

1a. 2 **1b.** 9

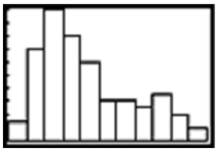
1c. *Hint:* Choose values that reflect the number of backpacks within each bin.

3a. 5 values

3b. 25th percentile **3c.** 95th percentile

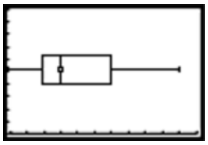
5a. The numbers of acres planted by farmers who plant more than the median number of acres vary more than the numbers of acres planted by farmers who plant fewer than the median number of acres.

5b.



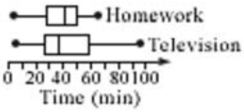
[0, 11, 1, 0, 20, 2]

5c. In a box plot, the part of the box to the left of the median would be smaller than the part to the right because there are more values close to 3 on the left than on the right.



[0, 11, 1, 0, 20, 2]

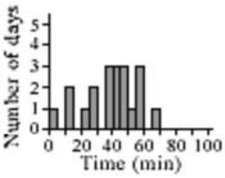
7a.



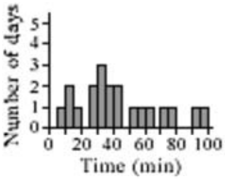
Television has the greater spread.

7b. Television will be skewed right. Neither will be mound shaped.

Homework

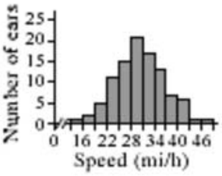


Television



7c. Homework: *median* = 40.5 min; *IQR* = 21.5 min;
$\bar{x} \approx 38.4$ min; $s \approx 16.7$ min.
Television: *median* = 36.5 min; *IQR* = 32 min;
$\bar{x} \approx 42.2$ min; $s \approx 26.0$ min. Answers will vary.

9a. Speed Limit Study



9b. between 37 mi/h and 39 mi/h

9c. possible answer: 35 mi/h

9d. Answers will vary.

11a. The sum of the deviations is 13, not 0.

11b. 20

11c. i. {747, 707, 669, 676, 767, 783, 789, 838}; $s \approx 59.1$; *median* = 757; *IQR* = 94.5

11c. ii. {850, 810, 772, 779, 870, 886, 892, 941}; $s \approx 59.1$; *median* = 860; *IQR* = 94.5

11d. *Hint:* How does translating the data affect the standard deviation and *IQR*?

13. Marissa runs faster.

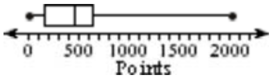
CHAPTER 2 REVIEW

1. Plot B has the greater standard deviation, because the data have more spread.

3a. mean: 553.6 points; median: 460 points; mode: none

3b. 5,167, 460, 645, 2019

3c. Points Scored by Los Angeles Lakers Players (2001–2002 Season)



skewed right

3d. 478 points

3e. Kobe Bryant (2019 points) and Shaquille O'Neal (1822 points)

5a. $\bar{x} \approx 118.3$°F; $s \approx 26.8$°F

5b. $\bar{x} \approx -60.0$°F; $s \approx 45.7$°F

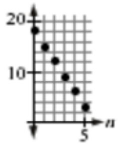
5c. Antarctica (59°F) is an outlier for the high temperatures. There are no outliers for the low temperatures.

7. Answers will vary. In general, the theory is supported by the statistics and graphs.

CHAPTER 3 • CHAPTER **3** CHAPTER 3 • CHAPTER

LESSON 3.1

1a.



1b. –3; The common difference is the same as the slope.

1c. 18; The y-intercept is the u_0-term of the sequence.

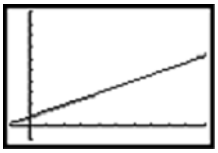
1d. $y = 18 - 3x$

3. $y = 7 + 3x$

5a. 1.7 **5b.** 1 **5c.** –4.5 **5d.** 0

7a. 190 mi **7b.** $y = 82 + 54x$

7c.



[–1, 10, 1, –100, 1000, 100]

7d. Yes, if only distances on the hour are considered. A line is continuous, whereas an arithmetic sequence is discrete.

Selected Answers

9a. $u_0 = -2$ **9b.** 5 **9c.** 50

9d. because you need to add 50 d's to the original height of u_0

9e. $u_n = u_0 + nd$

11a. *Hint:* The x-values must have a difference of 5.

11b. Graphs will vary; 4

11c. 7, 11, 15, 19, 23, 27

11d. *Hint:* The linear equation will have slope of 4. Find the y-intercept.

13. Although the total earnings are different at the end of the odd-numbered six-month periods, the total yearly income is always the same.

15a. \$93.49; \$96.80; \$7.55

15b. The median and mean prices indicate the mid-price and average price, respectively. The standard deviation indicates the amount of variation in prices. The median tells trend of prices better than the mean, which can be affected by outliers.

LESSON 3.2

1a. $\frac{3}{2} = 1.5$ **1b.** $-\frac{2}{3} \approx -0.67$ **1c.** –55

3a. $y = 14.3$ **3b.** $x = 6.5625$

3c. $a = -24$ **3d.** $b = -0.25$

5a. The equations have the same constant, –2. The lines share the same y-intercept. The lines are perpendicular, and their slopes are reciprocals with opposite signs.

5b. The equations have the same x-coefficient, –1.5. The lines have the same slope. The lines are parallel.

7a. Answer depends on data points used. Approximately 1.47 volts/battery.

7b. Answers will vary. The voltage increases by about 1.47 volts for every additional battery.

7c. Yes. There is no voltage produced from zero batteries, so the y-intercept should be 0.

9a. \$20, 497; \$17, 109 **9b.** \$847 per year

9c. in her 17th year

11a. Answer depends on data points used. Each additional ticket sold brings in about \$7.62 more in revenue.

11b. Answers will vary. Use points that are not too close together.

13a. $-10 + 3x$ **13b.** $1 - 11x$ **13c.** $28.59 + 5.4x$

15a. 71.7 beats/min

15b. 6.47 beats/min. The majority of the data falls within 6.47 beats/min of the mean.

17a. $a > 0, b < 0$ **17b.** $a < 0, b > 0$

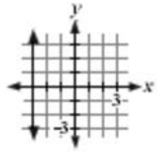
17c. $a > 0, b = 0$ **17d.** $a = 0, b < 0$

LESSON 3.3

1a. $y = 1 + \frac{2}{3}(x - 4)$ **1b.** $y = 2 - \frac{1}{5}(x - 1)$

3a. $u_n = 31$ **3b.** $t = -41.5$ **3c.** $x = 1.5$

5a.



possible answer: (–3, 0), (–3, 2)

5b. undefined **5c.** $x = 3$

5d. *Hint:* What can you say about the slope and the x- and y-intercepts of a vertical line?

7a. The y-intercept is about 1.7; (5, 4.6); $\hat{y} = 1.7 + 0.58x$.

7b. The y-intercept is about 7.5; (5, 3.75); $\hat{y} = 7.5 - 0.75x$.

7c. The y-intercept is about 8.6; (5, 3.9); $\hat{y} = 8.6 - 0.94x$.

9a. possible answer: [145, 200, 5, 40, 52, 1]

9b. possible answer: $\hat{y} = 0.26x + 0.71$

9c. On average, a student's forearm length increases by 0.26 cm for each additional 1 cm of height.

9d. The y-intercept is meaningless because a height of 0 cm should not predict a forearm length of 0.71 cm. The domain should be specified.

9e. 189.58 cm; 41.79 cm

11. 102

13a. 16

13b. Add 19.5 and any three numbers greater than 19.5.

LESSON 3.4

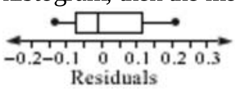
1a. 17, 17, 17 **1b.** 17, 16, 17

1c. 16, 15, 16 **1d.** 13, 12, 13

3. $y = 0.9 + 0.75(x - 14.4)$

5. $y = 3.15 + 4.7x$

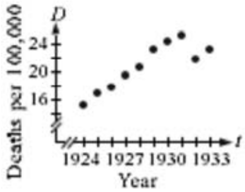
7. Answers will vary. If the residuals are small and have no pattern, as shown by the box plot and histogram, then the model is good.



9a. $\hat{y} = 997.12 - 0.389x$

9b. The world record for the 1-mile run is reduced by 0.389 s every year.

9c. 3:57.014. This prediction is 2.3 s faster than Roger Bannister actually ran.
9d. 4:27.745. This prediction is about 3.2 s slower than Walter Slade actually ran.
9e. This suggests that a world record for the mile in the year 0 would have been about 16.6 minutes. This is doubtful because a fast walker can walk a mile in about 15 minutes. The data are only approximately linear and only over a limited domain.
9f. A record of 3:43.13, 3.61 s slower than predicted was set by Hicham El Guerrouj in 1999.
11a.

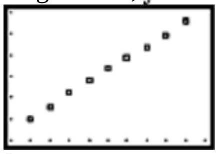


11b. $M_1(1925, 17.1)$, $M_2(1928.5, 22.05)$, $M_3(1932, 23.3)$
11c. $\hat{D} = -1687.83 + 0.886t$
11d. For each additional year, the number of deaths by automobile increases by 0.886 per hundred thousand population.
11e. Answers might include the fact that the United States was in the Great Depression and fewer people were driving.
11f. It probably would not be a good idea to extrapolate because a lot has changed in the automotive industry in the past 75 years. Many safety features
are now standard.
13. $y = 7 - 3x$
15. 2.3 g, 3.0 g, 3.0 g, 3.4 g, 3.6 g, 3.9 g

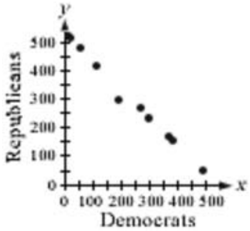
LESSON 3.5

1a. – 0.2 **1b.** – 0.4 **1c.** 0.6
3a. –2.74; –1.2; –0.56; –0.42; –0.18; 0.66; 2.3; 1.74; 0.98; 0.02; –0.84; –0.3; –0.26; –0.22; –0.78; –1.24; –0.536
3b. 1.22 yr
3c. In general, the life expectancy values predicted by the median-median line will be within 1.23 yr of the actual data values.

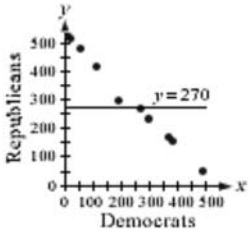
5a. Let x represent age in years, and let y represent height in cm; $\hat{y} = 82.5 + 5.6x$.



[4, 14, 1, 100, 160, 10]
5b. –1.3, –0.4, 0.3, 0.8, 0.8, 0.3, –0.4, –0.4, 1.1
5c. 0.83 cm
5d. In general, the mean height of boys ages 5 to 13 will be within 0.83 cm of the values predicted by the median-median line.
5e. between 165.7 cm and 167.3 cm
7. $\hat{y} = 29.8 + 2.4x$
9. Alex's method: 3.67; root mean square error method: approximately 3.21. Both methods give answers around 3, so Alex's method could be used as an alternate measure of accuracy.
11a.



11b. The points are nearly linear because the sum of electoral votes should be 538. The data are not perfectly linear because in a few of the elections, candidates other than the Democrats and Republicans received some electoral votes.
11c.

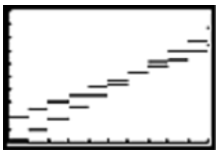


The points above the line are the elections in which the Republican Party's presidential candidate won.
11d. –218, 31, 250, –30, 219, 255, 156, –102, –111, 1
A negative residual means that the Democratic Party's presidential candidate won.
11e. a close election
13. *Hint:* The difference between the 2nd and 6th values is 12.

15a. $u_0 = 30$ and $u_n = u_{n-1}\left(1+\frac{0.07}{12}\right) + 30$ where $n \geq 1$
15b. i. deposited: \$360; interest: \$11.78
15b. ii. deposited: \$3,600; interest: \$1,592.54
15b. iii. deposited: \$9,000; interest: \$15,302.15
15b. iv. deposited: \$18,000; interest: \$145,442.13
15c. Sample answer: If you earn compound interest, in the long run the interest earned will far exceed the total amount deposited.

LESSON 3.6

1a. (1.8, –11.6) **1b.** (3.7, 31.9)
3. $y = 5 + 0.4(x - 1)$
5a. (4.125, -10.625) **5b.** (–3.16, 8.27)
5c. They intersect at every point; they are the same line.
7a. No. At $x = 25$, the cost line is above the income line.
7b. Yes. The profit is approximately \$120.
7c. About 120 pogo sticks. Look for the point where the cost and income lines intersect.
9a. Phrequent Phoner Plan: $y = 20 + 17([x] - 1)$; Small Business Plan: $y = 50 + 11([x] - 1)$
9b.

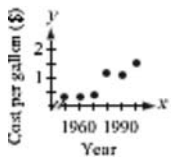


[0, 10, 1, 0, 200, 20]

9c. If the time of the phone call is less than 6 min, PPP is less expensive. For times between 6 and 7 min, the plans charge the same rate. If the time of the phone call is greater than or equal to 7 min, PPP is more expensive than SBP. (You could look at the calculator table to see these results.)
11a. Let l represent length in centimeters, and let w represent width in centimeters; $2l + 2w = 44$, $l = 2 + 2w$; $w = \frac{20}{3}$ cm, $l = \frac{46}{3}$ cm.
11b. Let l represent length of leg in centimeters, and let b represent length of base in centimeters; $2l + b = 40$, $b = l - 2$; $l = 14$ cm, $b = 12$ cm.
11c. Let f represent temperature in degrees Fahrenheit, and let c represent temperature in degrees Celsius; $f = 3c - 0.4$, $f = 1.8c + 32$; $c = 27$°C, $f = 80.6$°F.
13a. 51 **13b.** 3rd bin **13c.** 35%

LESSON 3.7

1a. $w = 11 + r$ **1b.** $h = \frac{18-2p}{3} = 6 - \frac{2}{3}p$
1c. $r = w - 11$ **1d.** $p = \frac{18-3h}{2} = 9 - \frac{3}{2}p$
3a. $5x - 2y = 12$; passes through the point of intersection of the original pair
3b. $-4y = 8$; passes through the point of intersection of the original pair and is horizontal
5a. $\left(-\frac{97}{182}, \frac{19}{7}\right) \approx (-0.5330, 2.7143)$
5b. $\left(8, -\frac{5}{2}\right) \approx (8, -2.5)$
5c. $\left(\frac{186}{59}, -\frac{4}{59}\right) \approx (3.1525, -0.0678)$
5d. $n = 26$, $s = -71$ **5e.** $d = -18$, $f = -49$
5f. $\left(\frac{44}{7}, -\frac{95}{14}\right) \approx (6.2857, -6.7857)$
5g. no solution
7. 80°F
9. *Hint:* Write two equations that pass through the point (–1.4, 3.6).
11a. $A = \frac{d^2}{2}$ **11b.** $P = I^2R$ **11c.** $A = \frac{C^2}{4\pi}$
13a. true **13b.** false; $(x - 4)(x + 4)$
15a.

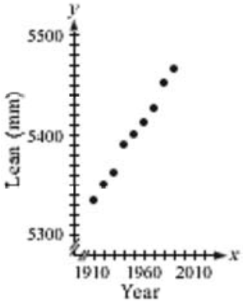


15b. $\hat{y} = \frac{213}{8000}x - 51.78$ or $\hat{y} = 0.027x - 51.78$
15c. If the same trend continues, the cost of gasoline in 2010 will be \$1.74. Answers will vary.
17a. i. 768, –1024; **ii.** 52, 61; **iii.** 32.75, 34.5
17b. i. geometric; **ii.** other; **iii.** arithmetic
17c. i. $u_1 = 243$ and $u_n = \left(-\frac{4}{3}\right)u_{n-1}$ where $n \geq 2$
17c. iii. $u_1 = 24$ and $u_n = u_{n-1} + 1.75$ where $n \geq 2$
17d. iii. $u_n = 1.75n + 22.25$

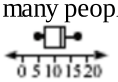
CHAPTER 3 REVIEW

1. $-\frac{975}{19}$
3a. approximately (19.9, 740.0)
3b. approximately (177.0, 740.0)
5a. Poor fit; there are too many points above the line.

5b. Reasonably good fit; the points are well-distributed above and below the line, and not clumped.
5c. Poor fit; there are an equal number of points above and below the line, but they are clumped to the left and to the right, respectively.
7a. (1, 0)
7b. every point; same line
7c. No intersection; the lines are parallel.
9a.



9b. $\hat{y} = 2088 + 1.7x$
9c. 1.7; for every additional year, the tower leans another 1.7 mm.
9d. 5474.4 mm
9e. Approximately 5.3 mm; the prediction in 9d is probably accurate within 5.3 mm. So the actual value will probably be between 5469.1 and 5479.7.
9f. $1173 \leq$ domain ≤ 1992 (year built to year retrofit began); $0 \leq$ range ≤ 5474.4 mm
11a. geometric; curved; 4, 12, 36, 108, 324
11b. shifted geometric; curved; 20, 47, 101, 209, 425
13a. Possible answer: $u_{2005} = 6486915022$ and $u_n = (1 + 0.015)u_{n-1}$ where $n \geq 2006$. The sequence is geometric.
13b. possible answer: 6,988,249,788 people
13c. On January 1, 2035, the population will be just above 10 billion. So the population will first exceed 10 billion late in 2034.
13d. Answers will vary. An increasing geometric sequence has no limit. But the model will not work for the distant future because there is a physical limit to how many people will fit on Earth.
15.



15a. skewed left **15b.** 12 **15c.** 6
15d. 50%; 25%; 0%
17a. $\left(\frac{110}{71}, -\frac{53}{213}\right)$ **17b.** $\left(-\frac{27}{20}, \frac{91}{20}\right)$
17c. $\left(\frac{46}{13}, \frac{9}{26}\right)$

19a. $u_1 = 6$ and $u_n = u_{n-1} + 7$ where $n \geq 1$
19b. $y = 6 + 7x$
19c. The slope is 7. The slope of the line is the same as the common difference of the sequence.
19d. 230; It's probably easier to use the equation from 19b.

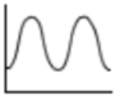
CHAPTER 4 • CHAPTER **4** CHAPTER 4 • CHAPTER

LESSON 4.1

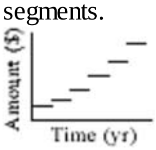
1a.

1b.

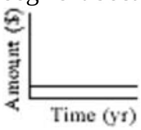
1c.



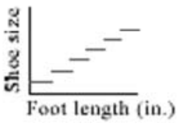
3a. A **3b.** C **3c.** D **3d.** B
5. *Hint:* Consider the rate and direction of change (increasing, decreasing, constant) of the various segments of the graph.
7a. Time in years is the independent variable; the amount of money in dollars is the dependent variable. The graph will be a series of discontinuous segments.



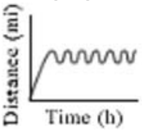
7b. Time in years is the independent variable; the amount of money in dollars is the dependent variable. The graph will be a continuous horizontal segment because the amount never changes.



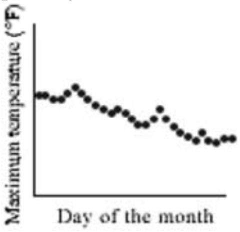
7c. Foot length in inches is the independent variable; shoe size is the dependent variable. The graph will be a series of discontinuous horizontal segments because shoe sizes are discrete.



7d. Time in hours is the independent variable; distance in miles is the dependent variable. The graph will be continuous because distance is changing continuously over time.



7e. The day of the month is the independent variable; the maximum temperature in degrees Fahrenheit is the dependent variable. The graph will be discrete points because there is just one temperature reading per day.

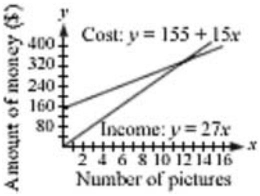


9a. Car A speeds up quickly at first and then less quickly until it reaches 60 mi/h. Car B speeds up slowly at first and then quickly until it reaches 60 mi/h.

9b. Car A will be in the lead because it is always going faster than Car B, which means it has covered more distance.

11a. Let x represent the number of pictures and let y represent the amount of money (either cost or income) in dollars; $y = 155 + 15x$.

11b. $y = 27x$



11c. 13 pictures

13a. $3x + 5y = -9$ **13b.** $6x - 3y = 21$

13c. $x = 2, y = -3$ **13d.** $x = 2, y = -3, z = 1$

LESSON 4.2

1a. Function; each x-value has only one y-value.

1b. Not a function; there are x-values that are paired with two y-values.

1c. Function; each x-value has only one y-value.

3. B

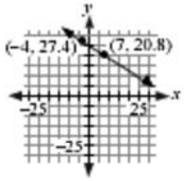
5a. The price of the calculator is the independent variable; function.

5b. The time the money has been in the bank is the independent variable; function.

5c. The amount of time since your last haircut is the independent variable; function.

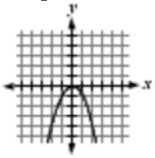
5d. The distance you have driven since your last fill-up is the independent variable; function.

7a, c, d.

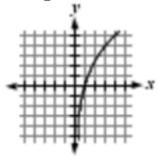


7b. 20.8 **7d.** – 4

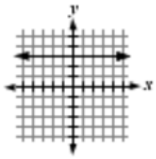
9a. possible answer:



9b. possible answer:



9c.

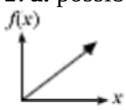


11. Let x represent the time since Kendall started moving and y represent his distance from the motion sensor. The graph is a function; Kendall can be at only one position at each moment in time, so there is only one y-value for each x-value.

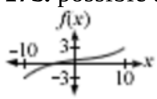
13a. 54 diagonals **13b.** 20 sides

15. *Hint:* Determine how many students fall into each quartile, and an average value for each quartile.

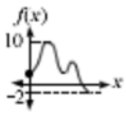
17a. possible answer:



17b. possible answer:



17c. possible answer:



LESSON 4.3

1. $y = -3 + \frac{2}{3}(x - 5)$

3a. $-2(x + 3)$ or $-2x - 6$

3b. $-3 + (-2)(x - 2)$ or $-2x + 1$

3c. $5 + (-2)(x + 1)$ or $-2x + 3$

5a. $y = -3 + 4.7x$ **5b.** $y = -2.8(x - 2)$

5c. $y = 4 - (x + 1.5)$ or $y = 2.5 - x$

7. $y = 47 - 6.3(x - 3)$

9a. $(1400, 733.\overline{3})$ **9b.** $(x + 400, y + 233.\overline{3})$

9c. 20 steps

11a. 12,500; The original value of the equipment is $12,500.

11b. 10; After 10 years the equipment has no value.

11c. –1250; Every year the value of the equipment decreases by $1250.

11d. $y = 12500 - 1250x$ **11e.** after 4.8 yr

13a. $x = 15$ **13b.** $x = 31$

13c. $x = -21$ **13d.** $x = 17.6$

LESSON 4.4

1a. $y = x^2 + 2$ **1b.** $y = x^2 - 6$

1c. $y = (x - 4)^2$ **1d.** $y = (x + 8)^2$

3a. translated down 3 units

3b. translated up 4 units

3c. translated right 2 units

3d. translated left 4 units

5a. $x = 2$ or $x = -2$ **5b.** $x = 4$ or $x = -4$

5c. $x = 7$ or $x = -3$

7a. $y = (x - 5)^2 - 3$ **7b.** $(5, -3)$

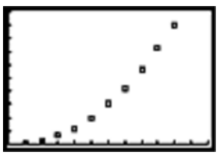
7c. (6, –2), (4, –2), (7, 1), (3, 1). If (x, y) are the coordinates of any point on the black parabola, then the coordinates of the corresponding point on the red parabola are $(x + 5, y - 3)$.

7d. 1 unit; 4 units

9a.

Number of teams (x)	1	2	3	4	5	6	7	8	9	10
Number of games (y)	0	2	6	12	20	30	42	56	72	90

9b. The points appear to be part of a parabola.

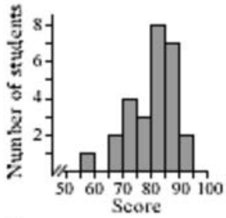


[0, 12, 1, 0, 100, 10]

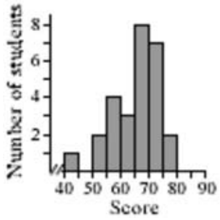
9c. $y = (x - 0.5)^2 - 0.25$

9d. 870 games

11a.

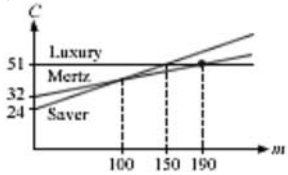


11b.



13a. Let m represent the miles driven and let C represent the cost of the one-day rental. Mertz: $C = 32 + 0.1m$; Saver: $C = 24 + 0.18m$; Luxury: $C = 51$.

13b.



13c. If you plan to drive less than 100 miles, then rent Saver. At exactly 100 miles, Mertz and Saver are the same. If you plan to drive between 100 miles and 190 miles, then rent Mertz. At exactly 190 miles, Mertz and Luxury are the same. If you plan to drive more than 190 miles, then rent Luxury.

15. Answers will vary.

LESSON 4.5

1a. $y = \sqrt{x} + 3$ **1b.** $y = \sqrt{x + 5}$

1c. $y = \sqrt{x + 5} + 2$ **1d.** $y = \sqrt{x - 3} + 1$

1e. $y = \sqrt{x - 1} - 4$

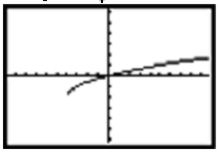
3a. $y = -\sqrt{x}$ **3b.** $y = -\sqrt{x} - 3$

3c. $y = -\sqrt{x + 6} + 5$ **3d.** $y = \sqrt{-x}$

3e. $y = \sqrt{-(x - 2)} - 3$, or $y = \sqrt{-x + 2} - 3$

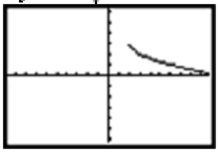
5a. possible answers: (– 4, –2), (–3, –1), and (0, 0)

5b. $y = \sqrt{x+4} - 2$



[– 9.4, 9.4, 1, –6.2, 6.2, 1]

5c. $y = -\sqrt{x-2} + 3$



[– 9.4, 9.4, 1, – 6.2, 6.2, 1]

7a. Neither parabola passes the vertical line test.

7b. i. $y = \pm\sqrt{x+4}$ **7b. ii.** $y = \pm\sqrt{x} + 2$

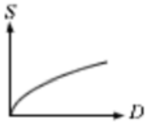
7c. i. $y^2 = x + 4$ **7c. ii.** $(y - 2)^2 = x$

9a. $y = -x^2$ **9b.** $y = -x^2 + 2$

9c. $y = -(x - 6)^2$ **9d.** $y = -(x - 6)^2 - 3$

11a. $S = 5.5\sqrt{0.7D}$

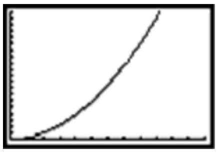
11b.



11c. approximately 36 mi/h

11d. $D = \frac{1}{0.7}\left(\frac{S}{5.5}\right)^2$; the minimum braking distance, when the speed is known.

11e.



[0, 60, 5, 0, 100, 5]

It is a parabola, but the negative half is not used because the distance cannot be negative.

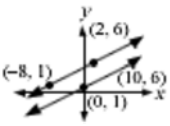
11f. approximately 199.5 ft

13a. $x = 293$ **13b.** no solution

13c. $x = 7$ or $x = -3$ **13d.** $x = -13$

15a. $y = \frac{1}{2}x + 5$

15b. $y = \frac{1}{2}(x - 8) + 5$



15c. $y = \left(\frac{1}{2}x + 5\right) - 4$ or $y + 4 = \frac{1}{2}x + 5$

15d. Both equations are equivalent to $y = \frac{1}{2}x + 1$.

LESSON 4.6

1a. $y = |x| + 2$ **1b.** $y = |x| - 5$

1c. $y = |x + 4|$ **1d.** $y = |x - 3|$

1e. $y = |x| - 1$ **1f.** $y = |x - 4| + 1$

1g. $y = |x + 5| - 3$ **1h.** $y = 3|x - 6|$

1i. $y = -\left|\frac{x}{4}\right|$ **1j.** $y = (x - 5)^2$

1k. $y = -\frac{1}{2}|x + 4|$ **1l.** $y = -|x + 4| + 3$

1m. $y = -(x + 3)^2 + 5$ **1n.** $y = \pm\sqrt{x-4} + 4$

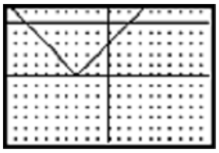
1p. $y = -2\left|\frac{x-3}{3}\right|$

3a. $y = 2(x - 5)^2 - 3$

3b. $y = 2\left|\frac{x+1}{3}\right| - 5$ **3c.** $y = -2\sqrt{\frac{x-6}{-3}} - 7$

5a. 1 and 7; $x = 1$ and $x = 7$

5b. $x = -8$ and $x = 2$



[–9.4, 9.4, 1, –6.2, 6.2, 1]

7a. (6, -2)

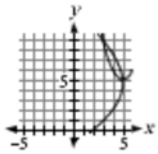
7b. (2, -3) and (8, -3)

7c. (2, -2) and (8, -2)

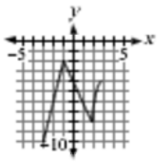
9a. possible answers: $x = 4.7$ or $y = 5$

9b. possible answers: $y = 4\left(\frac{x - 4.7}{1.9}\right)^2 + 5$ or $\left(\frac{y-5}{4}\right)^2 = \frac{x - 4.7}{-1.9}$

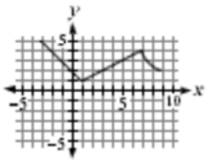
9c. There are at least two parabolas. One is oriented horizontally, and another is oriented vertically.



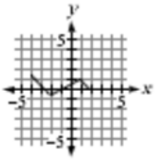
11a.



11b.



11c.

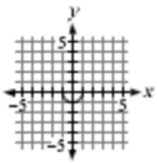


13a. $\bar{x} = 83.75$, $s = 7.45$
13b. $\bar{x} = 89.75$, $s = 7.45$
13c. By adding 6 points to each rating, the mean increases by 6, but the standard deviation remains the same.

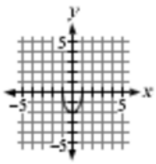
LESSON 4.7

1. 2nd row: Reflection, Across *x*-axis, N/A; 3rd row: Stretch, Horizontal, 4; 4th row: Shrink, Vertical, 0.4; 5th row: Translation, Right, 2; 6th row: Reflection, Across *y*-axis, N/A

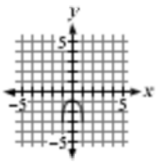
3a.



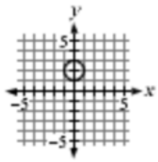
3b.



3c.

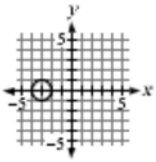


5a.



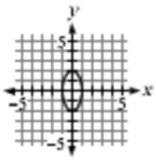
$y = \pm\sqrt{1-x^2} + 2$
or $x^2 + (y-2)^2 = 1$

5b.



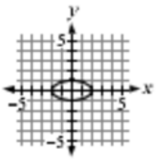
$y = \pm\sqrt{1-(x+3)^2}$
or $(x+3)^2 + y^2 = 1$

5c.



$y = \pm 2\sqrt{1-x^2}$
or $x^2 + \left(\frac{y}{2}\right)^2 = 1$

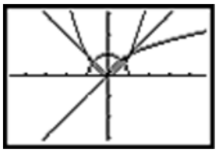
5d.



$y = \pm\sqrt{1-\left(\frac{x}{2}\right)^2}$
or $\frac{x^2}{4} + y^2 = 1$

7a. $x^2 + (2y)^2 = 1$ **7b.** $(2x)^2 + y^2 = 1$
7c. $\left(\frac{x}{2}\right)^2 + (2y)^2 = 1$

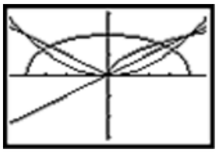
9a.



[-4.7, 4.7, 1, -3.1, 3.1, 1]
(0, 0) and (1, 1)

9b. The rectangle has width 1 and height 1. The width is the difference in *x*-coordinates, and the height is the difference in *y*-coordinates.

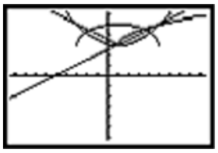
9c.



[-4.7, 4.7, 1, -3.1, 3.1, 1]
(0, 0) and (4, 2)

9d. The rectangle has width 4 and height 2. The width is the difference in *x*-coordinates, and the height is the difference in *y*-coordinates.

9e.



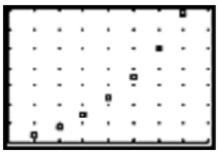
[-9.4, 9.4, 1, -6.2, 6.2, 1]
(1, 3) and (5, 5)

9f. The rectangle has width 4 and height 2. The difference in *x*-coordinates is 4, and the difference in *y*-coordinates is 2.

9g. The *x*-coordinate is the location of the right endpoint, and the *y*-coordinate is the location of the top of the transformed semicircle.

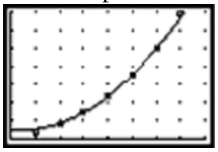
11. 625, 1562.5, 3906.25

13a.



[0, 80, 10, 0, 350, 50]

13b. Sample answer: $\hat{y} = 0.07(x - 3)^2 + 21$.



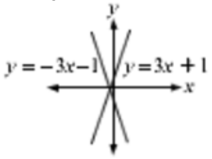
[0, 80, 10, 0, 350, 50]

13c. For the sample answer: residuals: -5.43, 0.77, 0.97, -0.83, -2.63, -0.43, 7.77; $s = 4.45$

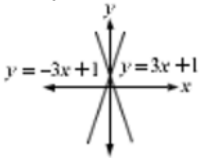
13d. approximately 221 ft

13e. 13d should be correct ±4.45 ft.

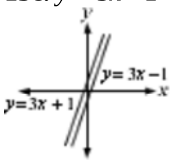
15a. $y = -3x - 1$



15b. $y = -3x + 1$



15c. $y = 3x - 1$



15d. The two lines are parallel.

LESSON 4.8

1a. 6 **1b.** 7 **1c.** 6 **1d.** 18

3a. approximately 1.5 m/s

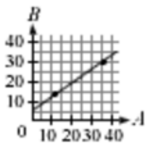
3b. approximately 12 L/min

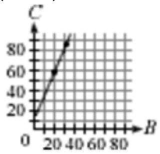
3c. approximately 15 L/min

5a. $y = |(x - 3)^2 - 1|$

5b. $f(x) = |x|$ and $g(x) = (x - 3)^2 - 1$

7a.





7b. approximately 41

7c. $B = \frac{2}{3}(A - 12) + 13$

7d. $C = \frac{9}{4}(B - 20) + 57$

7e. $C = \frac{9}{4}\left(\frac{2}{3}A + 5\right) + 12 = 1.5A + 23.25$

9a. 2 **9b.** – 1

9c. $g(f(x)) = x$ **9d.** $f(g(x)) = x$

9e. The two functions "undo" the effects of each other and thus give back the original value.

11. *Hint:* Use two points to find both parabola and semicircle equations for the curve. Then substitute a third point into your equations and decide which is most accurate.

13a. $x = -5$ or $x = 13$

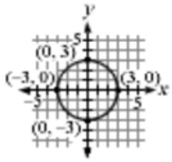
13b. $x = -1$ or $x = 23$

13c. $x = 64$

13d. $x = \pm\sqrt{1.5} \approx \pm 1.22$

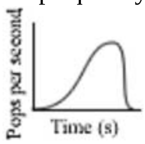
15a. $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$ or $x^2 + y^2 = 9$

15b.

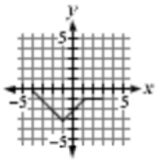


CHAPTER 4 REVIEW

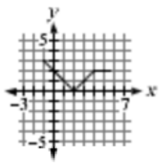
1. Sample answer: For a time there are no pops. Then the popping rate slowly increases. When the popping reaches a furious intensity, it seems to level out. Then the number of pops per second drops quickly until the last pop is heard.



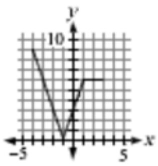
3a.



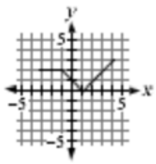
3b.



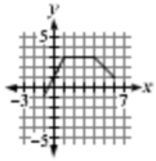
3c.



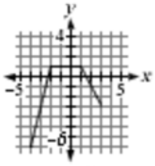
3d.



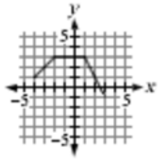
5a.



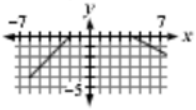
5b.



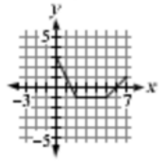
5c.



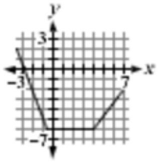
5d.



5e.



5f.



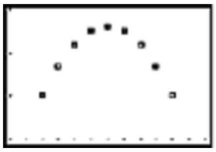
7a. $y = \frac{2}{3}x - 2$

7b. $y = \pm\sqrt{x+3} - 1$

7c. $y = \pm\sqrt{-(x-2)^2+1}$

9a. Number of passengers: 17000, 16000, 15000, 14000, 13000, 12000, 11000, 10000; Revenue: 18700, 19200, 19500, 19600, 19200, 18700, 18000

9b.



[0.8, 2, 0.1, 17000, 20000, 1000]

9c. (1.40, 19600). By charging $1.40 per ride, the company achieves the maximum revenue, $19,600.

9d. $\hat{y} = -10000(x - 1.4)^2 + 19600$

9d. i. $16,000 **9d. ii.** $0 or $2.80

CHAPTER 5 • CHAPTER 5 CHAPTER 5 • CHAPTER

LESSON 5.1

1a. $f(5) \approx 3.52738$ **1b.** $g(14) \approx 19{,}528.32$

1c. $h(24) \approx 22.9242$ **1d.** $j(37) \approx 3332.20$

3a. $f(0) = 125$, $f(1) = 75$, $f(2) = 45$; $u_0 = 125$ and $u_n = 0.6u_{n-1}$ where $n \geq 1$

3b. $f(0) = 3$, $f(1) = 6$, $f(2) = 12$; $u_0 = 3$ and $u_n = 2u_{n-1}$ where $n \geq 1$

5a. $u_0 = 1.151$ and $u_n = (1 + 0.015)u_{n-1}$ where $n \geq 1$

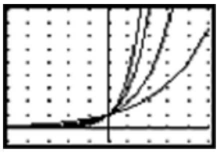
5b.

Year	Population (in billions)
1991	1.151
1992	1.168
1993	1.186
1994	1.204
1995	1.222
1996	1.240
1997	1.259
1998	1.277
1999	1.297
2000	1.316

5c. Let x represent the number of years since 1991, and let y represent the population in billions. $y = 1.151(1 + 0.015)^x$

5d. $y = 1.151(1 + 0.015)^{10} \approx 1.336$; the equation gives a population that is greater than the actual population. Sample answer: the growth rate of China's population has slowed since 1991.

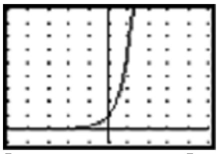
7a–d.



[–5, 5, 1, –1, 9, 1]

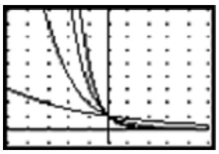
7e. As the base increases, the graph becomes steeper. The curves all intersect the y-axis at (0, 1).

7f. The graph of $y = 6^x$ should be the steepest of all of these. It will contain the points (0, 1) and (1, 6).



[–5, 5, 1, –1, 9, 1]

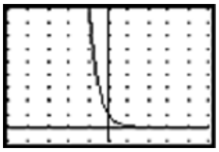
9a–d.



[–5, 5, 1, –1, 9, 1]

9e. As the base increases, the graph flattens out. The curves all intersect the y-axis at (0, 1).

9f. The graph of $y = 0.1^x$ should be the steepest of all of these. It will contain the points (0, 1) and (–1, 10).

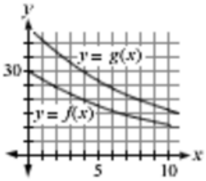


[–5, 5, 1, –1, 9, 1]

11a. $\frac{27}{30} = 0.9$

11b. $f(x) = 30(0.9)^x$

11c.

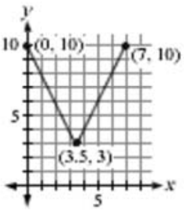


11d. $g(4) = 30$

11e. possible answer: $g(x) = 30(0.9)^{x-4}$

11f. *Hint:* Think about what x_1, y_1, and b represent.

13a. Let x represent time in seconds, and let y represent distance in meters.



13b. domain: $0 \leq x \leq 7$; range: $3 \leq y \leq 10$

13c. $y = 2\,|\,x - 3.5\,| + 3$

15a–c. *Hint:* One way to construct the circles is to duplicate circle M and change the radius in order to get the correct area.

15d. *Hint:* Recall that the area of a circle is given by the formula $A = \pi r^2$.

LESSON 5.2

1a. $\frac{1}{125}$ **1b.** – 36 **1c.** $-\frac{1}{81}$

1d. $\frac{1}{144}$ **1e.** $\frac{16}{9}$ **1f.** $\frac{7}{2}$

3a. false **3b.** false

3c. false **3d.** true

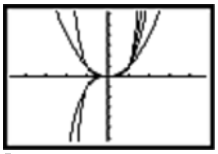
5a. $x \approx 3.27$ **5b.** $x = 784$

5c. $x \approx 0.16$ **5d.** $x \approx 0.50$

5e. $x \approx 1.07$ **5f.** $x = 1$

7. *Hint:* Is $(2 + 3)^2$ equivalent to $2^2 + 3^3$? Is $(2 + 3)^1$ equivalent to $2^1 + 3^1$? Is $(2 – 2)^3$ equivalent to $2^3 + (–2)^3$? Is $(2 – 2)^2$ equivalent to $2^2 + (–2)^2$?

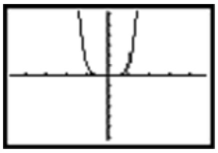
9a–d.



[–4.7, 4.7, 1, 6.2, 6.2, 1]

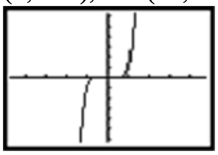
9e. Sample answer: As the exponents increase, the graphs get narrower horizontally. The even-power functions are U-shaped and always in the first and second quadrants, whereas the odd-power functions have only the right half of the U, with the left half pointed down in the third quadrant. They all pass through (0, 0) and (1, 1).

9f. Sample answer: The graph of $y = x^6$ will be U-shaped, will be narrower than $y = x^4$, and will pass through (0, 0), (1, 1), (–1, 1), (2, 64), and (–2, 64).



[– 4.7, 4.7, 1, –6.2, 6.2, 1]

9g. Sample answer: The graph of $y = x^7$ will fall in the first and third quadrants, will be narrower than $y = x^3$ or $y = x^5$, and will pass through (0, 0), (1, 1), (–1, –1), (2, 128), and (–2, –128).



[–4.7, 4.7, 1, –6.2, 6.2, 1]

11a. $47(0.9)\,(0.9)^{x-1} = 47(0.9)^1(0.9)^{x-1} = 47(0.9)^x$ by the product property of exponents; $42.3(0.9)^{x-1}$.

11b. $38.07(0.9)^{x-2}$

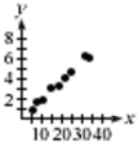
11c. The coefficients are equal to the values of Y1 corresponding to the number subtracted from x in the exponent. If (x_1, y_1) is on the curve, then any equation $y = y_1 \cdot b^{(x-x_1)}$ is an exponential equation for the curve.

13a. $x = 7$ **13b.** $x = -\frac{1}{2}$ **13c.** $x = 0$

15a. $x = 7$ **15b.** $x = – 4$

15c. $x = 4$ **15d.** $x = 4.61$

17a. Let x represent time in seconds, and let y represent distance in meters.



17b. All you need is the slope of the median-median line, which is determined by M_1 (8, 1.6) and M_3(31, 6.2). The slope is 0.2. The speed is approximately 0.2 m/s.

LESSON 5.3

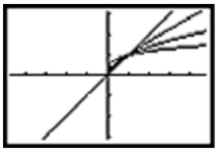
1. a—e—j; b—d—g; c—i; f—h

3a. $a^{1/6}$ **3b.** $b^{4/5}$, $b^{8/10}$, or $b^{0.8}$

3c. $c^{-1/2}$ or $c^{-0.5}$ **3d.** $d^{7/5}$ or $d^{1.4}$

5. 490 W/cm^2

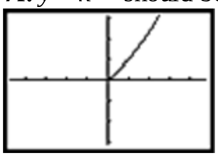
7a-d.



[– 4.7, 4.7, 1, –3.1, 3.1, 1]

7e. Each graph is steeper and less curved than the previous one. All of the functions go through (0, 0) and (1, 1).

7f. $y = x^{5/4}$ should be steeper and curve upward.



[– 4.7, 4.7, 1, –3.1, 3.1, 1]

9a. exponential **9b.** neither

9c. exponential **9d.** power

11a. $x = \left(\frac{13}{9}\right)^5 \approx 6.29$ **11b.** $x = 180^{1/4} \approx 3.66$

11c. $x = \left(\frac{\sqrt{35}}{4}\right)^{3/2} \approx 1.80$

13a. *Hint:* Solve for k. **13b.** $k = (40)(12.3) = 492$

13c. 8.2 L **13d.** 32.8 mm Hg

15a. $y = (x + 4)^2$ **15b.** $y = x^2 + 1$

15c. $y = -(x + 5)^2 + 2$ **15d.** $y = (x - 3)^2 - 4$

15e. $y = \sqrt{x+3}$ **15f.** $y = \sqrt{x} - 1$

15g. $y = \sqrt{x+2} + 1$ **15h.** $y = -\sqrt{x-1} - 1$

17a. $u_1 = 20$ and $u_n = 1.2u_{n-1}$ where $n \geq 2$

17b. $u_9 \approx 86$; about 86 rats

17c. Let x represent the year number, and let y represent the number of rats. $y = 20(1.2)^{x-1}$

LESSON 5.4

1a. $x = 50^{1/5} \approx 2.187$ **1b.** $x = 29.791$

1c. no real solution

3a. $9x^4$ **3b.** $8x^6$ **3c.** $216x^{-18}$

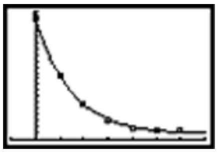
5a. She must replace y with $y - 7$ and y_1 with $y_1 - 7$; $y - 7 = (y_1 - 7) \cdot b^{x-x_1}$.

5b. $y - 7 = (105 - 7)b^{x-1}$; $\left(\frac{y-7}{98}\right)^{1/(x-1)} = b$

5c.

x	0	2	3	4	5	6
y	200	57	31	18	14	12
b	0.508	0.510	0.495	0.482	0.517	0.552

5d. Possible answer: The mean of the b-values is 0.511. $y = 7 + 98(0.511)^{x-1}$.



[–1, 7, 1, 0, 210, 10]

7a. 68.63 tons **7b.** 63.75 ft

9a. 1.9 g **9b.** 12.8%

11a. 0.9534, or 95.34% per year

11b. 6.6 g **11c.** $y = 6.6(0.9534)^x$

11d. 0.6 g **11e.** 14.5 yr

13. $x = -4.5$, $y = 2$, $z = 2.75$

LESSON 5.5

1. (–3, –2), (–1, 0), (2, 2), (6, 4)

3. Graph c is the inverse because the x- and y-coordinates have been switched from the original graph so that the graphs are symmetric across line $y = x$.

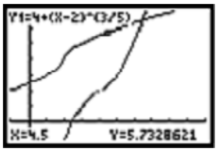
5a. $f(7) = 4$; $g(4) = 7$

5b. They might be inverse functions.

5c. $f(1) = -2$; $g(-2) = 5$

5d. They are *not* inverse functions, at least not over their entire domains and ranges.

5e. $f(x)$ for $x \geq 3$ and $g(x)$ for $x \geq -4$ are inverse functions.

7a. 

[–1, 10, 1, –1, 7, 1]

7b. The inverse function from 6b should be the same as the function drawn by the calculator.

7c. Find the composition of $f^{-1}(f(x))$. If it equals x, you have the correct inverse.

9a. i. $f^{-1}(x) = \dfrac{x+140}{6.34}$

9a. ii. $f(f^{-1}(15.75)) = 15.75$

9a. iii. $f^{-1}(f(15.75)) = 15.75$

9a. iv. $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

9b. i. $f^{-1}(x) = \dfrac{x-32}{1.8}$

9b. ii. $f(f^{-1}(15.75)) = 15.75$

9b. iii. $f^{-1}(f(15.75)) = 15.75$

9b. iv. $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

11a. $y = 100 - C$

11b. Solve $F = 1.8C + 32$ for C and substitute into $y = 100 - C$ to get $y = \dfrac{F-212}{-1.8} = \dfrac{212-F}{1.8}$.

13a. $c(x) = 7.18 + 3.98x$, where c is the cost and x is the number of thousand gallons.

13b. $39.02

13c. $g(x) = \dfrac{x-7.18}{3.98}$, where g is the number of thousands of gallons and x is the cost.

13d. 12,000 gal

13e. *Hint:* The compositions $g(c(x))$ and $c(g(x))$ should both be equivalent to x.

13f. about $13

13g. *Hint:* The product of length, width, and height should be equivalent to the volume of water, in cubic inches, saved in a month.

15. *Hint:* Solve $12.6(b)^3 = 42.525$ to find the base. Then use the point-ratio form.

17. *Hint:* Consider a vertically oriented parabola and a horizontally oriented parabola.

LESSON 5.6

1a. $10^x = 1000$

1b. $5^x = 625$

1c. $7^x = \sqrt{7}$

1d. $8^x = 2$

1e. $5^x = \dfrac{1}{25}$

1f. $6^x = 1$

3a. $x = \log_{10} 0.001$; $x = -3$

3b. $x = \log_5 100$; $x \approx 2.8614$

3c. $x = \log_{35} 8$; $x \approx 0.5849$

3d. $x = \log_{0.4} 5$; $x \approx -1.7565$

3e. $x = \log_{0.8} 0.03$; $x \approx 15.7144$

3f. $x = \log_{17} 0.5$; $x \approx -0.2447$

5a. false; $x = \log_6 12$

5b. false; $2^x = 5$

5c. false; $x = \dfrac{\log 5.5}{\log 3}$

5d. false; $x = \log_3 7$

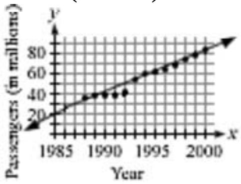
7a. 1980

7b. 13%

7c. 5.6 yr

9a. $y = 88.7(1.0077)^x$

9b. 23 or 24 clicks

11a. 

$\hat{y} = \dfrac{38}{9}x - 8359.91$

11b. 3.03, 0.71, –2.11, –6.43, –9.16, 0.02, 1.50, –0.52, –2.05, –3.17, –0.99, –0.51, –0.43

11c. 3.78 million riders. Most data are within 3.78 million of the predicted number.

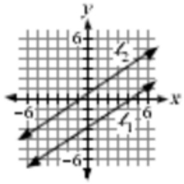
11d. 126.8 million riders

13a. $y + 1 = x - 3$ or $y = x - 4$

13b. $y + 4 = (x + 5)^2$ or $y = (x + 5)^2 - 4$

13c. $y - 2 = |x + 6|$ or $y = |x + 6| + 2$

13d. $y - 7 = \sqrt{x-2}$ or $y = \sqrt{x-2} + 7$

15a. 

They are parallel.

15b. Possible answer: $A(0, -3)$; $P(1, 1)$; $Q(4, 3)$

15c. Possible answer: Translate 1 unit right and 4 units up. $2(x - 1) - 3(y - 4) = 9$.

15d. Possible answer: Translate 4 units right and 6 units up. $2(x - 4) - 3(y - 6) = 9$.

15e. *Hint:* Distribute and combine like terms. You should find that the equations are equivalent.

LESSON 5.7

1a. $g^h \cdot g^k$; product property of exponents

1b. $\log st$; product property of logarithms

1c. f^{w-v}; quotient property of exponents

1d. $\log h - \log k$; quotient property of logarithms

1e. j^{st}; power property of exponents

1f. $g\log b$; power property of logarithms

1g. $k^{m/n}$; definition of rational exponents

1h. $\log_u t$; change-of-base property

1i. w^{t+s}; product property of exponents

1j. $\frac{1}{p^k}$; definition of negative exponents

3a. $a \approx 1.763$ **3b.** $b \approx 1.3424$

3c. $c \approx 0.4210$ **3d.** $d \approx 2.6364$

3e. $e \approx 2.6364$ **3f.** $f \approx 0.4210$

3g. $c = f$ and $d = e$ **3h.** $\log\frac{a}{b}$

3i. When numbers with the same base are divided, the exponents are subtracted.

5a. true

5b. false; possible answer: $\log 5 + \log 3 = \log 15$

5c. true

5d. true

5e. false; possible answer: $\log 9 - \log 3 = \log 3$

5f. false; possible answer: $\log\sqrt{7} = \frac{1}{2}\log 7$

5g. false; possible answer: $\log 35 = \log 5 + \log 7$

5h. true

5i. false; possible answer: $\log 3 - \log 4 = \log\frac{3}{4}$

5j. true

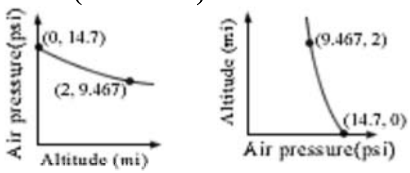
7a. $y = 261.6(2^{x/12})$

7b.

Note	Frequency (Hz)	Note	Frequency (Hz)
C4	261.6	G	392.0
C#	277.2	G#	415.3
D	293.6	A	440.0
D#	311.1	A#	466.1
E	329.6	B	493.8
F	349.2	C5	523.2
F#	370.0		

9a. $y = 14.7(0.8022078)^x$

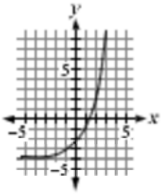
9b.



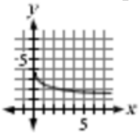
9c. $y = 8.91$ lb/in.2 **9d.** $x = 6.32$ mi

11. *Hint:* If more than one input value results in the same output value, then a function's inverse will not be a function. What does this mean about the graph of the function?

13a. The graph has been vertically stretched by a factor of 3, then translated to the right 1 unit and down 4 units.



13b. The graph has been horizontally stretched by a factor of 3, reflected across the x-axis, and translated up 2 units.



15a. Let h represent the length of time in hours, and let c represent the driver's cost in dollars. $c = 14h + 20$. The domain is the set of possible values of the number of hours, $h > 0$. The range is the set of possible values of the cost paid to the driver, $c > 20$.

15b. Let c represent the driver's cost in dollars, and let a represent the agency's charge in dollars. $a = 1.15c + 25$. The domain is the money paid to the driver if she had been booked directly, c. $c > 20$. The range is the amount charged by the agency, a. $a > 48$.

15c. $a = 1.15(14h + 20) + 25$, or $a = 16.1h + 48$

LESSON 5.8

1a. $\log(10^{n+p}) = \log((10^n)(10^p))$

$(n + p)\log 10 = \log 10^n + \log 10^p$

$(n + p)\log 10 = n\log 10 + p\log 10$

$(n + p)\log 10 = (n + p)\log 10$

1b. $\log\left(\frac{10^d}{10^e}\right) = \log\left(10^{d-e}\right)$

$\log 10^d - \log 10^e = \log(10^{d-e})$

$d\log 10 - e\log 10 = (d - e)\log 10$

$(d - e)\log 10 = (d - e)\log 10$

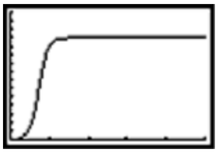
3. about 195.9 mo or about 16 yr 4 mo

5a. $f(20) \approx 133.28$; After 20 days 133 games have been sold.

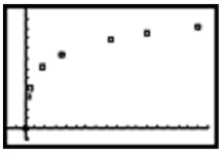
5b. $f(80) \approx 7969.17$; After 80 days 7969 games have been sold.

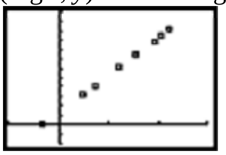
5c. $x = 72.09$; After 72 days 6000 games have been sold.

5d. $\dfrac{12000}{1+499(1.09)^{-x}} = 6000;\ 2 = 1 + 499(1.09)^{-x};$
$1 = 499(1.09)^{-x}$; $0.002 = (1.09)^{-x}$;
$\log(0.002) = \log(1.09)^{-x}$; $\log 0.002 = -x\log 1.09$;
$x = -\dfrac{\log 0.002}{\log 1.09} \approx 72.1$

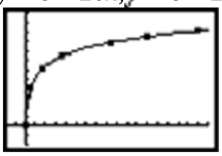
5e.

[0, 500, 100, 0, 15000, 1000]

Sample answer: The number of games sold starts out increasing slowly, then speeds up, and then slow down as everyone who wants the game has purchased one.

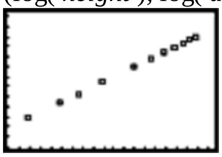
7a.

[–16, 180, 10, –5, 60, 5]

7b. ($\log x$, y) is a linear graph.

[–1, 3, 1, –10, 60, 5]

7c. $y = 6 + 20x$; $\hat{y} = 6 + 20\log x$

7d.

[–16, 180, 10, –10, 60, 5]

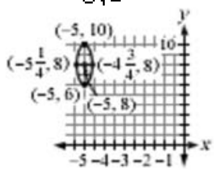
Sample answer: Yes; the graph shows that the equation is a good model for the data.

9a. The data are the most linear when viewed as (log(*height*), log(*distance*)).

[2.3, 4.2, 0.1, 1.5, 2.8, 0.1]

9b. $view = 3.589height^{0.49909}$

11a. $y = 18(\sqrt{2})^{x-4}$, $y = 144(\sqrt{2})^{x-10}$, or $y = 4.5(\sqrt{2})^{x}$

11b. $y = \dfrac{\log x - \log 18}{\log\sqrt{2}} + 4$,
$y = \dfrac{\log x - \log 144}{\log\sqrt{2}} + 10$, or $y = \dfrac{\log x - \log 4.5}{\log\sqrt{2}}$

13.


CHAPTER 5 REVIEW

1a. $\frac{1}{16}$ **1b.** $-\frac{1}{3}$ **1c.** 125 **1d.** 7 **1e.** $\frac{1}{4}$
1f. $\frac{27}{64}$ **1g.** –1 **1h.** 12 **1i.** 0.6

3a. $x = \dfrac{\log 28}{\log 4.7} \approx 2.153$

3b. $x = \pm\sqrt{\dfrac{\log 2209}{\log 4.7}} \approx \pm 2.231$

3c. $x = 2.9^{1/1.25} = 2.9^{0.8} \approx 2.344$

3d. $x = 3.1^{47} \approx 1.242 \times 10^{23}$

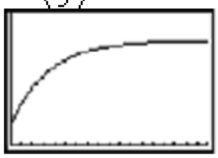
3e. $x = \left(\frac{101}{7}\right)^{1/2.4} \approx 3.041$

3f. $x = \dfrac{\log 18}{\log 1.065} \approx 45.897$

3g. $x = 10^{3.771} \approx 5902$ **3h.** $x = 47^{5/3} \approx 612$

5. about 39.9 h

7. $y = 5\left(\frac{32}{5}\right)^{(x-1)/6}$

9a.

[0, 18, 1, 0, 125, 0]

9b. domain: $0 \le x \le 120$; range: $20 \le y \le 100$

9c. Vertically stretch by a factor of 80; reflect across the *x*-axis; vertically shift by 100.

9d. 55% of the average adult size

9e. about 4 years old

11a. approximately 37 sessions

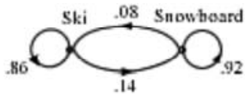
11b. approximately 48 wpm

11c. Sample answer: It takes much longer to improve your typing speed as you reach higher levels. 60 wpm is a good typing speed, and very few people type more than 90 wpm, so $0 \leq x \leq 90$ is a reasonable domain.

CHAPTER 6 • CHAPTER 6 CHAPTER 6 • CHAPTER

LESSON 6.1

1a.



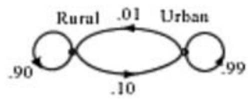
1b. $\begin{bmatrix} .86 & .14 \\ .08 & .92 \end{bmatrix}$

3. $\begin{bmatrix} .60 & .40 \\ .53 & .47 \end{bmatrix}$

5a. 20 girls and 25 boys **5b.** 18 boys

5c. 13 girls batted right-handed.

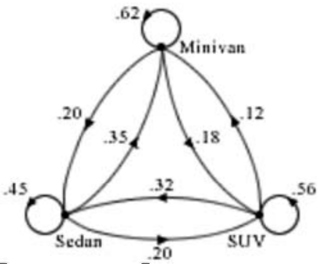
7a.



7b. $\begin{bmatrix} .99 & .01 \\ .10 & .90 \end{bmatrix}$

7c. [16.74 8.26]; [17.3986 7.6014]

9a.



9b. $\begin{bmatrix} .62 & .20 & .18 \\ .35 & .45 & .20 \\ .12 & .32 & .56 \end{bmatrix}$

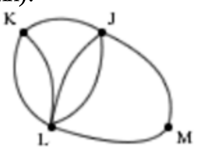
9c. The sum of each row is 1; percentages should sum to 100%.

11a. 5×5

11b. $m_{32} = 1$; there is one round-trip flight between City C and City B.

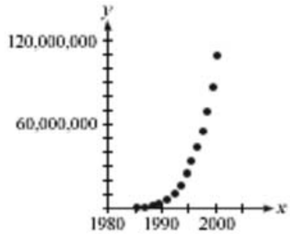
11c. City A has the most flights. From the graph, more paths have A as an endpoint than any other city. From the matrix, the sum of row 1 (or column 1) is greater than the sum of any other row (or column).

11d.



13. $7.4p + 4.7s = 100$

15a. Let x represent the year, and let y represent the number of subscribers.



15b. possible answer: $\hat{y} = 1231000(1.44)^{x-1987}$

15c. About 420,782,749 subscribers. Explanations will vary.

LESSON 6.2

1. [196.85 43.15]; 197 students will choose ice cream, 43 will choose frozen yogurt.

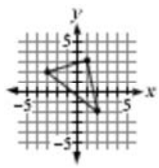
3a. $\begin{bmatrix} 7 & 3 & 0 \\ -19 & -7 & 8 \\ 5 & 2 & -1 \end{bmatrix}$ **3b.** $\begin{bmatrix} -2 & 5 \\ 8 & 7 \end{bmatrix}$ **3c.** [13 29]

3d. not possible because the inside dimensions do not match

3e. $\begin{bmatrix} 4 & -1 \\ 4 & -2 \end{bmatrix}$

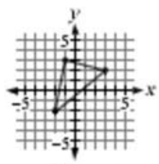
3f. not possible because the dimensions aren't the same

5a.



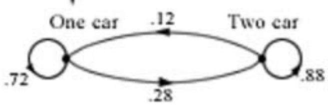
5b. $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 3 & -2 \end{bmatrix}$

5c.



5d. The original triangle is reflected across the y-axis.

7a.



7b. [4800 4200]

7c. $\begin{bmatrix} .72 & .28 \\ .12 & .88 \end{bmatrix}$

7d. $[4800 \quad 4200]\begin{bmatrix} .72 & .28 \\ .12 & .88 \end{bmatrix} = [3960 \quad 5040]$

7e. $[3456 \quad 5544]$

9a. $a = 3$, $b = 4$ **9b.** $a = 7$, $b = 4$

11. The probability that the spider is in room 1 after four room changes is .375. The long-run probabilities for rooms 1, 2, and 3 are $[.\overline{3} \quad .\overline{3} \quad .\overline{3}]$.

13a. $\begin{bmatrix} 3 \\ 1 \\ 3 \\ 1 \\ 3 \\ 1 \\ 3 \\ 1 \\ 3 \\ 1 \\ 3 \\ 1 \end{bmatrix}$

13b. The first and last UPCs are valid.

13c. For the second code, the check digit should be 7. For the third code, the check digit should be 5.

15. $\overline{CD}$: $y = -3 + \frac{2}{3}(x - 1)$ or $y = -1 + \frac{2}{3}(x - 4)$;
$\overline{AB}$: $y = 2 + \frac{2}{3}(x + 2)$ or $y = 4 + \frac{2}{3}(x - 1)$;
$\overline{AD}$: $y = 2 - \frac{5}{3}(x + 2)$ or $y = -3 - \frac{5}{3}(x - 1)$;
$\overline{BC}$: $y = 4 - \frac{5}{3}(x - 1)$ or $y = -1 - \frac{5}{3}(x - 4)$

17. $x = 2$, $y = \frac{1}{2}$, $z = -3$

LESSON 6.3

1a. $\begin{cases} 2x + 5y = 8 \\ 4x - y = 6 \end{cases}$ **1b.** $\begin{cases} x - y + 2z = 3 \\ x + 2y - 3z = 1 \\ 2x + y - z = 2 \end{cases}$

3a. $\left[\begin{array}{rrr|r} 1 & -1 & 2 & 3 \\ 0 & 3 & -5 & -2 \\ 2 & 1 & -1 & 2 \end{array}\right]$ **3b.** $\left[\begin{array}{rrr|r} 1 & -1 & 2 & 3 \\ 1 & 2 & -3 & 1 \\ 0 & 3 & -5 & -4 \end{array}\right]$

5a. $\left[\begin{array}{rrr|r} 1 & 0 & 0 & -31 \\ 0 & 1 & 0 & 24 \\ 0 & 0 & 1 & -4 \end{array}\right]$ **5b.** $\left[\begin{array}{rrr|r} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$

5c. cannot be reduced to row-echelon form (dependent system)

5d. cannot be reduced to row-echelon form (inconsistent system)

7. *Hint:* Define variables for the three angle measures and write a system of three equations. The statement of the exercise should help you write two equations. For the third equation, recall that in a triangle the sum of the angle measures is 180°.

9. *Hint:* Create a system of three equations by substituting each pair of coordinates for x and y. Then solve the system for a, b, and c.

11a. first plan: \$14,600; second plan: \$13,100

11b. Let x represent the number of tickets sold, and let y represent the income in dollars; $y = 12500 + 0.6x$.

11c. $y = 6800 + 1.8x$

11d. more than 4750 tickets

11e. The company should choose the first plan if they expect to sell fewer than 4750 tickets and the second if they expect to sell more than 4750 tickets.

13. $\overline{AB}$: $y = 6 - \frac{2}{3}(x - 4)$ or $y = 4 + \frac{2}{3}(x - 1)$;
$\overline{BC}$: $y = 4 - \frac{2}{3}(x - 7)$ or $y = 6 - \frac{2}{3}(x - 4)$;
$\overline{CD}$: $y = 1 + 3(x - 6)$ or $y = 4 + 3(x - 7)$;
$\overline{DE}$: $y = 1$; $\overline{AE}$: $y = 4 - 3(x - 1)$ or $y = 1 - 3(x - 2)$

LESSON 6.4

1a. $\begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ -8 \end{bmatrix}$

1b. $\begin{bmatrix} 1 & 2 & 1 \\ 3 & -4 & 5 \\ -2 & -8 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -11 \\ 1 \end{bmatrix}$

1c. $\begin{bmatrix} 5.2 & 3.6 \\ -5.2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 8.2 \end{bmatrix}$

1d. $\begin{bmatrix} \frac{1}{4} & -\frac{2}{5} \\ \frac{3}{8} & \frac{2}{5} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

3a. $\begin{bmatrix} 1a + 5c & 1b + 5d \\ 6a + 2c & 6b + 2d \end{bmatrix} = \begin{bmatrix} -7 & 33 \\ 14 & -26 \end{bmatrix}$;
$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & -7 \\ -2 & 8 \end{bmatrix}$

3b. $\begin{bmatrix} 1a + 5c & 1b + 5d \\ 6a + 2c & 6b + 2d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$;
$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -\frac{1}{14} & \frac{5}{28} \\ \frac{3}{14} & -\frac{1}{28} \end{bmatrix}$

5a. $\begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}$

5b. $\begin{bmatrix} -\frac{1}{6} & \frac{2}{3} & \frac{1}{9} \\ \frac{1}{2} & -1 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$ or $\begin{bmatrix} 0.167 & 0.667 & 0.111 \\ 0.5 & -1 & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$

5c. $\begin{bmatrix} \frac{7}{5} & -\frac{3}{5} \\ -2 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1.4 & -0.6 \\ -2 & 1 \end{bmatrix}$

5d. Inverse does not exist.

7a. Jolly rides cost \$0.50, Adventure rides cost \$0.85, and Thrill rides cost \$1.50.

7b. \$28.50

7c. Carey would have been better off buying a ticket book.

9. 20°, 50°, 110°

11. $x = 0.0016$, $y = 0.0126$, $z = 0.0110$

13a. $\begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}$

13b. $\begin{bmatrix} -\frac{5}{9} & \frac{13}{9} & \frac{1}{9} \\ \frac{1}{2} & -1 & 0 \\ -\frac{7}{6} & \frac{7}{3} & \frac{1}{3} \end{bmatrix}$ or

$\begin{bmatrix} -0.5555 & 1.4444 & 0.1111 \\ 0.5 & -1 & 0 \\ -1.6666 & 2.3333 & 0.3333 \end{bmatrix}$

15. *Hint:* You want to write a second equation that would result in the graph of the same line.

17a. $[A] = \begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

17b. It is 0 because there are zero roads connecting Murray to itself.

17c. The matrix has reflection symmetry across the main diagonal.

17d. 5; 10. The matrix sum is twice the number of roads; each road is counted twice in the matrix because it can be traveled in either direction.

17e. For example, if the road between Davis and Terre is one-way toward Davis, a_{34} changes from 1 to 0. The matrix is no longer symmetric.

LESSON 6.5

1a. $y < \frac{10 - 2x}{-5}$ or $y < -2 + 0.4x$

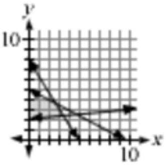
1b. $y < \frac{6 - 2x}{-12}$ or $y < -\frac{1}{2} + \frac{1}{6}x$

3a. $y < 2 - 0.5x$

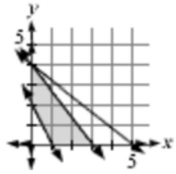
3b. $y \geq 3 + 1.5x$

3c. $y > 1 - 0.75x$

3d. $y \leq 1.5 + 0.5x$

5. 

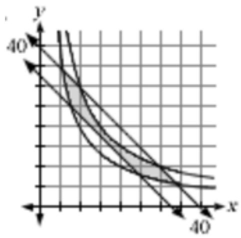
vertices: (0, 2), (0, 5), (2.752, 3.596), (3.529, 2.353)

7. 

vertices: (0, 4), (3, 0), (1, 0), (0, 2)

9a. Let x represent length in inches, and let y represent width in inches.

$$\begin{cases} xy \geq 200 \\ xy \leq 300 \\ x + y \geq 33 \\ x + y \leq 40 \end{cases}$$

9b. 

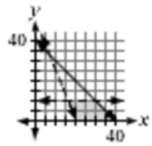
9c. i. no **9c. ii.** yes **9c. iii.** no

11a. $5x + 2y > 100$

11b. $y < 10$

11c. $x + y \leq 40$

11d. common sense: $x \geq 0$, $y \geq 0$



11e. (20, 0), (40, 0), (30, 10), (16, 10)

13. $a = 100$, $b \approx 0.7$

15a. 2 or 3 spores

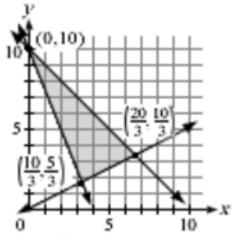
15b. about 1,868,302 spores

15c. $x = \dfrac{\log \frac{y}{2.68}}{\log 3.84}$

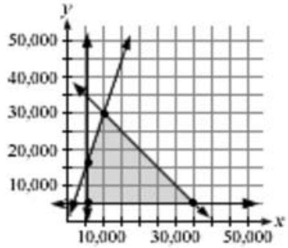
15d. after 14 hr 40 min

LESSON 6.6

1.



3.



vertices: (5500, 5000), (5500, 16500), (10000, 30000), (35000, 5000); (35000, 5000); maximum: 3300

5a. possible answer:

$$\begin{cases} y \geq 7 \\ y \leq \frac{7}{5}(x-3)+6 \\ y \leq -\frac{7}{12}x + 13 \end{cases}$$

5b. possible answer:

$$\begin{cases} x \geq 0 \\ y \geq 7 \\ y \geq \frac{7}{5}(x-3)+6 \\ y \leq -\frac{7}{12}x + 13 \end{cases}$$

5c. possible answer:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x \leq 11 \\ y \leq \frac{7}{5}(x-3)+6 \\ y \leq 7 \end{cases}$$

7. 5 radio minutes and 10 newspaper ads to reach a maximum of 155,000 people. This requires the assumption that people who listen to the radio are independent of people who read the newspaper, which is probably not realistic.

9. 3000 acres of coffee and 4500 acres of cocoa for a maximum total income of $289,800

11a. $x = -\frac{7}{11}, y = \frac{169}{11}$

11b. $x = -3.5$, $y = 74$, $z = 31$

13. $\begin{cases} x \geq 2 \\ y \leq 5 \\ x + y \geq 3 \\ 2x - y \leq 9 \end{cases}$

15. $y = -\left(\frac{x}{2}\right)^2 - \frac{3}{2}$ or $y = -\frac{1}{4}x^2 - \frac{3}{2}$

CHAPTER 6 REVIEW

1a. impossible because the dimensions are not the same

1b. $\begin{bmatrix} -4 & 7 \\ 1 & 2 \end{bmatrix}$

1c. $\begin{bmatrix} -12 & 4 & 8 \\ 8 & 12 & -8 \end{bmatrix}$

1d. $\begin{bmatrix} -3 & 1 & 2 \\ -11 & 11 & 6 \end{bmatrix}$

1e. impossible because the inside dimensions do not match

1f. $[-7 \quad -5 \quad 6]$

3a. $x = 2.5$, $y = 7$

3b. $x = 1.22$, $y = 6.9$, $z = 3.4$

5a. consistent and independent

5b. consistent and dependent

5c. inconsistent

5d. inconsistent

7. about 4.4 yr

9a. $\begin{bmatrix} .92 & .08 & 0 \\ .12 & .82 & .06 \\ 0 & .15 & .85 \end{bmatrix}$

9b. i. Mozart: 81; Picasso: 66; Hemingway: 63

9b. ii. Mozart: 82; Picasso: 70; Hemingway: 58

9b. iii. Mozart: 94; Picasso: 76; Hemingway: 40

11a. $a < 0$; $p < 0$; $d > 0$

11b. $a > 0$; $p > 0$; d cannot be determined

11c. $a > 0$; $p = 0$; $d < 0$

13. 20 students in second period, 18 students in third period, and 24 students in seventh period

15a. $x = 245$

15b. $x = 20$

15c. $x = -\frac{1}{2}$

15d. $x = \dfrac{\log\left(\frac{37000}{15}\right)}{\log 9.4} \approx 3.4858$

15e. $x = 21$

15f. $x = \dfrac{\log 342}{\log 36} \approx 1.6282$

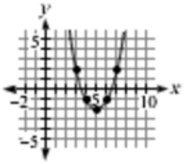
17a. $y = 50(0.72)^{x-4}$ or $y = 25.92(0.72)^{x-6}$

17b. 0.72; decay

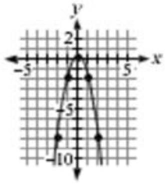
17c. approximately 186

17d. 0

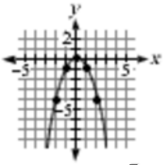
19a. a translation right 5 units and down 2 units



19b. a reflection across the x-axis and a vertical stretch by a factor of 2



19c. $-1 \cdot [P] = \begin{bmatrix} 2 & 1 & 0 & -1 & -2 \\ -4 & -1 & 0 & -1 & -4 \end{bmatrix}$;
This is a reflection across the x-axis and a reflection across the y-axis. However, because the graph is symmetric with respect to the y-axis, a reflection over that axis does not change the graph.



19d. $[P] + \begin{bmatrix} -2 & -2 & -2 & -2 & -2 \\ 3 & 3 & 3 & 3 & 3 \end{bmatrix} = \begin{bmatrix} -4 & -3 & -2 & -1 & 0 \\ 7 & 4 & 3 & 4 & 7 \end{bmatrix}$

CHAPTER 7 • CHAPTER 7 CHAPTER 7 • CHAPTER

LESSON 7.1

1a. 3 **1b.** 2 **1c.** 7 **1d.** 5
3a. no; {2.2, 2.6, 1.8, –0.2, –3.4}
3b. no; {0.007, 0.006, 0.008, 0.010}
3c. no; {150, 150, 150}
5a. D_1 = {2, 3, 4, 5, 6}; D_2 = {1, 1, 1, 1}; 2nd degree
5b. The polynomial is 2nd degree, and the D_2 values are constant.
5c. 4 points. You have to find the finite differences twice, so you need at least four data points to calculate two D_2 values that can be compared.
5d. $s = 0.5n^2 + 0.5n$; $s = 78$
5e. The pennies can be arranged to form triangles.

7a. i. D_1 = {15.1, 5.3, –4.5, –14.3, –24.1, –33.9}; D_2 = {-9.8, -9.8, -9.8, -9.8, -9.8}
7a. ii. D_1 = {59.1, 49.3, 39.5, 29.7, 19.9, 10.1}; D_2 = {–9.8, –9.8, –9.8, –9.8, –9.8}
7b. i. 2; **ii.** 2
7c. i. $h = -4.9t^2 + 20t + 80$; **ii.** $h = -4.9t^2 + 64t + 4$
9. Let x represent the energy level, and let y represent the maximum number of electrons; $y = 2x^2$.
11a. $x = 2.5$
11b. $x = 3$ or $x = -1$
11c. $x = \dfrac{\log 16}{\log 5} \approx 1.7227$
13. $\begin{cases} y \geq -\frac{1}{2}x + \frac{3}{2} \\ y \leq \frac{1}{2}x + \frac{9}{2} \\ y \leq -\frac{11}{6}x + \frac{97}{6} \end{cases}$

LESSON 7.2

1a. factored form and vertex form
1b. none of these forms
1c. factored form
1d. general form
3a. – 1 and 2 **3b.** – 3 and 2 **3c.** 2 and 5
5a. $y = x^2 - x - 2$
5b. $y = 0.5x^2 + 0.5x - 3$
5c. $y = -2x^2 + 14x - 20$
7a. $y = -0.5x^2 - hx - 0.5h^2 + 4$
7b. $y = ax^2 - 8ax + 16a$
7c. $y = ax^2 - 2ahx + ah^2 + k$
7d. $y = -0.5x^2 - (0.5r + 2)x - 2r$
7e. $y = ax^2 - 2ax - 8a$
7f. $y = ax^2 - a(r + s)x + ars$
9a. $y = (x + 2)(x - 1)$
9b. $y = -0.5(x + 2)(x - 3)$
9c. $y = \frac{1}{3}(x + 2)(x - 1)(x - 3)$
11a. lengths: 35, 30, 25, 20, 15; areas: 175, 300, 375, 400, 375
11b. $y = x(40 - x)$ or $y = -x^2 + 40x$
11c. 20 m; 400 m^2.
11d. 0 m and 40 m
13a. $12x^2 - 15x$ **13b.** $x^2 - 2x - 15$
13c. $x^2 - 49$ **13d.** $9x^2 - 6x + 1$
15a. $(x + 5)(x - 2)$ **15b.** $(x + 4)(x + 4)$
15c. $(x + 5)(x - 5)$

LESSON 7.3

1a. $(x-5)^2$ **1b.** $\left(x+\frac{5}{2}\right)^2$

1c. $(2x-3)^2$ or $4\left(x-\frac{3}{2}\right)^2$ **1d.** $(x-y)^2$

3a. $y=(x+10)^2-6$ **3b.** $y=(x-3.5)^2+3.75$

3c. $y=6(x-2)^2+123$ **3d.** $y=5(x+0.8)^2-3.2$

5. $(-4, 12)$

7a. Let x represent time in seconds, and let y represent height in meters; $y=-4.9(x-1.1)(x-4.7)$ or $y=-4.9x^2+28.42x-25.333$.

7b. 28.42 m/s **7c.** 25.333 m

9. Let x represent time in seconds, and let y represent height in meters;

$y=-4.9x^2+17.2x+50$.

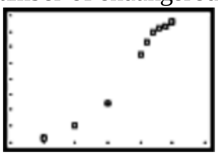
11a. $n=-2p+100$ **11b.** $R(p)=-2p^2+100p$

11c. Vertex form: $R(p)=-2(p-25)^2+1250$. The vertex is (25, 1250). This means that the maximum revenue is \$1250 when the price is \$25.

11d. between \$15 and \$35

13. $x=2$, $x=-3$, or $x=\frac{1}{2}$

15a. Let x represent the year, and let y represent the number of endangered species.



[1975, 2005, 5, 200, 1000, 100]

15b. $\hat{y}=45.64x-90289$

15c. approximately 1219 species in 2005; 3273 species in 2050

LESSON 7.4

1a. $x=7.3$ or $x=-2.7$ **1b.** $x=-0.95$ or $x=-7.95$

1c. $x=2$ or $x=-\frac{1}{2}$

3a. -0.102 **3b.** -5.898 **3c.** -0.243 **3d.** 8.243

5a. $y=(x-1)(x-5)$ **5b.** $y=(x+2)(x-9)$

5c. $y=5(x+1)(x+1.4)$

7a. $y=a(x-3)(x+3)$ for $a\neq 0$

7b. $y=a(x-4)(x+\frac{2}{5})$ or $y=a(x-4)(5x+2)$ for $a\neq 0$

7c. $y=a(x-r_1)(x-r_2)$ for $a\neq 0$

9. *Hint:* When will the quadratic formula result in no real solutions?

11a. $y=-4x^2-6.8x+49.2$

11b. 49.2 L **11c.** 2.76 min

13a. $x^2+14x+49=(x+7)^2$

13b. $x^2-10x+25=(x-5)^2$

13c. $x^2+3x+\frac{9}{4}=\left(x+\frac{3}{2}\right)^2$

13d. $2x^2+8x+8=2(x^2+4x+4)=2(x+2)^2$

15a. $y=2x^2-x-15$ **15b.** $y=-2x^2+4x+2$

17. $a=k=52.08\overline{3}$ ft; $b=j=33.\overline{3}$ ft; $c=i=18.75$ ft; $d=h=8.\overline{3}$ ft; $e=g=2.08\overline{3}$ ft; $f=0$; $229.1\overline{6}$ ft

LESSON 7.5

1a. $8+4i$ **1b.** 7

1c. $4-2i$ **1d.** $-2.56-0.61i$

3a. $5+i$ **3b.** $-1-2i$

3c. $2-3i$ **3d.** $-2.35+2.71i$

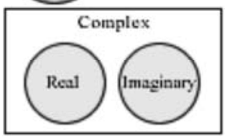
5a.

5b.

5c.

5d.

5e.



7a. $-i$ **7b.** 1 **7c.** i **7d.** -1

9. $0.2+1.6i$

11a. $y=x^2-2x-15$ **11b.** $y=x^2+7x+12.25$

11c. $y=x^2+25$ **11d.** $y=x^2-4x+5$

13a. $x=(5+\sqrt{34})i\approx 10.83i$ or $x=(5-\sqrt{34})i\approx -0.83i$

13b. $x=2i$ or $x=i$

13c. The coefficients of the quadratic equations are nonreal.

15a. 0, 0, 0, 0, 0, 0; remains constant at 0

15b. $0, i, -1+i, -i, -1+i, -i$; alternates between $-1+i$ and $-i$

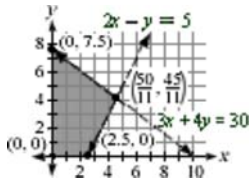
15c. $0, 1-i, 1-3i, -7-7i, 1+97i, -9407+193i$; no recognizable pattern in these six terms

15d. $0, 0.2+0.2i, 0.2+0.28i, 0.1616+0.312i, 0.12877056+0.3008384i, 0.1260781142+0.2774782585i$; approaches $0.142120634+0.2794237653i$

17a. Let x represent the first integer, and let y represent the second integer.

$$\begin{cases} x > 0 \\ y > 0 \\ 3x + 4y < 30 \\ 2x < y + 5 \end{cases}$$

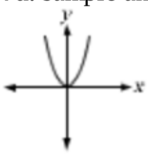
17b.



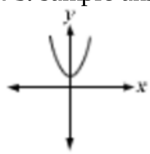
17c. (1, 1), (2, 1), (1, 2), (2, 2), (3, 2), (1, 3), (2, 3), (3, 3), (1, 4), (2, 4), (3, 4), (4, 4), (1, 5), (2, 5), (3, 5), (1, 6)

LESSON 7.6

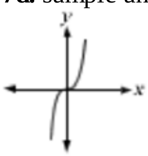
1a. x-intercepts: –1.5, –6; y-intercept: –2.25
1b. x-intercept: 4; y-intercept: 48
1c. x-intercepts: 3, –2, –5; y-intercept: 60
1d. x-intercepts: –3, 3; y-intercept: –135
3a. $y = x^2 - 10x + 24$ **3b.** $y = x^2 - 6x + 9$
3c. $y = x^3 - 64x$ **3d.** $y = 3x^3 + 15x^2 - 12x - 60$
5a. approximately 2.94 units; approximately 420 cubic units
5b. 5 and approximately 1.28
5c. The graph exists, but these x- and y-values make no physical sense for this context. If $x \geq 8$, there will be no box left after you take out two 8-unit square corners from the 16-unit width.
5d. The graph exists, but these x- and y-values make no physical sense for this context.
7a. sample answer:



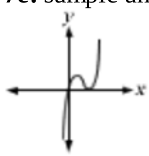
7b. sample answer:



7c. not possible
7d. sample answer:



7e. sample answer:



7f. not possible

9a. $(T + t)^2$ or $T^2 + 2Tt + t^2$

	T	t
T	TT	Tt
t	Tt	tt

9b. $(T + t)^2 = 1$ or $T^2 + 2Tt + t^2 = 1$
9c. $0.70 + t^2 = 1$
9d. $t \approx 0.548$ **9e.** $T \approx 0.452$
9f. $TT \approx 0.204$, or about 20% of the population
11. $y = 0.25(x + 2)^2 + 3$
13a. $f^{-1}(x) = \frac{3}{2}x - 5$
13b. $g^{-1}(x) = -3 + (x + 6)^{3/2}$
13c. $h^{-1}(x) = \log_2(7 - x)$

LESSON 7.7

1a. $x = -5$, $x = 3$, and $x = 7$
1b. $x = -6$, $x = -3$, $x = 2$, and $x = 6$
1c. $x = -5$ and $x = 2$
1d. $x = -5$, $x = -3$, $x = 1$, $x = 4$, and $x = 6$
3a. 3 **3b.** 4 **3c.** 2 **3d.** 5
5a. $y = a(x - 4)$ where $a \neq 0$
5b. $y = a(x - 4)^2$ where $a \neq 0$
5c. $y = a(x - 4)^3$ where $a \neq 0$; or $y = a(x - 4)(x - r_1)(x - r_2)$ where $a \neq 0$, and r_1 and r_2 are complex conjugates
7a. 4 **7b.** 5 **7c.** $y = -x(x + 5)^2(x + 1)(x - 4)$
9. The leading coefficient is equal to the y-intercept divided by the product of the zeros if the degree of the function is even, or the y-intercept divided by – 1 times the product of the zeros if the degree of the function is odd.
11a. i. $y = (x + 5)^2(x + 2)(x - 1)$
11a. ii. $y = -(x + 5)^2(x + 2)(x - 1)$
11a. iii. $y = (x + 5)^2(x + 2)(x - 1)^2$
11a. iv. $y = -(x + 5)(x + 2)^3(x - 1)$
11b. i. $x = -5$, $x = -5$, $x = -2$, and $x = 1$
11b. ii. $x = -5$, $x = -5$, $x = -2$, and $x = 1$
11b. iii. $x = -5$, $x = -5$, $x = -2$, $x = 1$, and $x = 1$
11b. iv. $x = -5$, $x = -2$, $x = -2$, $x = -2$, and $x = 1$
13. *Hint:* A polynomial function of degree n will have at most $n - 1$ extreme values and n x-intercepts.
15. $3 - 5\sqrt{2}$; $0 = a(x^2 - 6x - 41)$ where $a \neq 0$

17a. $\begin{bmatrix} 4 & 9 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{5}{2} \\ -\frac{2}{3} \end{bmatrix}$

17b. $\left[\begin{array}{cc|c} 4 & 9 & 4 \\ 2 & -3 & 7 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & \frac{5}{2} \\ 0 & 1 & -\frac{2}{3} \end{array}\right]$

LESSON 7.8

1a. $3x^2 + 7x + 3$ **1b.** $6x^3 - 4x^2$

3a. $a = 12$ **3b.** $b = 2$ **3c.** $c = 7$ **3d.** $d = -4$

5. $\pm 15, \pm 5, \pm 3, \pm 1, \pm\frac{15}{2}, \pm\frac{5}{2}, \pm\frac{3}{2}, \pm\frac{1}{2}$

7a. $2(3i)^3 - (3i)^2 + 18(3i) - 9 =$
$-54i + 9 + 54i - 9 = 0$

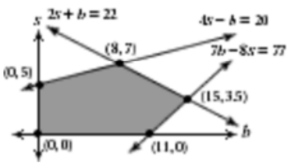
7b. $x = -3i$ and $x = \frac{1}{2}$

9. $y = (x - 3)(x + 5)(2x - 1)$ or
$y = 2(x-3)(x+5)\left(x - \frac{1}{2}\right)$

11a. $f(x) = 0.00639x^{3/2}$ **11b.** $f^{-1}(x) \approx (156x)^{2/3}$

11c. 33 in. **11d.** about 176 ft

13a.



13b. 15 baseball caps and 3 sun hats; $33

15a. $x = -3$ or $x = 1$ **15b.** $x = \frac{-3 \pm \sqrt{37}}{2}$

15c. $x = 1 \pm 2i$

CHAPTER 7 REVIEW

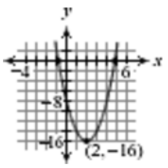
1a. $2(x - 2)(x - 3)$

1b. $(2x + 1)(x + 3)$ or $2(x + 0.5)(x + 3)$

1c. $x(x - 12)(x + 2)$

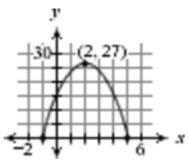
3. 1; 4; 10; $\frac{1}{6}n^3 - \frac{1}{2}n^2 + \frac{1}{3}n$

5a.



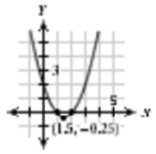
zeros: $x = -0.83$ and $x = 4.83$

5b.



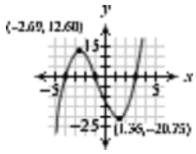
zeros: $x = -1$ and $x = 5$

5c.



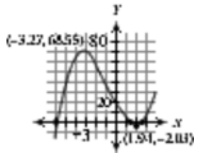
zeros: $x = 1$ and $x = 2$

5d.



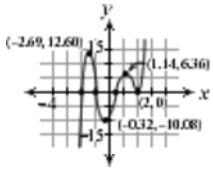
zeros: $x = -4$, $x = -1$, and $x = 3$

5e.



zeros: $x = -5.84$, $x = 1.41$, and $x = 2.43$

5f.



zeros: $x = -2$, $x = -1$, $x = 0.5$, and $x = 2$

7. 18 in. by 18 in. by 36 in.

9a. $y = 0.5x^2 + 0.5x + 1$

9b. 16 pieces; 56 pieces

11a. $\pm 1, \pm 3, \pm 13, \pm 39, \pm\frac{1}{3}, \pm\frac{13}{3}$

11b. $x = -\frac{1}{3}$, $x = 3$, $x = 2 + 3i$, and $x = 2 - 3i$

13. $2x^2 + 4x + 3$

CHAPTER 8 • CHAPTER 8 CHAPTER 8 • CHAPTER

LESSON 8.1

1a.

t	x	y
–2	–7	–3
–1	–4	–1
0	–1	1
1	2	3
2	5	5

1b.

t	x	y
–2	–1	4
–1	0	1
0	1	0
1	2	1
2	3	4

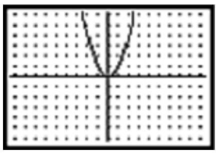
1c.

t	x	y
–2	4	1
–1	1	2
0	0	3
1	1	4
2	4	5

1d.

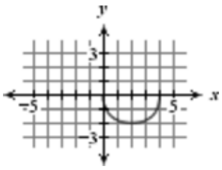
t	x	y
–2	–3	0
–1	–2	1.73
0	–1	2
1	0	1.73
2	1	0

3a.

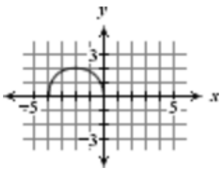


[–9.4, 9.4, 1, –6.2, 6.2, 1]

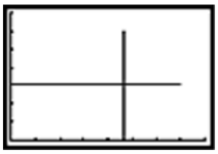
3b. The graph is translated right 2 units.
3c. The graph is translated down 3 units.
3d. The graph is translated right 5 units and up 2 units.
3e. The graph is translated horizontally a units and vertically b units.
5a. 15 s **5b.** 30 yd **5c.** –2 yd/s
5d. Sample answer: 65 yd is her starting position relative to the goal line, –2 yd/s is her velocity, and 50 yd is her position relative to the sideline.
5e. The graph simulation will produce the graphs pictured in the problem. A good window is [0, 100, 10, 0, 60, 10] with $0 \le t \le 15$.
5f. She crosses the 10-yard line after 27.5 s.
5g. $65 - 2t = 10$; 27.5 s
7a. The graph is reflected across the x-axis.



7b. The graph is reflected across the y-axis.



9a. $x = 0.4t$ and $y = 1$
9b. [0, 50, 5, 0, 3, 1]; $0 \le t \le 125$
9c. $x = 1.8(t - 100)$, $y = 2$
9d. The tortoise will win.
9e. The tortoise takes 125 s and the hare takes approximately 28 s, but because he starts 100 s later, he finishes at 128 s.
11a.

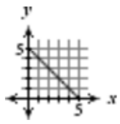


[0, 8, 1, 0, 7, 1]

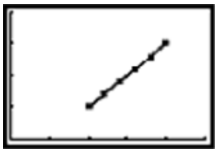
11b. 1.4 m/s is the velocity of the first walker, 3.1 m is the vertical distance between the walkers when they start, 4.7 m is the horizontal distance between the walkers when they start, and 1.2 m/s is the velocity of the second walker.
11c. (4.7, 3.1)
11d. No, the first walker arrives at (4.7, 3.1) at 3.357 s, and the second walker arrives there at 2.583 s.
13. (7, –3)
15a. $2.5n^2 - 5.5n - 3$ **15b.** 887
17. $y = -2x^2 + 5x - 2$

LESSON 8.2

1a. $t = x - 1$ **1b.** $t = \frac{x+1}{3}$
1c. $t = \pm\sqrt{x}$ **1d.** $t = x + 1$
3a. $y = \frac{x+7}{2}$ **3b.** $y = \pm\sqrt{x} + 1$
3c. $y = \frac{2x-4}{3}$ **3d.** $y = 2(x + 2)^2$
5.



7. $-2.5 \le t \le 2.5$
9a. $x = 20 + 2t$, $y = 5 + t$
9b.



[0, 50, 10, 0, 20, 5]
$0 \le t \le 10$

The points lie on the line.
9c. $y = \frac{1}{2}x - 5$
9d. The slope of the line in 9c is the ratio of the y-slope over the x-slope in the parametric equations.
11a. $x = 1$, $y = 1.5t$
11b. $x = 1.1$, $y = 12 - 2.5t$
11c. possible answer: [0, 2, 1, 0, 12, 1]; $0 \le t \le 3$
11d.

[0, 2, 1, 0, 12, 1]
$0 \le t \le 3$

They meet after hiking 3 h, when both are 4.5 mi north of the trailhead.
11e. $1.5t = 12 - 2.5t$; $t = 3$; substitute $t = 3$ into either y-equation to get $y = 4.5$.

13. $x = t^2$, $y = t$

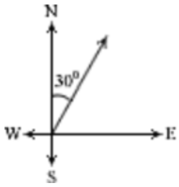
15. $y = \left(\frac{2}{3}(x-5) - 2\right) + 3$ or $y = \frac{2}{3}x - \frac{7}{3}$

LESSON 8.3

1. $\sin A = \frac{k}{j}$; $\sin B = \frac{h}{j}$;
$\sin^{-1}\left(\frac{k}{j}\right) = A$; $\sin^{-1}\left(\frac{h}{j}\right) = B$;
$\cos B = \frac{k}{j}$; $\cos A = \frac{h}{j}$;
$\cos^{-1}\left(\frac{k}{j}\right) = B$; $\cos^{-1}\left(\frac{h}{j}\right) = A$;
$\tan A = \frac{k}{h}$; $\tan B = \frac{h}{k}$;
$\tan^{-1}\left(\frac{k}{h}\right) = A$; $\tan^{-1}\left(\frac{h}{k}\right) = B$

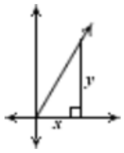
3a. $a \approx 17.3$ **3b.** $b \approx 22.8$ **3c.** $c \approx 79.3$

5.



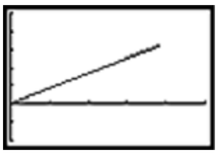
5a. 60°

5b.



5c. 180 mi east, 311.8 mi north

7a.



[0, 5, 1, –2, 5, 1]

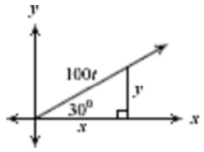
$0 \leq t \leq 1$

7b. It is a segment 5 units long, at an angle of 40° above the *x*-axis.

7c. This is the value of the angle in the equations.

7d. It makes the segment 5 units long when $t = 1$; the graph becomes steeper, and the segment becomes shorter; the graph becomes shorter; but the slope is the same as it was originally.

9a.

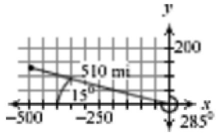


$x = 100t \cos 30°$, $y = 100t \sin 30°$

9b. $0 \leq t \leq 5$

9c. 100 represents the speed of the plane in miles per hour, *t* represents time in hours, 30° is the angle the plane is making with the *x*-axis, *x* is the horizontal position at any time, and *y* is the vertical position at any time.

11a.



11b. 23.2 h **11c.** 492.6 mi west, 132.0 mi north

11d. The paths cross at approximately 480 mi west and 129 mi north of St. Petersburg. No, the ships do not collide because Tanker A reaches this point after 24.4 h and Tanker B reaches this point after 22.6 h.

13a. $y = 5 + \frac{3}{4}(x-6)$ or $y = \frac{1}{2} + \frac{3}{4}x$

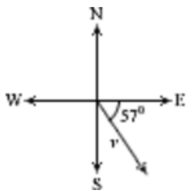
13b. $y = 5 + \frac{3(x-6)}{4}$. They are the same equation.

15. $(x - 2.6)^2 + (y + 4.5)^2 = 12.96$

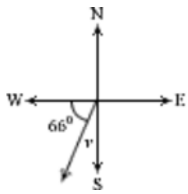
LESSON 8.4

1. $x = 10t \cos 30°$, $y = 10t \sin 30°$

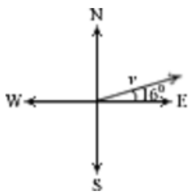
3a.



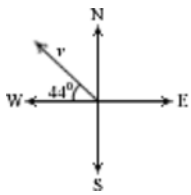
3b.



3c.

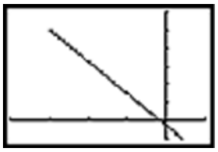


3d.



5a. (– 0.3, 0.5) **5b.** $x = -0.3 + 4t$

5c. $y = 0.5 - 7t$

5d. 

[– 0.4, 0.1, 0.1, – 0.1, 0.6, 0.1]

$0 \le t \le 0.1$

5e. At 0.075 h (4.5 min), the boat lands 0.025 km (25 m) south of the dock.

5f. 0.605 km

7a. $y = -5t$ **7b.** $x = st$ **7c.** $s = 10$ mi/h

7d. 4.47 mi **7e.** 0.4 h **7f.** 11.18 mi/h

7g. 63.4°

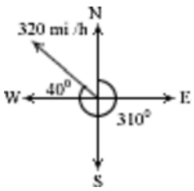
9a. $y = -20t \sin 45°$ **9b.** $x = 20t \cos 45°$

9c. Both the plane's motion and the wind contribute to the actual path of the plane, so you add the x-contributions and add the y-contributions to form the final equations.

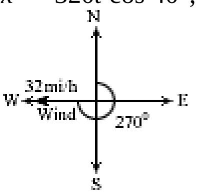
9d. possible answer:
[-1000, 0, 100, -100, 0, 10]; $0 \le t \le 5$

9e. 4.24. It takes the plane 4.24 h to fly 1000 mi west.

9f. 60 mi

11a. 

11b. $x = -320t \cos 40°$, $y = 320t \sin 40°$

11c. 

11d. $x = -32t$, $y = 0$

11e. $x = -320t \cos 40° - 32t$, $y = 320t \sin 40°$

11f. 1385.7 mi west and 1028.5 mi north

13a. x-component: $50 \cos 40° \approx 38.3$;
y-component: $50 \sin 40° \approx 32.1$

13b. x-component: $90 \cos 140° \approx -68.9$;
y-component: $90 \sin 140° \approx 57.9$

13c. x-component: – 30.6; y-component: 90.0

13d. 95.1 N **13e.** 109° **13f.** 95.1 N at 289°

15a. two real, rational roots

15b. two real, irrational roots

15c. no real roots **15d.** one real, rational root

LESSON 8.5

1a. the Moon; centimeters and seconds

1b. right 400 cm and up 700 cm

1c. up-left **1d.** 50 cm/s

3a. $x = 2t$, $y = -4.9t^2 + 12$

3b. $-4.9t^2 + 12 = 0$

3c. 1.56 s, 3.13 m from the cliff

3d. possible answer: [0, 4, 1, 0, 12, 1]

5a. possible answer: [0, 5, 1, 0, 3, 5, 1], $0 \le t \le 1.5$

5b. *Hint:* Describe the initial angle, velocity, and position of the projectile. Be sure to include units, and state what planet the motion occurred on.

7a. $x = 83t \cos 0°$, $y = -4.9t^2 + 83t \sin 0° + 1.2$

7b. No; it will hit the ground 28.93 m before reaching the target.

7c. The angle must be between 2.44° and 3.43°.

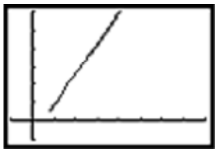
7d. at least 217 m/s

9. 46 ft from the end of the cannon

11a. $x = 122t \cos 38°$, $y = -16t^2 + 122t \sin 38°$

11b. 451 ft **11c.** 378 ft

13a. $x = 2.3t + 4$, $y = 3.8t + 3$



[–5, 40, 5, –5, 30, 5]

13b. 4.44 m/s on a bearing of 31°

15. $a(4x^3 + 8x^2 - 23x - 33) = 0$, where a is an integer, $a \ne 0$

LESSON 8.6

1. 9.7 cm

3. $X \approx 50.2°$ and $Z \approx 92.8°$

5a. $B = 25.5°$; $BC \approx 6.4$ cm; $AB \approx 8.35$ cm

5b. $J \approx 38.8°$; $L \approx 33.3°$; $KJ \approx 4.77$ cm

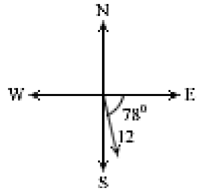
7a. 12.19 cm

7b. Because the triangle is isosceles, knowing the measure of one angle allows you to determine the measures of all three angles.

9. 2.5 km

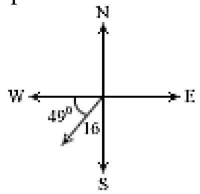
11a. 41° **11b.** 70° **11c.** 0°

13a.



x-component: $12\cos 78° \approx 2.5$;
y-component: $-12\sin 78° \approx -11.7$

13b.



x-component: $-16\cos 49° \approx -10.5$;
y-component: $-16\sin 49° \approx -12.1$

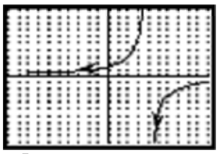
15a. \$26,376.31 **15b.** 20 years 11 months

LESSON 8.7

1. approximately 6.1 km
3a. $A \approx 41.4°$ **3b.** $b = 8$
5. 1659.8 mi
7. From point *A*, the underground chamber is at a 22° angle from the ground between *A* and *B*. From point *B*, the chamber is at a 120° angle from the ground. If the truck goes 1.5 km farther in the same direction, the chamber will be approximately 2.6 km directly beneath the truck.
9. 2.02 mi
11. 10.3 nautical mi
13. 1751 cm^2

CHAPTER 8 REVIEW

1a. $t = 3$: $x = -8$, $y = 0.5$; $t = 0$: $x = 1$, $y = 2$; $t = -3$: $x = 10$, $y = -1$
1b. $y = \frac{6}{11}$ **1c.** $x = \frac{5}{2}$
1d. When $t = -1$, the y-value is undefined.

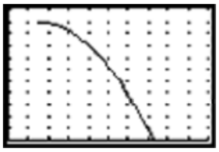


[–10, 10, 1, –10, 10, 1]

3a. $y = \frac{x+7}{2}$. The graph is the same.

3b. $y = \pm\sqrt{x-1} - 2$. The graph is the same except for the restrictions on t.
3c. $y = (2x-1)^2$. The graph is the same. The values of t are restricted, but endpoints are not visible within the calculator screen given.
3d. $y = x^2 - 5$. The graph is the same, except the parametric equations will not allow for negative values for x.

5a. $A \approx 43°$ **5b.** $B \approx 28°$ **5c.** $c \approx 23.0$
5d. $d \approx 12.9$ **5e.** $e \approx 21.4$ **5f.** $f \approx 17.1$
7. 7.2 m



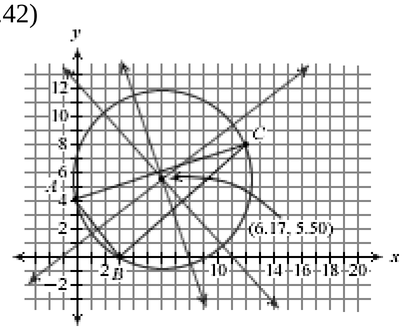
[0, 10, 1, 0, 11, 1]

9. She will miss it by 11.1 ft.
11a. $a \approx 7.8$ m, $c \approx 6.7$ m, $C = 42°$
11b. $A \approx 40°$, $b \approx 3.5$ cm, $C \approx 58°$

CHAPTER 9 • CHAPTER **9** CHAPTER 9 • CHAPTER

LESSON 9.1

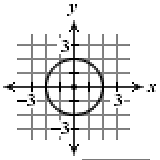
1a. 10 units **1b.** $\sqrt{74}$ units
1c. $\sqrt{85}$ units **1d.** $\sqrt{81+4d^2}$ units
3. $x = -1 \pm \sqrt{2160}$ or $x = -1 \pm 12\sqrt{15}$
5. approximately 25.34 units
7. approximately between the points (2.5, 2.134) and (2.5, 3.866)
9a. $y = \sqrt{10^2 + x^2} + \sqrt{(20-x)^2 + 13^2}$
9b. domain: $0 \le x \le 20$; range: $30 < y < 36$
9c. When the wire is fastened approximately 8.696 m from the 10 m pole, the minimum length is approximately 30.48 m.
11a. $d = \sqrt{(5-x)^2 + (0.5x^2 + 4)^2}$
11b. approximately 6.02 units; approximately (0.92, 1.42)
13a-d.



13b. All three perpendicular bisectors intersect at the same point. No, you could find the intersection by constructing only two perpendicular bisectors.
13c. Approximately (6.17, 5.50); this should agree with the answer to 12a.
13d. Regardless of which point is chosen, the circle passes through A, B, and C. Because the radius of the circle is constant, the distance from the recreation center to all three towns is the same.
15a. midpoint of $\overline{AB}$: (4.5, 1.5); midpoint of $\overline{BC}$: (2.5, 0); midpoint of $\overline{AC}$: (6, – 3.5)
15b. median from A to $\overline{BC}$:
$y = -0.\overline{36}x + 0.\overline{90}$ or $y = -\frac{4}{11}x + \frac{10}{11}$;
median from B to $\overline{AC}$: $y = -1.7x + 6.7$;
median from C to $\overline{AB}$: $y = 13x - 57$
15c. $(4.\overline{3}, -0.\overline{6})$ or $\left(4\frac{1}{3}, -\frac{2}{3}\right)$
17. approximately 44.6 nautical mi
19. $w = 74°$, $x = 50°$

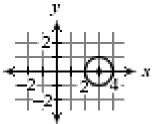
LESSON 9.2

1a. center: (0, 0); radius: 2



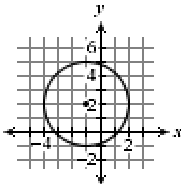
$y = \pm\sqrt{4 - x^2}$

1b. center: (3, 0); radius: 1



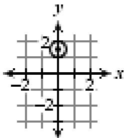
$y = \pm\sqrt{1 - (x-3)^2}$

1c. center: (–1, 2); radius: 3



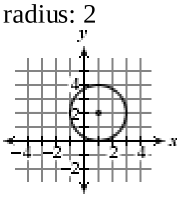
$y = \pm\sqrt{9 - (x+1)^2} + 2$

1d. center: (0, 1.5); radius: 0.5

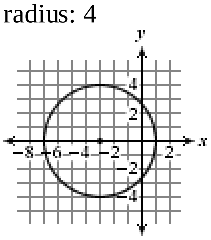


$y = \pm\sqrt{0.25 - x^2} + 1.5$

1e. center: (1, 2); radius: 2



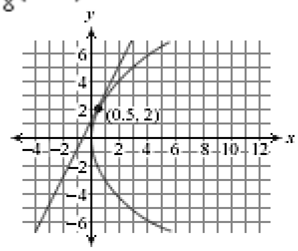
1f. center: (–3, 0); radius: 4



3a. $x = 5\cos t + 3$, $y = 5\sin t$
3b. $x = 3\cos t - 1$, $y = 3\sin t + 2$
3c. $x = 4\cos t + 2.5$, $y = 4\sin t + 0.75$
3d. $x = 0.5\cos t + 2.5$, $y = 0.5\sin t + 1.25$
5a. $x = 2\cos t$, $y = 2\sin t + 3$
5b. $x = 6\cos t - 1$, $y = 6\sin t + 2$
7a. $(\sqrt{27}, 0), (-\sqrt{27}, 0)$
7b. $(3, \sqrt{21}), (3, -\sqrt{21})$
7c. $(-1 + \sqrt{7}, 2), (-1 - \sqrt{7}, 2)$
7d. $(3 + \sqrt{27}, -1), (3 - \sqrt{27}, -1)$
9a. 1.0 m **9b.** 1.6 m
11a. 240 r/min **11b.** 18.6 mi/h **11c.** 6.3 mi/h
13. $y = -(x + 3)^2 + 2$
15. $y = 2x^2 - 24x + 117$

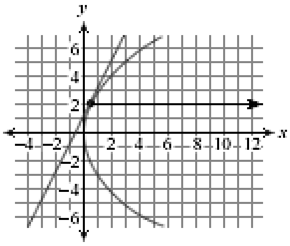
LESSON 9.3

1a. (1, 0.5) **1b.** $y = 8$ **1c.** (9, 2)
3a. focus: (0, 6); directrix: $y = 4$
3b. focus: (– 1.75, – 2); directrix: $x = -2.25$
3c. focus: (– 3, 0); directrix: $y = 1$
3d. focus: (3.875, 0); directrix: $x = 4.125$
3e. focus: (– 1, 5); directrix: $y = 1$
3f. focus: $\left(\frac{61}{12}, 0\right)$; directrix: $x = \frac{11}{12}$
5a. $x = t^2$, $y = t + 2$ **5b.** $x = t$, $y = -t^2 + 4$
5c. $x = 2t + 3$, $y = t^2 - 1$
5d. $x = -t^2 - 6$, $y = 3t + 2$
7. The path is parabolic. If you locate the rock at (0, 2) and the shoreline at $y = 0$, the equation is $y = \frac{1}{4}x^2 + 1$.
9. $y = \frac{1}{8}(x-1)^2 + 1$
11a, c.



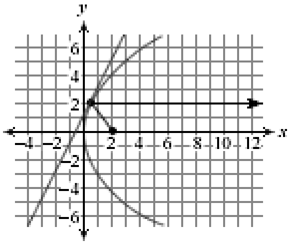
(0.5, 2); $m = 2$
11b. (2, 0)

11d.



$m = 0$

11e.



$m = -\frac{4}{3}$

11f. 63.4°; 63.4°; the angles are congruent.

13. $\frac{\sqrt{3}}{2}$; $\left(\pm\frac{\sqrt{2}}{2}, \frac{1}{2}\right)$

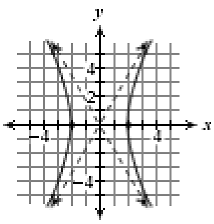
15a. $\pm1, \pm2, \pm5, \pm10, \pm\frac{1}{2}, \pm\frac{5}{2}$

15b. $\frac{1}{2}$ is the only rational root.

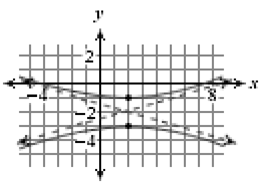
15c. $f(x) = (2x - 1)(x - 1 + 3i)(x - 1 - 3i)$

LESSON 9.4

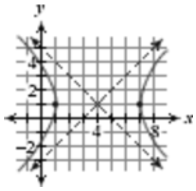
1a. vertices: $(-2, 0)$ and $(2, 0)$; asymptotes: $y = \pm 2x$



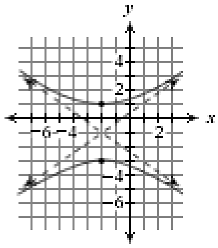
1b. vertices: $(2, -1)$ and $(2, -3)$; asymptotes: $y = \frac{1}{3}x - \frac{8}{3}$ and $y = -\frac{1}{3}x - \frac{4}{3}$



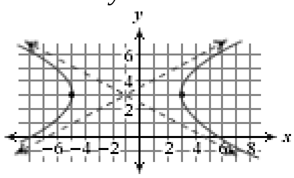
1c. vertices: $(1, 1)$ and $(7, 1)$; asymptotes: $y = x - 3$ and $y = -x + 5$



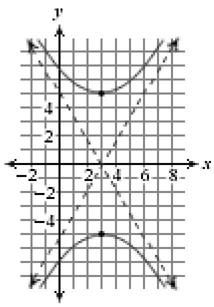
1d. vertices: $(-2, 1)$ and $(-2, -3)$; asymptotes: $y = \frac{2}{3}x + \frac{1}{3}$ and $y = -\frac{2}{3}x - \frac{7}{3}$



1e. vertices: $(-5, 3)$ and $(3, 3)$; asymptotes: $y = 0.5x + 3.5$ and $y = -0.5x + 2.5$



1f. vertices: $(3, 5)$ and $(3, -5)$; asymptotes: $y = \frac{5}{3}x - 5$ and $y = -\frac{5}{3}x + 5$



3a. $\left(\frac{x}{2}\right)^2 - \left(\frac{y}{1}\right)^2 = 1$

3b. $\left(\frac{y+3}{2}\right)^2 - \left(\frac{x-3}{2}\right)^2 = 1$

3c. $\left(\frac{x+2}{3}\right)^2 - \left(\frac{y-1}{4}\right)^2 = 1$

3d. $\left(\frac{y-1}{4}\right)^2 - \left(\frac{x+2}{3}\right)^2 = 1$

5a. $y = \pm 0.5x$

5b. $y = x - 6$ and $y = -x$

5c. $y = \frac{4}{3}x + \frac{11}{3}$ and $y = -\frac{4}{3}x - \frac{5}{3}$

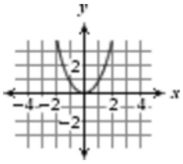
5d. $y = \frac{4}{3}x + \frac{11}{3}$ and $y = -\frac{4}{3}x - \frac{5}{3}$

7. $\left(\frac{x-1}{5}\right)^2 - \left(\frac{y-1}{\sqrt{11}}\right)^2 = 1$

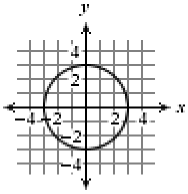
9a. possible answer: $\left(\frac{x-1}{2}\right)^2 - \left(\frac{y}{3}\right)^2 = 1$

9b. possible answer: $\left(\frac{y+2}{3}\right)^2 - \left(\frac{x+4.5}{2.5}\right)^2 = 1$

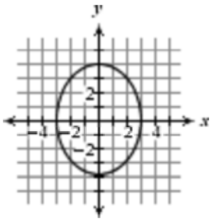
11a.



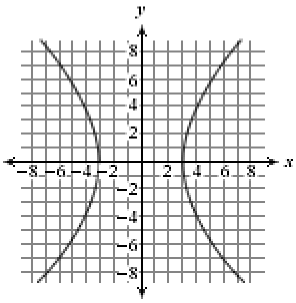
11b.



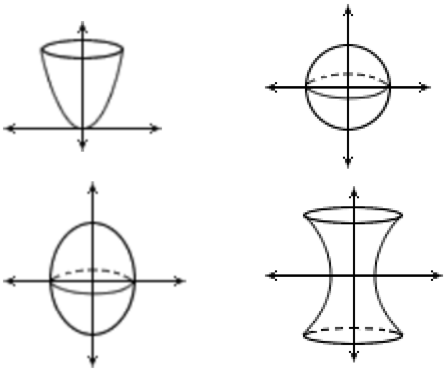
11c.



11d.



11e. The resulting shapes are a paraboloid, a sphere, an ellipsoid, and a hyperboloid.



13. $0 = 4 - (x-3)^2$; $x = 1$ or $x = 5$

15a. possible answer: $y = -\frac{1}{8}(x-10)^2 + 17.5$

15b. approximately 18.5 ft or 1.5 ft

17a. $s = s_0\left(\frac{1}{2}\right)^{t/1620}$

17b. 326 g

17c. 13,331 yr

LESSON 9.5

1a. $x^2 + 14x - 9y + 148 = 0$

1b. $x^2 + 9y^2 - 14x + 198y + 1129 = 0$

1c. $x^2 + y^2 - 2x + 6y + 5 = 0$

1d. $9x^2 - 4y^2 - 36x - 24y - 36 = 0$

3a. $\left(\frac{y-3}{6}\right)^2 - \left(\frac{x-8}{\sqrt{72}}\right)^2 = 1$; hyperbola

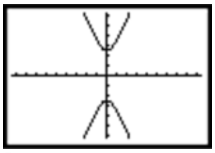
3b. $\left(\frac{x-3}{6}\right)^2 + \left(\frac{y-8}{\sqrt{72}}\right)^2 = 1$, or

$\left(\frac{x-3}{6}\right)^2 + \left(\frac{y-8}{6\sqrt{2}}\right)^2 = 1$; ellipse

3c. $\left(\frac{x+5}{\sqrt{5}}\right)^2 = \frac{(y-15.8)}{-3}$; parabola

3d. $(x+2)^2 + y^2 = 5.2$; circle

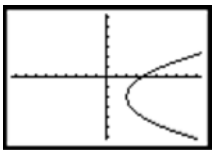
5a. $y = \frac{\pm\sqrt{400x^2+1600}}{-8}$ or $y = \mp\frac{5}{2}\sqrt{x^2+4}$



[–18.8, 18.8, 2, –12.4, 12.4, 2]

5b. $y = \frac{-16 \pm \sqrt{160x - 320}}{8}$ or

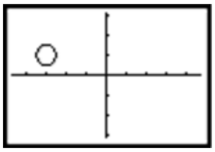
$y = \frac{-4 \pm \sqrt{10x - 20}}{2}$



[–9.4, 9.4, 1, –6.2, 6.2, 1]

5c. $y = \dfrac{8 \pm \sqrt{-64x^2 - 384x - 560}}{8}$

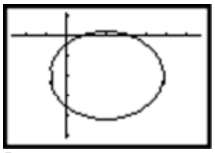
or $y = 1 \pm \dfrac{\sqrt{-16x^2 - 96x - 140}}{4}$



[– 4.7, 4.7, 1, – 3.1, 3.1, 1]

5d. $y = \dfrac{-20 \pm \sqrt{-60x^2 + 240x + 240}}{10}$

or $y = -2 \pm \dfrac{\sqrt{-15x^2 + 60x + 60}}{5}$



[–2.7, 6.7, 1, –5.1, 1.1, 1]

7. approximately 26.7 mi east and 13.7 mi north of the first station, or approximately 26.7 mi east and 13.7 mi south of the first station

9a, b. These constructions will result in a diagram similar to the one shown on page 532.

9c. $\triangle PAG$ is an isosceles triangle, so $PA = PG$. So $FP + GP$ remains constant because they sum to the radius.

9d. An ellipse. The sum of the distances to two points remains constant.

9e. Moving G within the circle creates other ellipses. The closer P is to G, the less eccentric the ellipse. Locations outside the circle produce hyperbolas.

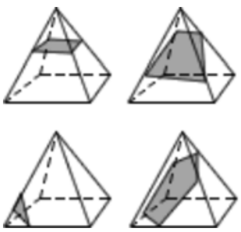
11. $x^2 + y^2 = 11.52$

13a. $\left(-2.5 + 2\sqrt{5}\right)$ and $\left(-2.5 - 2\sqrt{5}\right)$

13b. $\left(1 + \dfrac{\sqrt{3}}{2}, -2\right)$ and $\left(1 - \dfrac{\sqrt{3}}{2}, -2\right)$

15. 113°

17. square, trapezoid, kite, triangle, pentagon

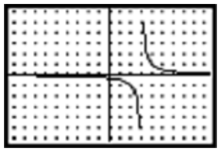


LESSON 9.6

1a. $f(x) = \frac{1}{x} + 2$

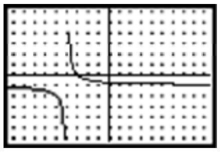
[–9.4, 9.4, 1, –6.2, 6.2, 1]

1b. $f(x) = \dfrac{1}{x-3}$



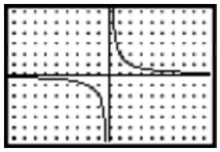
[–9.4, 9.4, 1, –6.2, 6.2, 1]

1c. $f(x) = \dfrac{1}{x+4} - 1$



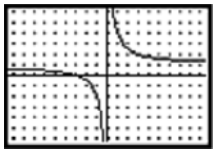
[–9.4, 9.4, 1, –6.2, 6.2, 1]

1d. $f(x) = 2\left(\frac{1}{x}\right)$ or $f(x) = \frac{2}{x}$



[–9.4, 9.4, 1, –6.2, 6.2, 1]

1e. $f(x) = 3\left(\frac{1}{x}\right) + 1$ or $f(x) = \frac{3}{x} + 1$



[– 9.4, 9.4, 1, – 6.2, 6.2, 1]

3a. $x = -4$

3b. $x = \frac{113}{18}$ or $x = 6.2\overline{7}$

3c. $x = 2.6$

3d. $x = -8.5$

5. 12 games

7a. 20.9 mL

7b. $f(x) = \dfrac{20.9 + x}{55 + x}$

7c. 39.72 mL

7d. The graph approaches $y = 1$.

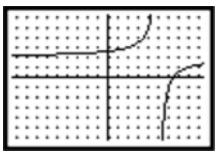
9a. i. $y = 2 + \dfrac{-3}{x-5}$

9a. ii. $y = 3 + \dfrac{2}{x+3}$

9b. i. Stretch vertically by a factor of – 3, and translate right 5 units and up 2 units.

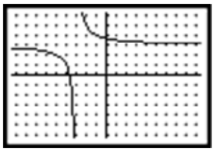
9b. ii. Stretch vertically by a factor of 2, and translate left 3 units and up 3 units.

9c. i.



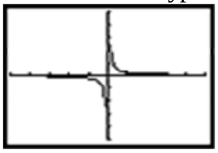
[–9.4, 9.4, 1, –6.2, 6.2, 1]

9c. ii.



[–9.4, 9.4, 1, –6.2, 6.2, 1]

11a. a rotated hyperbola



[– 5, 5, 1, – 5, 5, 1]

11b. The inverse variation function, $y=\frac{1}{x}$, can be converted to the form $xy = 1$, which is a conic section. Its graph is a rotated hyperbola.

11c. $xy - 3x - 2y + 5 = 0$; $A = 0$, $B = 1$, $C = 0$, $D = -3$, $E = -5$, $F = 5$

13. *Hint:* Graph $y = \frac{1}{x}$ and plot the foci $(\sqrt{2}, \sqrt{2})$ and $(-\sqrt{2}, -\sqrt{2})$. Measure the distance from both foci to points on the curve, and verify that the difference of the distances is constant.

15. $(x - 2)^2 + (y + 3)^2 = 16$

17a. 53° to the riverbank **17b.** 375m

17c. $x = 5t \cos 37°$, $y = 5t \sin 37° - 3t$

19a. $b = \sqrt{3}$, $c = 2$ **19b.** $a = 1$, $c = \sqrt{2}$

19c. $b = \frac{1}{2}$, $c = 1$ **19d.** $a = \frac{\sqrt{2}}{2}$, $c = 1$

19e. $\frac{\sqrt{2}}{2}$; $\frac{\sqrt{2}}{2}$; 1; $\frac{1}{2}$; $\frac{\sqrt{3}}{2}$; 1

LESSON 9.7

1a. $\frac{(x+3)(x+4)}{(x+2)(x-2)}$ **1b.** $\frac{x(x-7)(x+2)}{(x+1)(x+1)}$

3a. $\frac{7x-7}{x-2}$ **3b.** $\frac{-7x+12}{2x-1}$

5a. $y = \frac{x+2}{x+2}$ **5b.** $y = \frac{-2(x-3)}{x-3}$

5c. $y = \frac{(x+2)(x+1)}{x+1}$

7a. vertical asymptote $x = 0$, slant asymptote $y = x - 2$

7b. vertical asymptote $x = 1$, slant asymptote $y = -2x + 3$

7c. hole at $x = 2$

7d. For 7a: $y = \frac{x^2-2x+1}{x}$. The denominator is 0 and the numerator is nonzero when $x = 0$, so the vertical asymptote is $x = 0$.

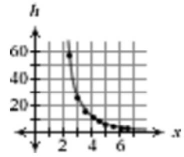
For 7b: $y = \frac{-2x^2+5x-1}{x-1}$. The denominator is 0 and the numerator is nonzero when $x = 1$, so the vertical asymptote is $x = 1$.

For 7c: $y = \frac{3x-6}{x-2}$. A zero occurs once in both the numerator and denominator when $x = 2$. This causes a hole in the graph.

9. $y = \frac{-(x+2)(x-6)}{3(x-2)}$

11a. $x = 3 \pm \sqrt{2}$ **11b.** $x = \frac{3 \pm i\sqrt{7}}{2}$

13a.



13b. The height gets larger as the radius gets smaller. The radius must be greater than 2.

13c. $V = \pi x^2 h - 4\pi h$ **13d.** $h = \frac{V}{\pi(x^2 - 4)}$

13e. approximately 400 units^3

15a. $83\frac{1}{3}$ g; approximately 17% almonds and 43% peanuts

15b. 50 g; approximately 27.3% almonds, 27.3% cashews, and 45.5% peanuts

LESSON 9.8

1a. $\frac{x(x+2)}{(x-2)(x+2)} = \frac{x}{x-2}$

1b. $\frac{(x-1)(x-4)}{(x+1)(x-1)} = \frac{x-4}{x+1}$

1c. $\frac{3x(x-2)}{(x-4)(x-2)} = \frac{3x}{x-4}$

1d. $\frac{(x+5)(x-2)}{(x+5)(x-5)} = \frac{x-2}{x-5}$

3a. $\frac{(2x-3)(x+1)}{(x+3)(x-2)(x-3)}$

3b. $\frac{-x^2+6}{(x+2)(x+3)(x-2)}$

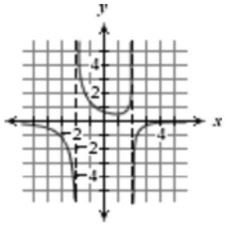
3c. $\frac{2x^2-x+9}{(x-3)(x+2)(x+3)}$

3d. $\frac{2x^2-5x+6}{(x+1)(x-2)(x-1)}$

5a. $\frac{2(x-2)}{x+1}$ **5b.** 1

7a. $x = 3$ is a zero, because that value causes the numerator to be 0. The vertical asymptotes are $x = 2$ and $x = -2$, because these values make the denominator 0 and do not also make the numerator 0. The horizontal asymptote is $x = 0$, because this is the value that y approaches when $|x|$ is large.

7b.



9a. Answers will vary

9b. $-x^2 + xy - y = 0$; yes

9c. no

9d. not possible; no

9e. After reducing common factors, the degree of the numerator must be less than or equal to 2, and the degree of the denominator must be 1.

11a. $x = 3$, $y = 1$

11b. translation right 3 units and up 1 unit

11c. -2

11d. $y = 1 - \frac{2}{x-3}$ or $y = \frac{x-5}{x-3}$

11e. x-intercept: 5; y-intercept: $\frac{5}{3}$

13a. \$370.09

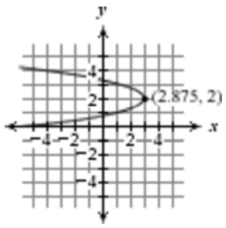
13b. \$382.82

13c. \$383.75

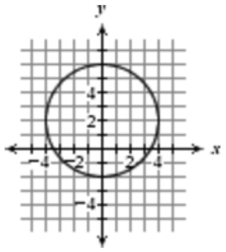
13d. \$383.99

CHAPTER 9 REVIEW

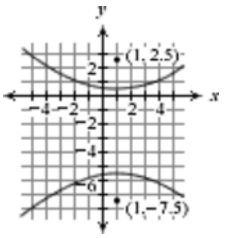
1a.



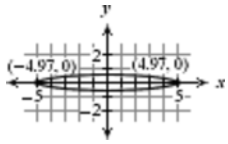
1b.



1c.



1d.



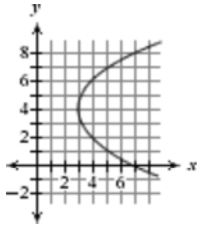
3a. $y = \pm 0.5x$

3b. $x^2 - 4y^2 - 4 = 0$

3c. $d = 0.5x - \sqrt{\frac{x^2}{4} - 1}$

3d. 1, 0.101, 0.050, 0.010; As x-values increase, the curve gets closer to the asymptote.

5. $(y-4)^2 = \frac{x-3}{0.25}$; vertex: (3, 4), focus: (4, 4), directrix: $x = 2$



7a. $y = 1 + \frac{1}{x+2}$ or $y = \frac{x+3}{x+2}$

7b. $y = -4 + \frac{1}{x}$ or $y = \frac{-4x+1}{x}$

9. Multiply the numerator and denominator by the factor $(x + 3)$.

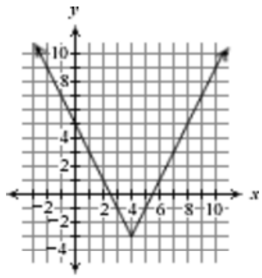
$$y = \frac{(2x-14)(x+3)}{(x-5)(x+3)}$$

11a. $\frac{3x^2+8x+3}{(x-2)(x+1)(x+2)}$

11b. $\frac{3x}{x+1}$

11c. $\frac{(x+1)^2(x-1)}{x(x-2)}$

13.



13a. $y = 2\,|x|$

13b. $y = 2\,|x - 4|$

13c. $y = 2\,|x - 4| - 3$

15a. Not possible. The number of columns in $[A]$ must match the number of rows in $[B]$.

15b. Not possible. To add matrices, they must have the same dimensions.

15c. $\begin{bmatrix} -3 & 1 \\ 1 & -5 \end{bmatrix}$ **15d.** $\begin{bmatrix} -2 & 6 \\ -10 & 0 \\ -7 & 3 \end{bmatrix}$

15e. $\begin{bmatrix} 5 & -3 \\ -1 & 9 \end{bmatrix}$

17a. 7.5 yd/s **17b.** 27.5°

17c. $x = 7.5t \cos 27.5°$, $y = 7.5t \sin 27.5°$

17d. $x = 100 - 7.5t \cos 27.5°$, $y = 7.5t \sin 27.5°$

17e. midfield (50, 26), after 7.5 s

19a. $\left(\frac{y}{5}\right)^2 - \left(\frac{x}{2}\right)^2 = 1$; hyperbola

19b. $(y+2)^2 = \frac{(x-2)}{\frac{2}{5}}$; parabola

19c. $(x+3)^2 + (y-1)^2 = \frac{1}{4}$; circle

19d. $\left(\frac{x-2}{\sqrt{8}}\right)^2 + \left(\frac{y+2}{\sqrt{4.8}}\right)^2 = 1$

or $\left(\frac{x-2}{2\sqrt{2}}\right)^2 + \left(\frac{y+2}{2\sqrt{1.2}}\right)^2 = 1$; ellipse

21a. $x \approx 1.64$ **21b.** $x \approx -0.66$

21c. $x = 15$ **21d.** $x \approx 2.57$

21e. $x \approx 17.78$ **21f.** $x = 3$

21g. $x = 2$ **21h.** $x = 495$

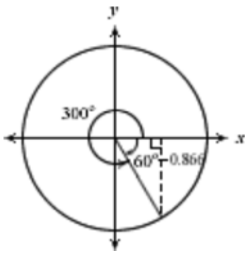
21i. $x \approx \pm 4.14$

23. Bases are home (0, 0), first (90, 0), second (90, 90), and third (0, 90).
Deanna: $x = 90$, $y = 28t + 12$
ball: $x = 125(t - 1.5) \cos 45°$, $y = 125(t - 1.5) \sin 45°$ Deanna reaches second base after 2.79 s, ball reaches second base after 2.52 s. Deanna is out.

CHAPTER 10 • CHAPTER 10 CHAPTER 10 • CHAPTER

LESSON 10.1

1.



approximately – 0.866 m

3a. 2 **3b.** 4

5a. periodic, 180° **5b.** not periodic

5c. periodic, 90° **5d.** periodic, 180°

7. Quadrant I: $\cos \theta$ and $\sin \theta$ are positive;
Quadrant II: $\cos \theta$ is negative and $\sin \theta$ is positive;
Quadrant III: $\cos \theta$ and $\sin \theta$ are negative;
Quadrant IV: $\cos \theta$ is positive and $\sin \theta$ is negative.

9. $x = \{-270°, -90°, 90°, 270°\}$

11a. $\theta = -15°$ **11b.** $\theta = 125°$

11c. $\theta = -90°$ **11d.** $\theta = 48°$

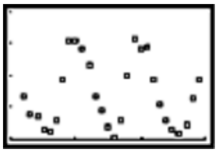
13a. $\theta = 150°$ and $\theta = 210°$

13b. $\theta = 135°$ and $\theta = 225°$

13c. $\theta \approx 217°$ and $\theta \approx 323°$

13d. $\theta = 90°$

15a.



[1970, 2000, 10, 0, 200, 50]

The data are cyclical and appear to have a shape like a sine or cosine curve.

15b. 10–11 yr **15c.** in about 2001

17a. 43,200 s **17b.** 4.4 ft/s

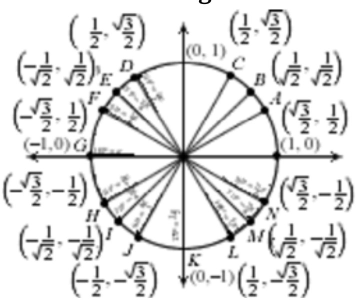
19a. $\frac{3}{x-4}$ **19b.** 2 **19c.** $\frac{2(3+a)}{6-a}$

LESSON 10.2

1a. $\frac{4\pi}{9}$ **1b.** $\frac{19\pi}{6}$ **1c.** – 240° **1d.** 220°

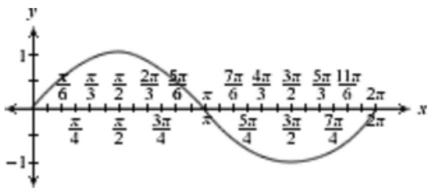
1e. –135° **1f.** 540° **1g.** -5π **1h.** 150°

3a, b.



5. Less than; one rotation is 2π, which is more than 6.

7a, b.



7c. $\frac{\pi}{2}; \frac{3\pi}{2}$; 0,$\pi$, and 2π

9a. $\frac{3}{2}; \frac{3}{2}$ **9b.** –2; –2

9c. They are equal. **9d.** approximately 2.414

11a. $\frac{4\pi}{3}$ **11b.** $\frac{7\pi}{4}$ **11c.** $\frac{\pi}{3}$

13a. $A \approx 57.54$ cm^2 **13b.** $\frac{A}{64\pi} = \frac{\frac{4\pi}{7}}{2\pi}$

13 c. $A = 64\pi \cdot \frac{\frac{4\pi}{7}}{2\pi} \approx 57.54$ cm^2

15a. 1037 mi **15b.** 61.17° **15c.** 2660 mi

17a. $y = -2(x + 1)^2$ **17b.** $y + 4 = (x - 2)^2$

17c. $y + 2 = \left|\frac{x+1}{2}\right|$ **17d.** $-\frac{y-2}{2} = |x-3|$

19a. 18 cm **19b.** 169 cm

21. *Hint:* Construct $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$. $\triangle APC$ is isosceles because $\overline{AP}$ and $\overline{CP}$ are radii of the same circle. $\angle ABP$ measures 90° because the angle is inscribed in a semicircle. Use these facts to prove that $\triangle ABP \cong \triangle CBP$.

LESSON 10.3

1a. $y = \sin x + 1$ **1b.** $y = \cos x - 2$

1c. $y = \sin x - 0.5$ **1d.** $y = -3 \cos x$

1e. $y = -2 \sin x$ **1f.** $y = 2 \cos x + 1$

3a. The k-value vertically translates the graph of the function.

3b. The b-value vertically stretches or shrinks the graph of the function. The absolute value of b represents the amplitude. When b is negative, the curve is reflected across the x-axis.

3c. The a-value horizontally stretches or shrinks the graph of the function. It also determines the period with the relationship $2\pi a = period$.

3d. The h-value horizontally translates the graph of the function. It represents the phase shift.

5. translate $y = \sin x$ left $\frac{\pi}{2}$ units

7a. Let x represent the number of days after a full moon (today), and let y represent the percentage of lit surface that is visible.
$y = 0.5 + 0.5 \cos\left(\frac{2\pi x}{28}\right)$

7b. 72% **7c.** day 5

9. first row: $1; \frac{\sqrt{3}}{2}; \frac{\sqrt{2}}{2}; 0; -\frac{\sqrt{2}}{2}; -1; -\frac{1}{2}; \frac{1}{2}; \frac{\sqrt{3}}{2}$;
second row: $0; \frac{1}{2}; \frac{\sqrt{2}}{2}; 1; \frac{\sqrt{2}}{2}; 0; -\frac{\sqrt{3}}{2}; -\frac{\sqrt{3}}{2}; -\frac{1}{2}$;
third row: 0; $\frac{1}{\sqrt{3}}$; 1; undefined; – 1; 0; $\sqrt{3}$; $-\sqrt{3}$; $-\frac{1}{\sqrt{3}}$

11a. $y = 1.5 \cos 2\left(x + \frac{\pi}{2}\right)$

11b. $y = -3 + 2 \sin 4\left(x - \frac{\pi}{4}\right)$

11c. $y = 3 + 2 \cos \frac{x-\pi}{3}$

13a. 0.79 m **13b.** 0.74 m **13c.** 0.81 m

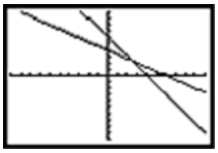
15. See below.

17a. i. $y = -\frac{2}{3}x + 4$

17a. ii. $y = \pm\sqrt{x+4} - 2$

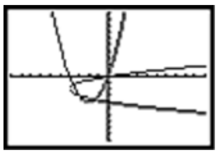
17a. iii. $y = \frac{\log(x+8)}{\log 1.3} - 6$

17b. i.



[–10, 10, 1, –10, 10, 1]

17b. ii.

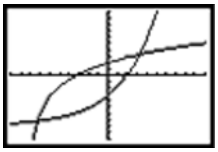


[–10, 10, 1, –10, 10, 1]

15. (*Lesson 10.3*)

Degrees	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
Radians	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
Degrees	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°	
Radians	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{17\pi}{12}$	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π	

17b. iii.

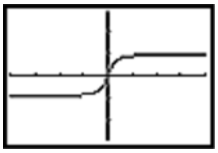


[–10, 10, 1, –10, 10, 1]

17c. The inverses of i and iii are functions.

LESSON 10.4

1a. 27.8° and 0.49 **1b.** –14.3° and –0.25
1c. 144.2° and 2.52 **1d.** 11.3° and 0.20
3a–d. *Hint:* Graph $y = \sin x$ or $y = \cos x$ for $-2\pi \leq x \leq 2\pi$. Plot all points on the curve that have a y-value equal to the y-value of the expression on the right side of the equation. Then find the x-value at each of these points.
5. $-1 \leq \sin x \leq 1$. There is no angle whose sine is 1.28.
7a. $x \approx 0.485$ or $x \approx 2.656$
7b. $x \approx -2.517$ or $x \approx -3.766$
9. 106.9°
11a. *Hint:* Use your calculator.
11b. The domain is all real numbers. The range is $-\frac{\pi}{2} \leq \pi \leq \frac{\pi}{2}$. See graph for 11d.
11c. The function $y = \tan^{-1} x$ is the portion of $x = \tan y$, such that $-\frac{\pi}{2} < y < \frac{\pi}{2}$ (or $-90° < y < 90°$).
11d.

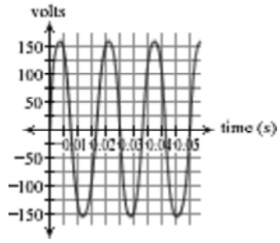


[–20, 20, 5, –3π/2, 3π/2, π/2]

13. 650°
15a. $8.0 \cdot 10^{-4}$ W/m^2; $6.0 \cdot 10^{-4}$W/m^2
15b. $\theta = 45°$ **15c.** $\theta = 90°$
17a. $y = \tan\dfrac{x + \frac{\pi}{2}}{2}$
17a. $y = 1 - 0.5\tan\left(x - \frac{\pi}{2}\right)$
19a. Ellipse with center at origin, horizontal major axis of length 6 units, and vertical minor axis of length 4 units. The parametric equations are $x = 3\cos t$ and $y = 2\sin t$.
19b. $\left(\frac{x+1}{3}\right)^2 + \left(\frac{y-2}{2}\right)^2 = 1$; $x = 3\cos t - 1$ and $y = 2\sin t + 2$
19c. approximately (1.9, 1.5) and (– 2.9, 0.5)
19d. (1.92, 1.54) and (– 2.92, 0.46)

LESSON 10.5

1a. $x = \left\{\frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}\right\}$
1b. $x = \left\{\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}\right\}$
3a. 5; 5 **3b.** 7; – 2; 12; 7
3c. $\frac{11}{2\pi}$; 11 **3d.** 9; 9
5. $y = 1.2\sin\frac{2\pi t}{8} + 2$ or $y = 1.2\sin\frac{\pi t}{4} + 2$
7a. possible answer: $v = 155.6 \sin(120\pi t)$
7b.



9a. $y_1 = -3\cos\left(\dfrac{2\pi(t + 0.17)}{\frac{2}{3}}\right), y_2 = -4\cos\left(\dfrac{2\pi t}{\frac{2}{3}}\right)$
9b. at 0.2, 0.6, 0.9, 1.2, 1.6, 1.9 s
11a. about 9.6 h
11b. March 21 and September 21 or 22
13. Construct a circle and its diameter for the main rotating arm. Construct a circle with a fixed radius at each end of the diameter. Make a point on each of these two circles. Animate them and one endpoint of the diameter.
15. The sector has the larger area. The triangle's area is 10.8 cm^2; the sector's area is 12.5 cm^2.
17a. $\pm 1, \pm 5, \pm\frac{1}{2}, \pm\frac{5}{2}$
17b. $x = \frac{1}{2}$
17c. $x = \pm\sqrt{5}$
17d. $P(x) = 2\left(x - \frac{1}{2}\right)\left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)$

LESSON 10.6

1. Graph $y = \frac{1}{\tan x}$.

3. *Hint:* Use the distributive property to rewrite the left side of the equation. Use a reciprocal trigonometric identity to rewrite cot *A*, then simplify. Use a Pythagorean identity to complete the proof.

5. A trigonometric equation may be true for some, all, or none of the defined values of the variable. A trigonometric identity is a trigonometric equation that is true for all defined values of the variable.

7a. *Hint:* Replace $\cos 2A$ with $\cos^2 A - \sin^2 A$. Rewrite $\cos^2 A$ using a Pythagorean identity. Then combine like terms.

7b. *Hint:* Replace $\cos 2A$ with $\cos^2 A - \sin^2 A$. Rewrite $\sin^2 A$ using a Pythagorean identity. Then combine like terms.

9a. $y = \sin x$ **9b.** $y = \cos x$ **9c.** $y = \cot x$
9d. $y = \cos x$ **9e.** $y = -\sin x$ **9f.** $y = -\tan x$
9g. $y = \sin x$ **9h.** $y = -\sin x$ **9i.** $y = \tan x$

11a–c. *Hint:* Use the reciprocal trigonometric identities to graph each equation on your calculator, with window $[0, 4\pi, \pi/2, -2, 2, 1]$.

13a. 2; undefined when θ equals 0 or π

13b. $3\cos\theta$; undefined when θ equals $0, \frac{\pi}{2}, \pi$, or $\frac{3\pi}{2}$

13c. $\tan^2\theta + \tan\theta$; undefined when θ equals $\frac{\pi}{2}$ or $\frac{3\pi}{2}$

13d. $\sec\theta$; undefined when θ equals $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, or 2π

15a. $1505.12

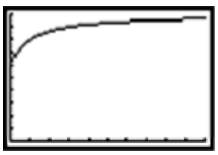
15b. another 15 years, or until he's 32

17a. $c(x) = \frac{60 + x}{100 + x}$

17b. $66\frac{2}{3}\%$

17c. 300 mL

17d.



[0, 1000, 100, 0, 1, 0.1]

The asymptote is the line $y = 1$. The more pure medicine that is added the closer the concentration will get to 100%, but it will never actually become 100%.

17e. Use the diluting function to obtain concentrations less than 60%. Use the concentrating function to obtain concentrations greater than 60%.

LESSON 10.7

1a. not an identity **1b.** not an identity
1c. not an identity **1d.** not an identity

3a. cos 1.1 **3b.** cos 2.8
3c. sin 1.7 **3d.** sin 0.7

5. $\frac{4\sqrt{5}}{9}$

7. *Hint:* Begin by writing $\sin(A - B)$ as $\sin(A + (-B))$. Then use the sum identity given in Exercise 6. Next, use the identities $\cos(-x) = \cos x$ and $\sin(-x) = -\sin x$ to simplify further.

9. *Hint:* Begin by writing $\sin 2A$ as $\sin(A + A)$. Then use a sum identity to expand, and simplify by combining like terms.

11. *Hint:* Show that $\tan(A + B) \neq \tan A + \tan B$ by substituting values for *A* and *B* and evaluating. To find an identity for $\tan(A + B)$, first rewrite as $\frac{\sin(A+B)}{\cos(A+B)}$. Then use sum identities to expand. Divide both the numerator and denominator by $\cos A \cos B$, and rewrite each occurrence of $\frac{\sin\theta}{\cos\theta}$ as $\tan\theta$.

13a. *Hint:* Solve $\cos 2A = 1 - 2\sin^2 A$ for $\sin^2 A$.

13b. *Hint:* Solve $\cos 2A = 2\cos^2 A - 1$ for $\cos^2 A$.

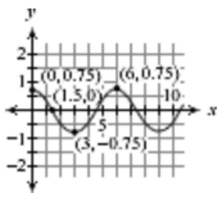
15a. Period: 8π, 12π, 20π, 24π, 12π

15b. The period is 2π multiplied by the least common multiple of *a* and *b*.

15c. 48π; multiply 2π by the least common multiple of 3, 4, and 8, which is 24.

17a. $x \approx -1.2361$ **17b.** $x \approx 1.1547$
17c. $x \approx 1.0141$ **17d.** $x = 0$

19a. Let *x* represent time in minutes, and let *y* represent height in meters above the surface of the water if it was calm.



19b. $y = 0.75\cos\frac{\pi}{3}x$

19c. $y = 0.75\sin\left(\frac{\pi}{3}(x + 1.5)\right)$

CHAPTER 10 REVIEW

1a. I; 420°; $\frac{\pi}{3}$ **1b.** III; $\frac{10\pi}{3}$; 240°
1c. IV; –30°; $\frac{11\pi}{6}$ **1d.** IV; $\frac{7\pi}{4}$; –45°

3. Other equations are possible.

3a. period $=\frac{2\pi}{3}$; $y=-2\cos\left(3\left(x-\frac{2\pi}{3}\right)\right)$

3b. period $=\frac{\pi}{2}$; $y=3\sin\left(4\left(x-\frac{\pi}{8}\right)\right)$

3c. period $=\pi$; $y=\csc\left(2\left(x+\frac{\pi}{4}\right)\right)$

3d. period $=\frac{\pi}{2}$; $y=\cot\left(2\left(x-\frac{\pi}{4}\right)\right)+1$

5a. $y=-2\sin(2x)-1$

5b. $y=\sin(0.5x)+1.5$

5c. $y=0.5\tan\left(x-\frac{\pi}{4}\right)$

5d. $y=0.5\sec(2x)$

7. $\cos y=x$: domain: $-1\le x\le 1$; range: all real numbers.
$y=\cos^{-1}x$: domain: $-1\le x\le 1$; range: $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$

9. $y=-3\sin\left(\frac{x-\frac{\pi}{2}}{4}\right)$

11. 0.174 s, 0.659 s, 1.008 s, 1.492 s, 1.841 s, 2.325 s, 2.675 s

CHAPTER 11 • CHAPTER 11 CHAPTER 11 • CHAPTER

LESSON 11.1

1. –3, –1.5, 0, 1.5, 3; $u_1=-3$, $d=1.5$

3a. 3 + 4 + 5 + 6; 18 **3b.** – 2 + 1 + 6; 5

5. $S_{75}=5700$

7a. $u_{46}=229$

7b. $u_n=5n-1$, or $u_1=4$ and $u_n=u_{n-1}+5$ where $n\ge 2$

7c. $S_{46}=5359$

9a. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

9b. $u_1=3$ and $u_n=u_{n-1}+3$ where $n\ge 2$

9c. 3384 cans **9d.** 13 rows with 15 cans left over

11. $S_x=x^2+64x$

13a. $u_1=4.9$ and $u_n=u_{n-1}+9.8$ where $n\ge 2$

13b. $u_n=9.8n-4.9$ **13c.** 93.1 m **13d.** 490 m

13e. $S_n=4.9n^2$ **13f.** approximately 8.2 s

15a. 576,443 people **15b.** 641,676 people

17a. 81, 27, 9, 3, 1, $\frac{1}{3}$

17b. $u_1=81$ and $u_n=\frac{1}{3}u_{n-1}$ where $n\ge 2$

19a. $\frac{\sqrt{6}+\sqrt{2}}{4}$ **19b.** $\frac{\sqrt{6}-\sqrt{2}}{4}$

LESSON 11.2

1a. 0.4 + 0.04 + 0.004 + . . .

1b. $u_1=0.4$, $r=0.1$

1c. $S=\frac{4}{9}$

3a. 0.123 + 0.000123 + 0.000000123 + . . .

3b. $u_1=0.123$, $r=0.001$

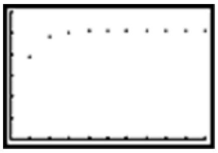
3c. $S=\frac{123}{999}=\frac{41}{333}$

5. $u_1=32768$

7a. 96, 24, 6, 1.5, 0.375. 0.09375, 0.0234375, 0.005859375, 0.00146484375, 0.0003662109375

7b. $S_{10}\approx 128.000$

7c.



[0, 10, 1, 0, 150, 25]

7d. $S=128$

9a. $25,000,000 **9b.** $62,500,000

9c. 2.5 **9d.** 44.4%

11a. $\sqrt{2}$ in. **11b.** 0.125 in.^{2}

11c. approximately 109.25 in.

11d. 128 in.^{2}

13. 88 gal

15a. $56,625 **15b.** 43 wk

LESSON 11.3

1a. $u_1=12$, $r=0.4$, $n=8$

1b. $u_1=75$, $r=1.2$, $n=15$

1c. $u_1=40$, $r=0.8$, $n=20$

1d. $u_1=60$, $r=2.5$, $n=6$

3a. $S_5=92.224$ **3b.** $S_{15}\approx 99.952$ **3c.** $S_{25}\approx 99.999$

5a. 3069 **5b.** 22 **5c.** 2.8 **5d.** 0.95

7a. $S_{10}=15.984375$

7b. $S_{20}\approx 15.99998474$

7c. $S_{30}\approx 15.99999999$

7d. They continue to increase, but by a smaller amount each time.

9a. i. 128 **9a. ii.** more than 9×10^{18}

9a. iii. 255 **9a. iv.** more than 1.8×10^{19}

9b. $\sum_{n=1}^{64}2^{n-1}$

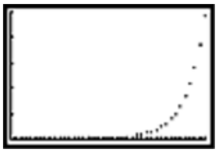
11a. 5, 15, 35, 75, 155, 315, 635

11b. No, they form a shifted geometric sequence.
11c. not possible
13a. 1 + 4 + 9 + 16 + 25 + 36 + 49 = 140
13b. 9 + 16 + 25 + 36 + 49 = 135
15. $637.95
17. Yes. The long-run height is only 24 in.

CHAPTER 11 REVIEW

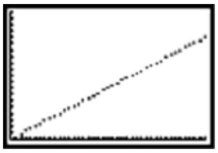
1a. $u_{128} = 511$ **1b.** $u_{40} = 159$
1c. $u_{20} = 79$ **1d.** $S_{20} = 820$
3a. 144; 1728; 20,736; 429,981,696
3b. $u_1 = 12$ and $u_n = 12\ u_{n-1}$ where $n \geq 2$
3c. $u_n = 12^n$ **3d.** approximately 1.2×10^{14}
5a. approximately 56.49 ft **5b.** 60 ft
7a. $S_{10} \approx 12.957$; $S_{40} \approx 13.333$
7b. $S_{10} \approx 170.478$; $S_{40} \approx 481571.531$
7c. $S_{10} = 40$; $S_{40} = 160$
7d. For $r = 0.7$ For $r = 1.3$

[0, 40, 1, 0, 20, 1]



[0, 40, 1, 0, 500000, 100000]

For $r = 1$



[0, 40, 1, 0, 200, 10]

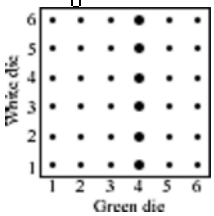
7e. 0.7

CHAPTER 12 • CHAPTER **12** CHAPTER 12 • CHAPTER

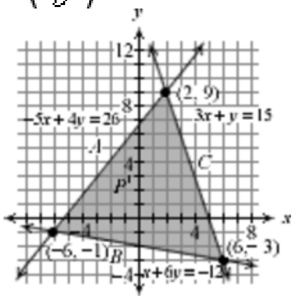
LESSON 12.1

1a. $\frac{6}{15} = .4$ **1b.** $\frac{7}{15} \approx .467$ **1c.** $\frac{2}{15} \approx .133$
3a. $\frac{4}{14} \approx .286$ **3b.** $\frac{10}{14} \approx .714$ **3c.** $\frac{7.5}{14} \approx .536$
3d. $\frac{1.5}{14} \approx .107$ **3e.** $\frac{2}{14} \approx .143$
5a. experimental **5b.** theoretical
5c. experimental
7. *Hint:* Consider whether each of the integers 0–9 are equally likely. Each of the procedures has shortcomings, but 7iii is the best method.

9a. 36
9b. $6; \frac{1}{6} \approx .167$



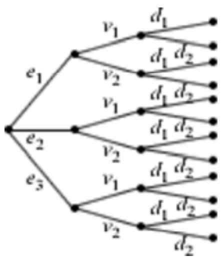
9c. $12; \frac{12}{36} \approx .333$ **9d.** $3; \frac{3}{36} \approx .083$
11a. 144 square units **11b.** 44 square units
11c. $\frac{44}{144}$ **11d.** $\frac{44}{144} \approx .306$
11e. $\frac{100}{144} \approx .694$ **11f.** 0; 0
13a. 270 **13b.** 1380
13c. $\frac{270}{1380} \approx .196$ **13d.** $\frac{1110}{1380} \approx .804$
15a. 53 pm, at point *C*
15b. 0 pm, at point *A*, the nucleus
15c. The probability starts at zero at the nucleus, increases and peaks at a distance of 53 pm, and then decreases quickly, then more slowly, but never reaches zero.
17. $\log\left(\frac{ac^2}{b}\right)$
19a.



19b. (2, 9), (– 6, – 1), (6, – 3)
19c. 68 square units
21a. Set i should have a larger standard deviation because the values are more spread out.
21b. i. $\bar{x} = 35$, $s \approx 22.3$; **ii.** $\bar{x} = 117$, $s \approx 3.5$
21c. The original values of $\bar{x}$ and s are multipied by 10.
21c. i. $\bar{x} = 350$, $s \approx 223.5$
21c. ii. $\bar{x} = 1170$, $s \approx 35.4$
21d. The original values of $\bar{x}$ are increased by 10, and the original values of s are unchanged.
21d. i. $\bar{x} = 45$, $s \approx 22.3$ **21d. ii.** $\bar{x} = 127$, $s \approx 3.5$

Selected Answers

LESSON 12.2

1. 

3. $P(a) = .7$; $P(b) = .3$; $P(c) = .18$; $P(d) = .4$; $P(e) = .8$; $P(f) = .2$; $P(g) = .08$

5. For the first choice, the probability of choosing a sophomore is $\frac{14}{21}$, and the probability of choosing a junior is $\frac{7}{21}$. Once the first student is chosen, the class total is reduced by 1 and either the junior or sophomore portion is reduced by 1.

7a. 24 **7b.** .25 **7c.** $\frac{2}{24} \approx .083$

7d. $\frac{1}{24} \approx .042$ **7e.** $\frac{23}{24} \approx .958$ **7f.** $\frac{12}{24} = .5$

9a. 4 **9b.** 8 **9c.** 16

9d. 32 **9e.** 1024 **9f.** 2^n

11a. $P(M3) = .45$; $P(G \mid M1) = .95$; $P(D \mid M2) = .08$; $P(G \mid M3) = .93$; $P(M1 \text{ and } D) = .01$; $P(M1 \text{ and } G) = .19$; $P(M2 \text{ and } D) = .028$; $P(M2 \text{ and } G) = .332$; $P(M3 \text{ and } D) = .0315$; $P(M3 \text{ and } G) = .4185$

11b. .08 **11c.** .0695 **11d.** .4029

13. $\frac{6}{16} = .375$

15a. 100,000 **15b.** 1,000,000,000

15c. 17,576,000 **15d.** 7,200,000

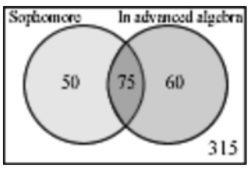
17a. $-3 + 2i$ **17b.** $2 + 24i$

17c. $\frac{18}{29} + \frac{16}{29}i$

19a. $\frac{50}{110} \approx .455$ **19b.** $\frac{120}{230} \approx .522$

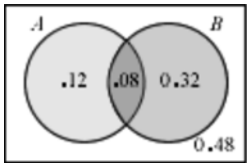
LESSON 12.3

1. 10% of the students are sophomores and not in advanced algebra. 15% are sophomores in advanced algebra. 12% are in advanced algebra but are not sophomores. 62% are neither sophomores nor in advanced algebra.

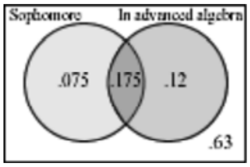
3. 

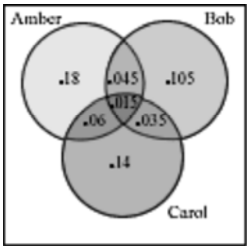
5a. Yes, because they do not overlap.

5b. No. $P(\text{A and B}) = 0$. This would be the same as $P(\text{A}) \cdot P(\text{B})$ if they were independent.

7a. 

7b. i. .08 **7b. ii.** .60 **7b. iii.** .48

9. 

11a. 

11b. .015 **11c.** .42

13. approximately 77

15a. $3\sqrt{2}$ **15b.** $3\sqrt{6}$ **15c.** $2xy^2\sqrt{15xy}$

LESSON 12.4

1a. Yes; the number of children will be an integer, and it is based on a random process.

1b. No; the length may be a non-integer.

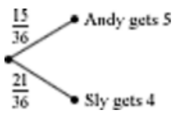
1c. Yes; there will be an integer number of pieces of mail, and it is based on random processes of who sends mail when.

3a. approximately .068

3b. approximately .221

5a. Answers will vary. Theoretically, after 10 games Sly should get about 23 points, and Andy should get 21.

5b. Answers will vary. Theoretically, it should be close to .47.

5c. 

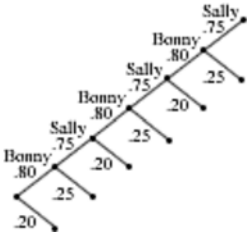
5d. –0.25

5e. Answers will vary. One possible answer is 5 points for Sly if the sum of the dice is less than 8 and 7 points for Andy if the sum of the dice is greater than 7.

7a. \$25 **7b.** .67 **7c.** \$28.33

9a. .2 **9b.** .12

3c.



.072

9d. .392 **9e.** geometric; $u_1 = .20$, $r = .6$

9f. .476672 **9g.** .5

11a. .580

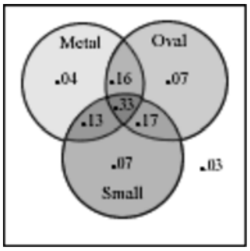
11b. 0; 0.312; 0.346; 0.192; 0.124; 0

11c. 0.974

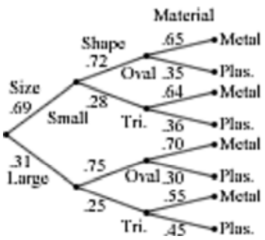
11d. On average, the engineer should expect to find 0.974 defective radio in a sample of 5.

13. 1

15a.



15b.



17. 44

LESSON 12.5

1a. Yes. Different arrangements of scoops are different.

1b. No. The order is not the same, so the arrangements should be counted separately if they are permutations.

1c. No. Repetition is not allowed in permutations.

1d. No. Repetition is not allowed in permutations.

3a. 210 **3b.** 5040

3c. $\frac{(n+2)!}{2}$ **3d.** $\frac{n!}{2}$

5a. 10000; $27.\overline{7}$ hr

5b. 100000; approximately 11.57 days

5c. 10

7. r factors

9a. 40,320 **9b.** 5040 **9c.** .125

9d. Sample answer: There are eight possible positions for Volume 5, all equally likely. So $P(5 \text{ in rightmost slot}) = \frac{1}{8} = .125$.

9e. .5; sample answer: there are four books that can be arranged in the rightmost position. Therefore, the number of ways the books can be arranged is $7! \cdot 4 = 20{,}160$.

9f. 1 **9g.** 40,319 **9h.** $\frac{1}{40320} \approx .000025$

11a. approximately .070 **11b.** approximately .005

11c. approximately .155 **11d.** \$3.20

13a. $\frac{30}{50} = .6$ **13b.** $\frac{16}{30} \approx .533$

15a. $\frac{1}{8} = .125$ **15b.** $\frac{3}{8} = .375$ **15c.** $\frac{1}{2} = .5$

17a. 41 **17b.** about 808.3 in.^{2}

LESSON 12.6

1a. 120 **1b.** 35 **1c.** 105 **1d.** 1

3a. $\frac{{}_7P_2}{2!} = {}_7C_2$ **3b.** $\frac{{}_7P_3}{3!} = {}_7C_3$

3c. $\frac{{}_7P_4}{4!} = {}_7C_4$ **3d.** $\frac{{}_7P_7}{7!} = {}_7C_7$

3e. $\frac{{}_nP_r}{r!} = {}_nC_r$

5. $n = 7$ and $r = 3$, or $n = 7$ and $r = 4$, or $n = 35$ and $r = 1$, or $n = 35$ and $r = 34$

7a. 35 **7b.** $\frac{20}{35} \approx .571$

9a. 4 **9b.** 8 **9c.** 16

9d. The sum of all possible combinations of n things is 2^n; $2^5 = 32$.

11a. 6 **11b.** 10 **11c.** 36

11d. ${}_nC_2 = \frac{n!}{2(n-2)!}$

13a. $x^2 + 2xy + y^2$

13b. $x^3 + 3x^2y + 3xy^2 + y^3$

13c. $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

15a. .0194 is the probability that someone is healthy and tests positive.

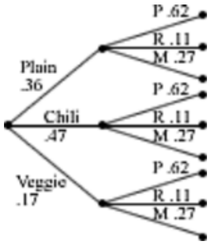
15b. .02 is the probability that a healthy person tests positive.
15c. .0491 is the probability that a person tests positive.
15d. .395 is the probability that a person who tests positive is healthy.
17. approximately 19.5 m; approximately 26.2 m

LESSON 12.7

1a. x^{47} **1b.** $5{,}178{,}066{,}751x^{37}y^{10}$
1c. $62{,}891{,}499x^7y^{40}$ **1d.** $47xy^{46}$
3a. .299 **3b.** .795, .496 **3c.** .203, .502, .791
3d. Both the "at most" and "at least" numbers include the case of "exactly." For example, if "exactly" 5 birds (.165) is subtracted from "at least" 5 birds (.203), the result (.038) is the same as 1 – .962 ("at most" 5 birds).
3e. The probability that at least 5 birds survive is 20.3%.
5. $p < \frac{23}{33}$
7a. $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
7b. $p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$
7c. $8x^3 + 36x^2 + 54x + 27$
7d. $81x^4 - 432x^3 + 864x^2 - 768x + 256$
9a. .401 **9b.** .940
9c. $Y_1 = {}_{30}C_x(.97)^{30-x}(.03)^x$
9d. .940
11a. .000257 **11b.** .446 **11c.** .983
13. Answers will vary. This event will happen in 15.6% of trials.
15a. 2; 2.25; ≈ 2.370; ≈ 2.441
15b. $f(10) \approx 2.5937$, $f(100) \approx 2.7048$, $f(1000) \approx 2.7169$, $f(10000) \approx 2.7181$
15c. There is a long-run value of about 2.718.
17. 37.44 cm^2
19a. *Hint:* Graph data in the form (*distance*, *period*), (log (*distance*), *period*), (*distance*, log(*period*)), and (log (*distance*), log (*period*)), and identify which is the most linear. Find an equation to fit the most linear data you find, then substitute the appropriate variables (*distance*, *period*, log(*distance*), log (*period*)) for *x* and *y*, and solve for *y*. You should find that $period \approx distance^{1.50} \times 10^{-9.38}$.
19b. *Hint:* Substitute the period and distance values given in the table into your equation from 19a. Errors are most likely due to rounding.
19c. $period^2 = 10^{-18.76}\,distance^3$

CHAPTER 12 REVIEW

1. Answers will vary. You might number 10 chips or slips of paper and select one. You might look at a random-number table and select the first digit of each number. You could alter the program Generate to :Int 10Rand + 1.
3a. .5 **3b.** 17.765 square units
5a.

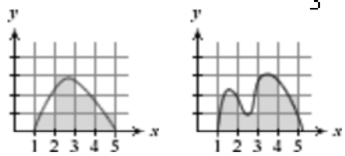


5b. .0517 **5c.** .8946 **5d.** .3501
7. 110.5
9. .044

CHAPTER 13 • CHAPTER 13 CHAPTER 13 • CHAPTER

LESSON 13.1

1a. $\frac{1}{8}$ **1b.** $\frac{1}{12}$ **1c.** $\frac{1}{10}$ **1d.** $\frac{1}{10}$
3. Answers will vary.
3a. $\sqrt{8} = 2\sqrt{2}$ **3b.** $\sqrt{12} = 2\sqrt{3}$
3c. 2.5 **3d.** $3\frac{1}{3}$
5.

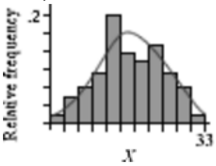


7a. true **7b.** true
7c. False; if the distribution is symmetric, then they can all be the same.
9. *Hint:* For 9a, create a random list of 100 numbers from 0 to 1, and store it in L_1. Enter values of $(L_1)^2$ in L_2, and graph a histogram of these values. You may want to rerandomize L_1 several times, and then generalize the shape of the histogram. See Calculator Notes 1L and 13D for help with these calculator functions. Use a similar process for 9b and 9c.
11. Answers will vary.
13a.

x	0–3	3-6	6-9	9-12	12-15	15-18
$P(x)$	.015	.046	.062	.092	.2	.138

x	18-21	21-24	24-27	27-30	30-33
$P(x)$	.123	.154	.092	.062	.015

13b, d.



13c. 15–18 min

15. .022

17. *QR* is 0 when *P* overlaps *R*, then grows larger and larger without bound until *P* reaches the *y*-axis, at which point *Q* is undefined because $\overleftrightarrow{PO}$ and $\overleftrightarrow{RT}$ are parallel and do not intersect. As *P* moves through Quadrant II, *QR* decreases to 0. Then, as *P* moves through Quadrant III, *QR* increases, again without bound. When *P* reaches the *y*-axis, point *Q* is again undefined. As *P* moves through Quadrant IV, *QR* again decreases to 0. These patterns correspond to the zeros and vertical asymptotes of the graph in Exercise 16.

19. In all cases, the area remains the same. The relationship holds for all two-dimensional figures.

LESSON 13.2

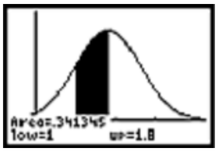
1a. *Hint:* Enter the expressions as Y1 and Y2. Then graph or create a table of values, and confirm that they are the same.

1b. $y \approx .242$ and $n(x, 0, 1) \approx .242$

3a. $\mu = 18, \sigma \approx 2.5$ **3b.** $\mu = 10, \sigma \approx 0.8$

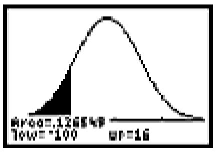
3c. $\mu \approx 68, \sigma \approx 6$ **3d.** $\mu \approx 0.47, \sigma \approx 0.12$

5.



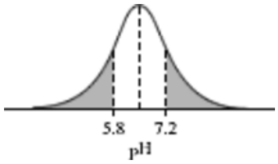
[– 0.6, 4.2, 1, – 0.1, 0.6, 0]

7a.

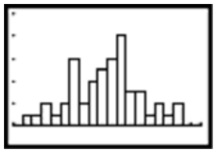


[14.7, 18.9, 1, – 0.1, 0.6, 0]

7b. 12.7%. Sample answer: No, more than 10% of boxes do not meet minimum weight requirements.

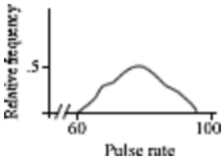


11a. $\mu = 79.1, \sigma \approx 7.49$



[60, 100, 2, – 1, 10, 2]

11b. sample answer:



11c. $y = \frac{1}{7.49\sqrt{2\pi}}\left(\sqrt{e}\right)^{-((x-79.1)/7.49)^2}$

11d. Answers will vary. The data do not appear to be normally distributed. They seem to be approximately symmetrically distributed with several peaks.

13a. *Hint:* Consider what the mean and standard deviation tell you about the distribution of test scores. Can you be sure which test is more difficult?

13b. The French exam, because it has the greatest standard deviation

13c. *Hint:* Determine how many standard deviations each student's score is from the mean. This will tell you how each student scored relative to other test-takers.

15. 25344

LESSON 13.3

1. *Hint:* See page 746.

3a. 122.6 **3b.** 129.8

3c. 131.96 **3d.** 123.8

5a. $z = 1.8$ **5b.** $z = -0.67$

5c. approximately .71

7a. (3.058, 3.142) **7b.** (3.049, 3.151)

7c. (3.034, 3.166)

9a. decrease **9b.** increase

9c. stay the same size **9d.** increase

11a. between 204.6 and 210.6 passengers

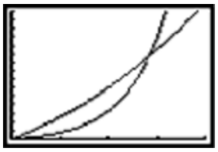
11b. .07

13a. $a = 0.0125$ **13b.** .6875
13c. .1875 **13d.** 0
13e. 0 **13f.** $18\frac{1}{3}$

15a. Let n represent the number of months, and let S_n represent the cumulated total.

Plan 1: $S_n = 398n + 2n^2$; Plan 2: $S_n = \frac{75(1-1.025^n)}{1-1.025}$

15b.

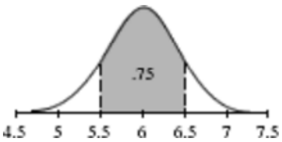


[0, 200, 50, 0, 150000, 10000]

15c. If you stay 11 years 9 months or less, choose Plan 2. If you stay longer, choose Plan 1.

LESSON 13.4

1a. 33.85% **1b.** 20.23%
1c. 10.56% **1d.** 0.6210%
3. 31.28%
5a. $224 < \mu < 236$
5b. $227.6 < \mu < 232.4$
5c. $228 < \mu < 232$
7. $s = 0.43$



9a. 5 samples **9b.** $\bar{x} = 0.56$, $s = 0.20$
9c. approximately 0
9d. probably, because these results are highly unlikely if the site is contaminated
11. The graph appears to sit on a horizontal line. The graph is skewed; it doesn't have a line of symmetry.
13. $y = 4.53x + 12.98$; 6.2

LESSON 13.5

1a. .95 **1b.** –.95 **1c.** – .6 **1d.** .9
3a. – 33 **3b.** 17; 4
3c. 6; 2.1213 **3d.** – .9723
3e. There is a strong negative correlation in the data.
3f. *Hint:* Do the points seem to decrease linearly?

5a. correlation; weight gain probably has more to do with amount of physical activity than television ownership

5b. correlation; the age of the children may be the variable controlling both size of feet and reading ability

5c. correlation; the size of a fire may be the variable controlling both the number of firefighters and the length of time

7. $r \approx .915$. There is a strong positive correlation between the number of students and the number of faculty.

9a. $r = -1$. This value of r implies perfect negative correlation, which is consistent with the data.

9b. $r \approx .833$. This value of r implies strong positive correlation, but the data suggest negative correlation with one outlier.

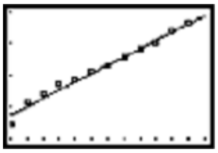
9c. $r = 0$. This value of r implies no correlation, but the data suggest negative correlation with one outlier.

9d. Yes, one outlier can drastically affect the value of r.

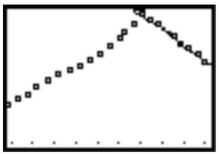
13. possible answer: $y = -1.5x + 6$
15. 60 km/h

LESSON 13.6

1a. $\bar{x} = 1975$ **1b.** $\bar{y} = 40.15$ **1c.** $s_x = 18.71$
1d. $s_y = 8.17$ **1e.** $r \approx .9954$
3a. 0.3166, –0.2292, 0.1251, 0.1794, –1.3663, 0.9880
3b. 0.01365
3c. 0.1002, 0.0525, 0.0157, 0.0322, 1.8667, 0.9761
3d. 3.0435 **3e.** 0.8723
5a.



[1978, 1990, 1, 85, 105, 5]
$\hat{y} = 1.3115x - 2505.3782$
5b. 124.2 ppt
5c.



[1979, 1998, 1, 85, 105, 5]
$\hat{y} = -0.7714x + 1639.4179$

5d. 92.8 ppt. This is 31.4 ppt lower than the amount predicted in 5b.

7a. $\hat{y} = -1.212x + 110.2$

7b. possible answer: 10°N to 60°N

7c. The cities that appear not to follow the pattern are Denver, which is a high mountainous city; Mexico City, which is also a high mountainous city; Phoenix, which is in desert terrain; Quebec, which is subject to the Atlantic currents; and Vancouver, which is subject to the Pacific currents.

7d. Answers will vary.

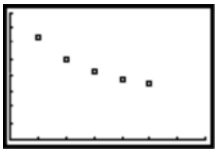
9. *Hint:* You may want to consider how many points are used to calculate each line of fit, whether each is affected by outliers, and which is easier to calculate by hand.

11a. $y = 1000$ **11b.** $y = \frac{100}{\sqrt{x}}$

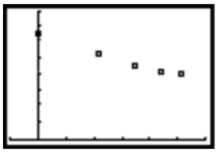
13. $y = -6(x - 1)(x + 2)(x + 5)$

15. The length will increase without bound.

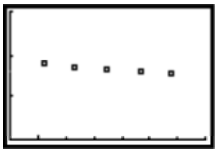
LESSON 13.7

1a. 

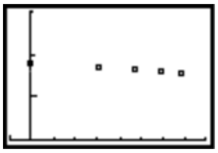
[0, 7, 1, 0, 80, 10]

1b. 

[–0.1, 0.8, 0.1, 0, 80, 10]

1c. 

[0, 6, 1, 0, 3, 1]

1d. 

[–0.1, 0.8, 0.1, 0, 3, 1]

1e. It is difficult to tell visually. But (log x, log y) has the strongest correlation coefficient, $r \approx -.99994$.

3a. $\hat{y} = 67.7 - 7.2x$

3b. $\hat{y} = 64 - 43.25 \log x$

3c. $\hat{y} = 54.4 \cdot 0.592^x + 20$

3d. $\hat{y} = 46.33x^{-0.68076} + 20$

5a. $\hat{y} = -3.77x^3 + 14.13x^2 + 8.23x - 0.01$

5b. 0.079 **5c.** 12.52 m^3

5d. Because the root mean square is 0.079, you can expect the predicted volume to be within approximately 0.079 cubic meter of the true value.

7a. 4.2125 **7b.** 43.16875

7c. 6.28525 **7d.** .8544

7e. .90236; the cubic model is a better fit.

7f. $\hat{y} = -0.07925x + 8.175$; $R^2 \approx .582$. The values of R^2 and r^2 are equal for the linear model.

9a. 86.2 **9b.** 79.8 **9c.** 89.4

11a. approximately 1910

11b. approximately 847

11c. approximately 919

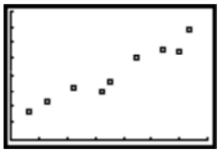
CHAPTER 13 REVIEW

1a. $0.5(20)(.1) = 1$ **1b.** $20 - 5\sqrt{6} \approx 7.75$

1c. .09 **1d.** $\frac{77}{300} \approx .257$

3a. $\bar{x} = 10.55$ lb; $s = 2.15$ lb

3b. 6.25 lb to 14.85 lb

5a. 

[0, 11, 1, 0, 80, 10]

5b. yes; $r \approx .965$, indicating a relationship that is close to linear

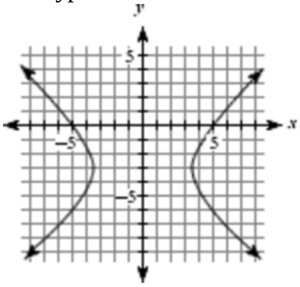
5c. $\hat{y} = 5.322x + 10.585$

5d. The rolling distance increases 5.322 in. for every additional inch of wheel diameter. The skateboard will skid approximately 10.585 in. even if it doesn't have any wheels.

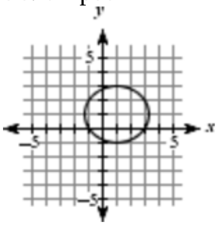
5e. 7.5 in.

7. approximately .062

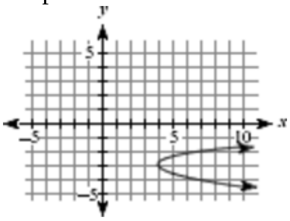
9a. hyperbola



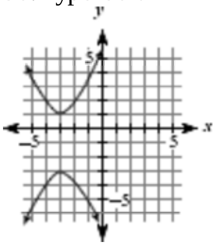
9b. ellipse



9c. parabola



9d. hyperbola



11a. $S_{12} = 144$ **11b.** $S_{20} = 400$ **11c.** $S_n = n^2$

13a. $y = -20x^2 + 332x$ **13b.** \$8.30; \$1377.80

15. Row 1: .72, .08; Row 2: .18, .02; Out of 100 people with the symptoms, the test will accurately confirm that 72 do not have the disease while mistakenly suggesting 8 do have the disease. The test will accurately indicate 18 do have the disease and make a mistake by suggesting 2 do not have the disease who actually have the disease.

16a. possible answer: 3.8% per year

16b. possible answer: $\hat{y} = 5.8(1 + 0.038)^{x - 1970}$

16c. possible answer: 31.1 million

16d. The population predicted by the equation is much higher.

17a. seats versus cost: $r = .9493$; speed versus cost: $r = .8501$

17b. The number of seats is more strongly correlated to cost. Sample answer: The increase in number of seats will cause an increase in weight (both passengers and luggage) and thus cause an increase in the amount of fuel needed.

19a. $\frac{11\pi}{36}$ **19b.** approximately 3.84 cm

19c. approximately 7.68 cm^2

21a. (1, 4) **21b.** (– 5.5, 0.5)

23a. domain: $x \geq \frac{3}{2}$; range: $y \geq 0$

23b. domain: any real number; range: $y \geq 0$

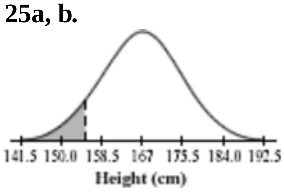
23c. $f(2) = 1$

23d. $x = \pm\sqrt{\frac{1}{3}}$ or $x \approx \pm 0.577$

23e. $g(f(3)) = 18$

23f. $f(g(x)) = \sqrt{12x^2 - 3}$

25a, b.



25c. approximately 7.9%

Glossary

The number in parentheses at the end of each definition gives the page where each word or phrase is first used in the text. Some words and phrases are introduced more than once, either because they have different applications in different chapters or because they first appeared within features such as Project or Take Another Look; in these cases, there may be multiple page numbers listed.

A

ambiguous case A situation in which more than one possible solution exists. (472)

amplitude Half the difference of the maximum and minimum values of a periodic function. (584)

angular speed The amount of rotation, or angle traveled, per unit of time. (577)

antilog The inverse function of a logarithm. (279)

arithmetic mean See **mean.**

arithmetic sequence A sequence in which each term after the starting term is equal to the sum of the previous term and a common difference. (31)

arithmetic series A sum of terms of an arithmetic sequence. (631)

asymptote A line that a graph approaches, but does not reach, as x- or y-values increase in the positive or negative direction. (516)

augmented matrix A matrix that represents a system of equations. The entries include a column for the coefficients of each variable and a final column for the constant terms. (318)

B

base The base of an exponential expression, b^x, is b. The base of a logarithmic expression, $\log_b x$, is b. (245)

bearing An angle measured clockwise from north. (439)

bin A column in a histogram that represents a certain interval of possible data values. (94)

binomial A polynomial with two terms. (360)

Binomial Theorem For any binomial $(p + q)$ and any positive integer n, the binomial expansion is $(p+q)^n = {}_nC_n\, p^n q^0 + {}_nC_{(n-1)}\, p^{n-1} q^1 + {}_nC_{(n-2)}\, p^{n-2} q^2 + \cdots + {}_nC_0\, p^0 q^n$. (712)

bisection method A method of finding an x-intercept of a function by calculating successive midpoints of segments with endpoints above and below the zero. (417)

bivariate sampling The process of collecting data on two variables per case. (763)

Boolean algebra A system of logic that combines algebraic expressions with "and" (multiplication), "or" (addition), and "not" (negative) and produces results that are "true" (1) or "false" (0). (232)

box plot A one-variable data display that shows the five-number summary of a data set. (79)

box-and-whisker plot See **box plot.**

C

center (of a circle) See **circle.**

center (of an ellipse) The point midway between the foci of an ellipse. (501)

center (of a hyperbola) The point midway between the vertices of a hyperbola. (514)

Central Limit Theorem If several samples containing n data values are taken from a population, then the means of the samples form a distribution that is approximately normal, the population mean is approximately the mean of the distribution of sample means, and the standard deviation of the sample means is approximately the population's standard deviation divided by the square root of n. Each approximation is better for larger values of n. (753)

circle A locus of points in a plane that are located a constant distance, called the radius, from a fixed point, called the center. (447, 497, 498)

coefficient of determination (R^2) A measure of how well a given curve fits a set of nonlinear data. (786)

combination An arrangement of choices in which the order is unimportant. (704, 705)

common base property of equality For all real values of a, m, and n, if $a^n = a^m$, then $n = m$. (246)

common difference The constant difference between consecutive terms in an arithmetic sequence. (31)

common logarithm A logarithm with base 10, written $\log x$, which is shorthand for $\log_{10} x$. (274)

common ratio The constant ratio between consecutive terms in a geometric sequence. (33)

complements Two events that are mutually exclusive and make up all possible outcomes. (682)

completing the square A method of converting a quadratic equation from general form to vertex form. (380, 527)

complex conjugate A number whose product with a complex number produces a nonzero real number. The complex conjugate of $a + bi$ is $a - bi$. (391)

complex number A number with a real part and an imaginary part. A complex number can be written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit, $\sqrt{-1}$. (391, 392)

complex plane A coordinate plane used for graphing complex numbers, where the horizontal axis is the real axis and the vertical axis is the imaginary axis. (394)

composition of functions The process of using the output of one function as the input of another function. The composition of f and g is written $f(g(x))$. (225)

compound event A sequence of simple events. (669)

compound interest Interest charged or received based on the sum of the original principal and accrued interest. (40)

conditional probability The probability of a particular dependent event, given the outcome of the event on which it depends. (672)

confidence interval A p% confidence interval is an interval about $\bar{x}$ in which you can be p% confident that the population mean, μ, lies. (748)

conic section Any curve that can be formed by the intersection of a plane and an infinite double cone. Circles, ellipses, parabolas, and hyperbolas are conic sections. (496)

conjugate pair A pair of complex numbers whose product is a nonzero real number. The complex numbers $a + bi$ and $a - bi$ form a conjugate pair. (391)

consistent (system) A system of equations that has at least one solution. (317)

constraint A limitation in a linear programming problem, represented by an inequality. (337)

continuous random variable A quantitative variable that can take on any value in an interval of real numbers. (724)

convergent series A series in which the terms of the sequence approach a long-run value, and the partial sums of the series approach a long-run value as the number of terms increases. (637)

correlation A linear relationship between two variables. (763)

correlation coefficient (r) A value between –1 and 1 that measures the strength and direction of a linear relationship between two variables. (763)

cosecant The reciprocal of the sine ratio. If A is an acute angle in a right triangle, then the cosecant of angle A is the ratio of the length of the hypotenuse to the length of the opposite leg, or $\csc A = \frac{hyp}{opp}$. See **trigonometric function.** (609)

cosine If A is an acute angle in a right triangle, then the cosine of angle A is the ratio of the length of the adjacent leg to the length of the hypotenuse, or $\cos A = \frac{adj}{hyp}$. See **trigonometric function.** (440)

cotangent The reciprocal of the tangent ratio. If A is an acute angle in a right triangle, then the cotangent of angle A is the ratio of the length of the adjacent leg to the length of the opposite leg, or $\cot A = \frac{adj}{opp}$. See **trigonometric function.** (609)

coterminal Describes angles in standard position that share the same terminal side. (569)

counting principle When there are n_1 ways to make a first choice, n_2 ways to make a second choice, n_3 ways to make a third choice, and so on, the product $n_1 \cdot n_2 \cdot n_3 \cdot \ldots$ represents the total number of different ways in which the entire sequence of choices can be made. (695)

Glossary

cubic function A polynomial function of degree 3. (399)

curve straightening A technique used to determine whether a relationship is logarithmic, exponential, power, or none of these. See **linearizing.** (287)

cycloid The path traced by a fixed point on a circle as the circle rolls along a straight line. (628)

D

degree In a one-variable polynomial, the power of the term that has the greatest exponent. In a multivariable polynomial, the greatest sum of the powers in a single term. (360)

dependent (events) Events are dependent when the probability of occurrence of one event depends on the occurrence of the other. (672)

dependent (system) A system with infinitely many solutions. (317)

dependent variable A variable whose values depend on the values of another variable. (123)

determinant The difference of the products of the entries along the diagonals of a square matrix. For any 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is $ad - bc$. (357)

deviation For a one-variable data set, the difference between a data value and some standard value, usually the mean. (87)

dilation A transformation that stretches or shrinks a function or graph both horizontally and vertically by the same scale factor. (309)

dimensions (of a matrix) The number of rows and columns in a matrix. A matrix with m rows and n columns has dimensions $m \times n$. (302)

directrix See **parabola.**

discontinuity A jump, break, or hole in the graph of a function. (185)

discrete graph A graph made of distinct, nonconnected points. (52)

discrete random variable A random variable that can take on only distinct (not continuous) values. (688)

distance formula The distance, d, between points (x_1, y_1) and (x_2, y_2), is given by the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. (489)

domain The set of input values for a relation. (123)

double root A value r is a double root of an equation $f(x) = 0$ if $(x - r)^2$ is a factor of $f(x)$. (409)

doubling time The time needed for an amount of a substance to double. (240)

E

e A transcendental number related to continuous growth, with a value of approximately 2.718. (293)

eccentricity A measure of how elongated an ellipse is. (502)

elimination A method for solving a system of equations that involves adding or subtracting multiples of the equations to eliminate a variable. (158)

ellipse A shape produced by stretching or shrinking a circle horizontally or vertically. The shape can be described as a locus of points in a plane for which the sum of the distances to two fixed points, called the foci, is constant. (217, 499, 500)

ellipsoid A three-dimensional shape formed by rotating an ellipse about one of its axes. (503)

end behavior The behavior of a function $y = f(x)$ for x-values that are large in absolute value. (405)

entry Each number in a matrix. The entry identified as a_{ij} is in row i and column j. (302)

even function A function that has the y-axis as a line of symmetry. For all values of x in the domain of an even function, $f(-x) = f(x)$. (235, 612)

event A specified set of outcomes. (659)

expanded form The form of a repeated multiplication expression in which every occurrence of each factor is shown. For example, $4^3 \cdot 5^2 = 4 \cdot 4 \cdot 4 \cdot 5 \cdot 5$. (245)

expansion An expression that is rewritten as a single polynomial. (711)

expected value An average value found by multiplying the value of each possible outcome by its probability, then summing all the products. (688, 689)

experimental probability A probability calculated based on trials and observations, given by the ratio of the number of occurrences of an event to the total number of trials. (659)

explanatory variable In statistics, the variable used to predict (or explain) the value of the response variable. (765)

explicit formula A formula that gives a direct relationship between two discrete quantities. A formula for a sequence that defines the *n*th term in relation to *n*, rather than the previous term (s). (114)

exponent The exponent of an exponential expression, b^x, is x. The exponent tells how many times the base, b, is a factor. (245)

exponential function A function with a variable in the exponent, typically used to model growth or decay. The general form of an exponential function is $y = ab^x$, where the coefficient, a, is the y-intercept and the base, b, is the ratio. (239, 240)

extraneous solution An invalid solution to an equation. Extraneous solutions are sometimes found when both sides of an equation are raised to a power. (206)

extrapolation Estimating a value that is outside the range of all other values given in a data set. (131)

extreme values Maximums and minimums. (405)

F

Factor Theorem If $P(r) = 0$, then r is a zero and $(x - r)$ is a factor of the polynomial function $y = P(x)$. This theorem is used to confirm that a number is a zero of a function. (413)

factored form The form $y = a(x - r_1)(x - r_2) \cdots (x - r_n)$ of a polynomial function, where $a \neq 0$. The values $r_1, r_2, \ldots, r_n$ are the zeros of the function, and a is the vertical scale factor. (370)

factorial For any integer n greater than 1, n factorial, written $n!$, is the product of all the consecutive integers from n decreasing to 1. (697)

fair Describes a coin that is equally likely to land heads or tails. Can also apply to dice and other objects. (657)

family of functions A group of functions with the same parent function. (194)

feasible region The set of points that is the solution to a system of inequalities. (337)

Fibonacci sequence The sequence of numbers 1, 1, 2, 3, 5, 8, . . . , each of which is the sum of the two previous terms. (37, 59)

finite A limited quantity. (630)

finite differences method A method of finding the degree of a polynomial that will model a set of data, by analyzing differences between data values corresponding to equally spaced values of the independent variable. (361)

first quartile (Q_1) The median of the values less than the median of a data set. (79)

five-number summary The minimum, first quartile, median, third quartile, and maximum of a one-variable data set. (79)

focus (plural **foci**) A fixed point or points used to define a conic section. See **ellipse, hyperbola,** and **parabola.**

fractal The geometric result of infinitely many applications of a recursive procedure or calculation. (32, 397)

frequency (of a data set) The number of times a value appears in a data set, or the number of values that fall in a particular interval. (94)

frequency (of a sinusoid) The number of cycles of a periodic function that can be completed in one unit of time. (602)

function A relation for which every value of the independent variable has at most one value of the dependent variable. (178)

function notation A notation that emphasizes the dependent relationship between the variables used in a function. The notation $y = f(x)$ indicates that values of the dependent variable, y, are explicitly defined in terms of the independent variable, x, by the function f. (178)

Glossary

Glossary

G

general form (of a polynomial) The form of a polynomial in which the terms are ordered such that the degrees of the terms decrease from left to right. (360)

general form (of a quadratic function) The form $y = ax^2 + bx + c$, where $a \neq 0$. (368)

general quadratic equation An equation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where A, B, and C do not all equal zero. (525)

general term The nth term, u_n, of a sequence. (29)

geometric probability A probability that is found by calculating a ratio of geometric characteristics, such as lengths or areas. (661)

geometric random variable A random variable that represents the number of trials needed to get the first success in a series of independent trials. (688)

geometric sequence A sequence in which each term is equal to the product of the previous term and a common ratio. (33)

geometric series A sum of terms of a geometric sequence. (637)

golden ratio The ratio of two numbers (larger to smaller) whose ratio to each other equals the ratio of their sum to the larger number. Or, the positive number whose square equals the sum of itself and 1. The number $\frac{1+\sqrt{5}}{2}$, or approximately 1.618, often represented with the lowercase Greek letter phi, ϕ. (60, 389)

golden rectangle A rectangle in which the ratio of the length to the width is the golden ratio. (60, 389)

greatest integer function The function $f(x) = [x]$ that returns the largest integer that is less than or equal to a real number, x. (155, 185)

H

half-life The time needed for an amount of a substance to decrease by one-half. (238)

histogram A one-variable data display that uses bins to show the distribution of values in a data set. (94)

hole A missing point in the graph of a relation. (544)

hyperbola A locus of points in a plane for which the difference of the distances to two fixed points, called the foci, is constant. (514, 518)

hyperboloid A three-dimensional shape formed by rotating a hyperbola about the line through its foci or about the perpendicular bisector of the segment connecting the foci. (496)

hypothesis testing The process of creating a hypothesis about one or more population parameters, and either rejecting the hypothesis or letting it stand, based on probabilities. (755)

I

identity An equation that is true for all values of the variables for which the expressions are defined. (609)

identity matrix The square matrix, symbolized by $[I]$, that does not alter the entries of a square matrix $[A]$ under multiplication. Matrix $[I]$ must have the same dimensions as matrix $[A]$, and it has entries of 1's along the main diagonal (from top left to bottom right) and 0's in all other entries. (327, 328)

image A graph of a function or point(s) that is the result of a transformation of an original function or point(s). (188)

imaginary axis See **complex plane.**

imaginary number A number that is the square root of a negative number. An imaginary number can be written in the form bi, where b is a real number ($b \neq 0$) and i is the imaginary unit, $\sqrt{-1}$. (391)

imaginary unit The imaginary unit, i, is defined by $i^2 = -1$ or $i = \sqrt{-1}$. (391)

inconsistent (system) A system of equations that has no solution. (317)

independent (events) Events are independent when the occurrence of one has no influence on the occurrence of the other. (671)

independent (system) A system of equations that has exactly one solution. (317)

independent variable A variable whose values are not based on the values of another variable. (123)

inequality A statement that one quantity is less than, less than or equal to, greater than, greater than or equal to, or not equal to another quantity. (336)

inference The use of results from a sample to draw conclusions about a population. (755)

infinite A quantity that is unending, or without bound. (637)

infinite geometric series A sum of infinitely many terms of a geometric sequence. (637)

inflection point A point where a curve changes between curving downward and curving upward. (739)

intercept form The form $y = a + bx$ of a linear equation, where a is the y-intercept and b is the slope. (121)

interpolation Estimating a value that is within the range of all other values given in a data set. (131)

interquartile range (*IQR*) A measure of spread for a one-variable data set that is the difference between the third quartile and the first quartile. (82)

inverse The relationship that reverses the independent and dependent variables of a relation. (268)

inverse matrix The matrix, symbolized by $[A]^{-1}$, that produces an identity matrix when multiplied by [A]. (327, 328)

inverse variation A relation in which the product of the independent and dependent variables is constant. An inverse variation relationship can be written in the form $xy = k$, or $y = \frac{k}{x}$. (537)

L

Law of Cosines For any triangle with angles A, B, and C, and sides of lengths a, b, and c (a is opposite $\angle A$, b is opposite $\angle B$, and c is opposite $\angle C$), these equalities are true:
$a^2 = b^2 + c^2 - 2bc \cos A$, $b^2 = a^2 + c^2 - 2ac \cos B$, and $c^2 = a^2 + b^2 - 2ab \cos C$. (477)

Law of Sines For any triangle with angles A, B, and C, and sides of lengths a, b, and c (a is opposite $\angle A$, b is opposite $\angle B$, and c is opposite $\angle C$), these equalities are true: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. (470)

least squares line A line of fit for which the sum of the squares of the residuals is as small as possible. (772)

limit A long-run value that a sequence or function approaches. The quantity associated with the point of stability in dynamic systems. (47)

line of fit A line used to model a set of two-variable data. (128)

line of symmetry A line that divides a figure or graph into mirror-image halves. (194)

linear In the shape of a line or represented by a line, or an algebraic expression or equation of degree 1. (52)

linear equation An equation characterized by a constant rate of change. The graph of a linear equation in two variables is a straight line. (114)

linear programming A method of modeling and solving a problem involving constraints that are represented by linear inequalities. (344)

linearizing A method of finding an equation to fit data by graphing points in the form ($\log x$, y), (x, $\log y$), or ($\log x$, $\log y$), and looking for a linear relationship. (781)

local maximum A value of a function or graph that is greater than other nearby values. (405)

local minimum A value of a function or graph that is less than other nearby values. (405)

locus A set of points that fit a given condition. (490)

logarithm A value of a logarithmic function, abbreviated log. For $a > 0$ and $b > 0$, $\log_b a = x$ means that $a = b^x$. (274)

logarithm change-of-base property For $a > 0$ and $b > 0$, $\log_a x$ can be rewritten as $\frac{\log_b x}{\log_b a}$. (275, 282)

logarithmic function The logarithmic function $y = \log_b x$ is the inverse of $y = b^x$, where $b > 0$ and $b \neq 1$. (274)

Glossary

logistic function A function used to model a population that grows and eventually levels off at the maximum capacity supported by the environment. A logistic function has a variable growth rate that changes based on the size of the population. (67)

lurking variable A variable that is not included in an analysis but which could explain a relationship between the other variables being analyzed. (767)

M

major axis The longer dimension of an ellipse. Or the line segment with endpoints on the ellipse that has this dimension. (500)

matrix A rectangular array of numbers or expressions, enclosed in brackets. (300)

matrix addition The process of adding two or more matrices. To add matrices, you add corresponding entries. (313)

matrix multiplication The process of multiplying two matrices. The entry c_{ij} in the matrix $[C]$ that is the product of two matrices, $[A]$ and $[B]$, is the sum of the products of corresponding entries in row i of matrix $[A]$ and column j of matrix $[B]$. (313)

maximum The greatest value in a data set or the greatest value of a function or graph. (79, 373, 377)

mean ($\bar{x}$ or μ) A measure of central tendency for a one-variable data set, found by dividing the sum of all values by the number of values. For a probability distribution, the mean is the sum of each value of x times its probability, and it represents the x-coordinate of the centroid or balance point of the region. (78, 727)

measure of central tendency A single number used to summarize a one-variable data set, commonly the mean, median, or mode. (78)

median A measure of central tendency for a one-variable data set that is the middle value, or the mean of the two middle values, when the values are listed in order. For a probability distribution, the median is the number d such that the line $x = d$ divides the area into two parts of equal area. (78, 727)

median-median line A line of fit found by dividing a data set into three groups, finding three points (M_1, M_2, and M_3) based on the median x-value and the median y-value for each group, and writing the equation that best fits these three points. (135, 137)

minimum The least value in a data set or the least value of a function or graph. (79, 373, 377)

minor axis The shorter dimension of an ellipse. Or the line segment with endpoints on the ellipse that has this dimension. (500)

mode A measure of central tendency for a one-variable data set that is the value(s) that occur most often. For a probability distribution, the mode is the value(s) of x at which the graph reaches its maximum value. (78, 727)

model A mathematical representation (sequence, expression, equation, or graph,) that closely fits a set of data. (52)

monomial A polynomial with one term. (360)

multiplicative identity The number 1 is the multiplicative identity because any number multiplied by 1 remains unchanged. (327)

multiplicative inverse Two numbers are multiplicative inverses, or reciprocals, if they multiply to 1. (327)

mutually exclusive (events) Two outcomes or events are mutually exclusive when they cannot both occur simultaneously. (679)

N

natural logarithm A logarithm with base e, written $\ln x$, which is shorthand for $\log_e x$. (293)

negative exponents For $a > 0$, and all real values of n, the expression a^{-n} is equivalent to $\frac{1}{a^n}$ and $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$. (246, 282).

nonrigid transformation A transformation that produces an image that is not congruent to the original figure. Stretches, shrinks, and dilations are nonrigid transformations (unless the scale factor is 1 or –1). (211)

normal curve The graph of a normal distribution. (735)

Glossary

normal distribution A symmetric bell-shaped distribution. The equation for a normal distribution with mean μ and standard deviation σ is $\frac{1}{\sigma\sqrt{2\pi}}\left(\sqrt{e}\right)^{-((x-\mu)/\sigma)^2}$.

null hypothesis A statement that a given hypothesis is not true. (755)

O

oblique (triangle) A triangle that does not contain a right angle. (468)

odd function A function that is symmetric about the origin. For all values of x in the domain of an odd function, $f(-x) = -f(x)$. (235, 612)

one-to-one function A function whose inverse is also a function. (268)

outcome A possible result of one trial of an experiment. (659)

outlier A value that stands apart from the bulk of the data. (89, 91)

P

parabola A locus of points in a plane that are equidistant from a fixed point, called the focus, and a fixed line, called the directrix. (194, 508, 510)

paraboloid A three-dimensional shape formed by rotating a parabola about its line of symmetry. (507)

parameter (in parametric equations) See **parametric equations.**

parameter (statistical) A number, such as the mean or standard deviation, that describes an entire population. (724)

parametric equations A pair of equations used to separately describe the x-and y-coordinates of a point as functions of a third variable, called the parameter. (424)

parent function The most basic form of a function. A parent function can be transformed to create a family of functions. (194)

partial sum A sum of a finite number of terms of a series. (630)

Pascal's triangle A triangular arrangement of numbers containing the coefficients of binomial expansions. The first and last numbers in each row are 1's, and each other number is the sum of the two numbers above it. (710)

percentile rank The percentage of values in a data set that are below a given value. (97)

perfect square A number that is equal to the square of an integer, or a polynomial that is equal to the square of another polynomial. (378)

period The time it takes for one complete cycle of a cyclical motion to take place. Also, the minimum amount of change of the independent variable needed for a pattern in a periodic function to repeat. (213, 566)

periodic function A function whose graph repeats at regular intervals. (566)

permutation An arrangement of choices in which the order is important. (697, 698)

phase shift The horizontal translation of a periodic graph. (584)

point-ratio form The form $y = y_1 \cdot b^{x-x_1}$ of an exponential function equation, where the curve passes through the point (x_1, y_1) and has ratio b. (254)

point-slope form The form $y = y_1 + b(x - x_1)$ of a linear equation, where (x_1, y_1) is a point on the line and b is the slope. (129)

polar coordinates A method of representing points in a plane with ordered pairs in the form (r, θ), where r is the distance of the point from the origin and θ is the angle of rotation of the point from the positive x-axis. (622)

polynomial A sum of terms containing a variable raised to different powers, often written in the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x^1 + a_0$, where x is a variable, the exponents are nonnegative integers, and the coefficients are real numbers. (360)

polynomial function A function in which a polynomial expression is set equal to a second variable, such as y or $f(x)$. (360)

population A complete set of people or things being studied. (713, 724)

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power function A function that has a variable as the base. The general form of a power function is $y = ax^n$, where a and n are constants. (247)

power of a power property For $a > 0$, and all real values of m and n, $(a^m)^n$ is equivalent to a^{mn}. (246, 282)

power of a product property For $a > 0$, $b > 0$, and all real values of m, $(ab)^m$ is equivalent to $a^m b^m$. (246, 282)

power of a quotient property For $a > 0$, $b > 0$, and all real values of n, $\left(\frac{a}{b}\right)^n$ is equivalent to $\frac{a^n}{b^n}$. (246, 282)

power property of equality For all real values of a, b, and n, if $a = b$, then $a^n = b^n$. (246)

power property of logarithms For $a > 0$, $x > 0$, and $n > 0$, $\log_a x^n$ can be rewritten $n \log_a x$. (282)

principal The initial monetary balance of a loan, debt, or account. (40)

principal value The one solution to an inverse trigonometric function that is within the range for which the function is defined. (597)

probability distribution A continuous curve that shows the values and the approximate frequencies of the values of a continuous random variable for an infinite set of measurements. (725)

product property of exponents For $a > 0$ and $b > 0$, and all real values of m and n, the product $a^m \cdot a^n$ is equivalent to a^{m+n}. (246, 282)

product property of logarithms For $a > 0$, $x > 0$, and $y > 0$, $\log_a xy$ is equivalent to $\log_a x + \log_a y$. (282)

projectile motion The motion of an object that rises or falls under the influence of gravity. (377)

Q

quadratic curves The graph of a two-variable equation of degree 2. Circles, parabolas, ellipses, and hyperbolas are quadratic curves. (525)

quadratic formula If a quadratic equation is written in the form $ax^2 + bx + c = 0$, the solutions of the equation are given by the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. (386)

quadratic function A polynomial function of degree 2. Quadratic functions are in the family with parent function $y = x^2$. (194, 368)

quotient property of exponents For $a > 0$ and $b > 0$, and all real values of m and n, the quotient $\frac{a^m}{a^n}$ is equivalent to a^{m-n}. (246, 282)

quotient property of logarithms For $a > 0$, $x > 0$, and $y > 0$, the expression $\log_a \frac{x}{y}$ can be rewritten as $\log_a x - \log_a y$. (282)

R

radian An angle measure in which one full rotation is 2π radians. One radian is the measure of an arc, or the measure of the central angle that intercepts that arc, such that the arc's length is the same as the circle's radius. (574)

radical A square root symbol. (205)

radius See **circle.**

raised to the power A term used to connect the base and the exponent in an exponential expression. For example, in the expression b^x, the base, b, is raised to the power x. (245)

random number A number that is as likely to occur as any other number within a given set. (658)

random process A process in which no individual outcome is predictable. (656)

random sample A sample in which not only is each person (or thing) equally likely, but all groups of persons (or things) are also equally likely. (78, 756)

random variable A variable that takes on numerical values governed by a chance experiment. (688)

range (of a data set) A measure of spread for a one-variable data set that is the difference between the maximum and the minimum. (79)

range (of a relation) The set of output values of a relation. (123)

rational Describes a number or an expression that can be expressed as a fraction or ratio. (252)

rational exponent An exponent that can be written as a fraction. The expression $a^{m/n}$ can be rewritten as $\left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$, for $a > 0$. (253, 282)

rational function A function that can be written as a quotient, $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial expressions and $q(x)$ is of degree 1 or higher. (537)

Rational Root Theorem If the polynomial equation $P(x) = 0$ has rational roots, they are of the form $\frac{p}{q}$, where p is a factor of the constant term and q is a factor of the leading coefficient. (414)

real axis See **complex plane.**

recursion Applying a procedure repeatedly, starting with a number or geometric figure, to produce a sequence of numbers or figures. Each term or stage builds on the previous term or stage. (28)

recursive formula A starting value and a recursive rule for generating a sequence. (29)

recursive rule Defines the *n*th term of a sequence in relation to the previous term(s). (29)

reduced row-echelon form A matrix form in which each row is reduced to a 1 along the diagonal, and a solution, and the rest of the matrix entries are 0's. (318)

reference angle The acute angle between the terminal side of an angle in standard position and the x-axis. (567)

reference triangle A right triangle that is drawn connecting the terminal side of an angle in standard position to the x-axis. A reference triangle can be used to determine the trigonometric ratios of an angle. (567)

reflection A transformation that flips a graph across a line, creating a mirror image. (202, 220)

regression analysis The process of finding a model with which to make predictions about one variable based on values of another variable. (772)

relation Any relationship between two variables. (178)

relative frequency histogram A histogram in which the height of each bin shows proportions (or relative frequencies) instead of frequencies. (725)

residual For a two-variable data set, the difference between the y-value of a data point and the y-value predicted by the equation of fit. (142)

response variable In statistics, the outcome (dependent) variable that is predicted by the explanatory variable. (765)

rigid transformation A transformation that produces an image that is congruent to the original figure. Translations, reflections, and rotations are rigid transformations. (211)

root mean square error (*s*) A measure of spread for a two-variable data set, similar to standard deviation for a one-variable data set. It is calculated by the formula $s = \sqrt{\frac{\sum_{i=1}^{n}(y_1 - \hat{y})^2}{n-2}}$. (145)

roots The solutions of an equation in the form $f(x) = 0$. (370)

row reduction method A method that transforms an augmented matrix into a solution matrix in reduced row-echelon form. (318)

S

sample A part of a population selected to represent the entire population. Sampling is the process of selecting and studying a sample from a population in order to make conjectures about the whole population. (713, 724)

scalar A real number, as opposed to a matrix or vector. (308)

scalar multiplication The process of multiplying a matrix by a scalar. To multiply a scalar by a matrix, you multiply the scalar by each value in the matrix. (308)

scale factor A number that determines the amount by which a graph is stretched or shrunk, either horizontally or vertically. (211)

secant The reciprocal of the cosine ratio. If A is an acute angle in a right triangle, the secant of angle A is the ratio of the length of the hypotenuse to the length of the adjacent leg, or $\sec A = \frac{hyp}{adj}$. See **trigonometric function.** (609)

sequence An ordered list of numbers. (29)

series A sum of terms of a sequence. (630)

shape (of a data set) Describes how the data are distributed relative to the position of a measure of central tendency. (80)

shifted geometric sequence A geometric sequence that includes an added term in the recursive rule. (47)

Glossary

shrink A transformation that compresses a graph either horizontally or vertically. (209, 213, 220)

simple event An event consisting of just one outcome. A simple event can be represented by a single branch of a tree diagram. (669)

simple random sample See **random sample.**

simulation A procedure that uses a chance model to imitate a real situation. (659)

sine If *A* is an acute angle in a right triangle, then the sine of angle *A* is the ratio of the length of the opposite leg to the length of the hypotenuse, or $\sin A = \frac{opp}{hyp}$. See **trigonometric function.** (440)

sine wave A graph of a sinusoidal function. See **sinusoid.** (583)

sinusoid A function or graph for which $y = \sin x$ or $y = \cos x$ is the parent function. (583)

skewed (data) Data that are spread out more on one side of the center than on the other side. (80)

slope The steepness of a line or the rate of change of a linear relationship. If (x_1, y_1) and (x_2, y_2) are two points on a line, then the slope of the line is $\frac{y_2 - y_1}{x_2 - x_1}$, where $x_2 \neq x_1$. (115, 121)

spread The variability in numerical data. (85)

square root function The function that undoes quaring, giving only the positive square root (that is, the positive number that, when multiplied by itself, gives the input). The square root function is written $y = \sqrt{x}$. (201)

standard deviation (s) A measure of spread for a one-variable data set that uses squaring to eliminate the effect of the different signs of the individual deviations. It is the square root of the variance, or $s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$. (88)

standard form (of a conic section) The form of an equation for a conic section that shows the transformations of the parent equation. (498, 499, 510, 518)

standard form (of a linear equation) The form $ax + by = c$ of a linear equation. (191)

standard normal distribution A normal distribution with mean 0 and standard deviation 1. (736)

standard position An angle positioned with one side on the positive *x*-axis. (567)

standardizing the variable The process of converting data values (*x*-values) to their images (*z*-values) when a normal distribution is transformed into the standard normal distribution. (746)

statistic A numerical measure of a data set or sample. (77)

statistics A collection of numerical measures, or the mathematical study of data collection and analysis. (77)

stem-and-leaf-plot A one-variable data display in which the left digit(s) of the data values, called the stems, are listed in a column on the left side of the plot, while the remaining digits, called the leaves, are listed in order to the right of the corresponding stem. (104)

step function A function whose graph consists of a series of horizontal lines. (185)

stretch A transformation that expands a graph either horizontally or vertically. (209, 213, 220)

substitution A method of solving a system of equations that involves solving one of the equations for one variable and substituting the resulting expression into the other equation. (153)

symmetric (data) Data that are balanced, or nearly so, about the center. (80)

synthetic division An abbreviated form of dividing a polynomial by a linear factor. (415, 416)

system of equations A set of two or more equations with the same variables that are solved or studied simultaneously. (151)

T

tangent If *A* is an acute angle in a right triangle, then the tangent of angle *A* is the ratio of the length of the opposite leg to the length of the adjacent leg, or $\tan A = \frac{opp}{adj}$. See **trigonometric function.** (440)

term (algebraic) An algebraic expression that represents only multiplication and division between variables and constants. (360)

term (of a sequence) Each number in a sequence. (29)

terminal side The side of an angle in standard position that is not on the positive i-axis. (567)

theoretical probability A probability calculated by analyzing a situation, rather than by performing an experiment, given by the ratio of the number of different ways an event can occur to the total number of equally likely outcomes possible. (659)

third quartile (Q_3) The median of the values greater than the median of a data set. (79)

transcendental number An irrational number that, when represented as a decimal, has infinitely many digits with no pattern, such as π or e, and is not the solution of a polynomial equation with integer coefficients. (293)

transformation A change in the size or position of a figure or graph. (194, 220)

transition diagram A diagram that shows how something changes from one time to the next. (300)

transition matrix A matrix whose entries are transition probabilities. (300)

translation A transformation that slides a figure or graph to a new position. (186, 188, 220)

tree diagram A diagram whose branches show the possible outcomes of an event, and sometimes probabilities. (668)

trigonometric function A periodic function that uses one of the trigonometric ratios to assign values to angles with any measure. (583)

trigonometric ratios The ratios of lengths of sides in a right triangle. The three primary trigonometric ratios are sine, cosine, and tangent. (439)

trigonometry The study of the relationships between the lengths of sides and the measures of angles in triangles. (439)

trinomial A polynomial with three terms. (360)

U

unit circle A circle with radius of one unit. The equation of a unit circle with center (0, 0) is $x^2 + y^2 = 1$. (217)

unit hyperbola The parent equation for a hyperbola, $x^2 - y^2 = 1$ or $y^2 - x^2 = 1$. (515)

V

variance (s^2) A measure of spread for a one-variable data set that uses squaring to eliminate the effect of the different signs of the individual deviations. It is the sum of the squares of the deviations divided by one less than the number of values, or $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$. (88)

vector A quantity with both magnitude and direction. (455)A

velocity A measure of speed and direction. Velocity can be either positive or negative. (426)

Venn diagram A diagram of overlapping circles that shows the relationships among members of different sets. (395)

vertex (of a conic section) The point or points where a conic section intersects the axis of symmetry that contains the focus or foci. (194, 514)

vertex (of a feasible region) A corner of a feasible region in a linear programming problem. (337)

vertex form The form $y = a(x - h)^2 + k$ of a quadratic function, where $a \neq 0$. The point (h, k) is the vertex of the parabola, and a is the vertical scale factor. (368)

Z

zero exponent For all values of a except 0, $a^0 = 1$. (246)

zero-product property If the product of two or more factors equals zero, then at least one of the factors must equal zero. A property used to find the zeros of a function without graphing. (369)

zeros (of a function) The values of the independent variable (x-values) that make the corresponding values of the function ($f(x)$-values) equal to zero. Real zeros correspond to x-intercepts of the graph of a function. See **roots.** (369)

***z*-value** The number of standard deviations that a given x-value lies from the mean in a normal distribution. (746)

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