Hands-On Long Division with Skittles for Students with Learning Disabilities

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Abstract

This article explains how to teach long division using hands-on materials (skittles, base ten blocks, colored counters, numeral cards) and investigates reviewing division word problems with middle school students (N=27) with learning disabilities in mathematics by this approach compared to spending the same amount of time on the typical practices of computerized drill and paper/pencil work. Student performance related to division was assessed by an identical pretest/posttest. Both groups participated in ten thirty-minute lessons during which the experimental group used place value color-coded numeral cards, counters and skittles, and the control group reviewed division with computerized practice and worksheet activities. A significant difference was found between the average gain scores of the two groups, favoring the experimental (17.1% versus 4.5%). The experimental group scored as well or better than the control group on all test sections**.**

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Introduction

Two years prior to becoming a college faculty member in education, I (Audrey Rule) accepted a teaching position at an urban public elementary school that became a Montessori magnet school. All of the teachers at our school underwent four hundred hours of instruction to become certified Montessori teachers. There were many wonderful effects of this new philosophy and curriculum at our school. Our racially integrated students interacted more respectfully and peacefully, teachers learned to individualize instruction for children in their multiage classes, and our standardized test scores rose fifteen percentile points in a two-year period.

My colleagues and I agreed that the hands-on Montessori mathematics materials were a large contributor to the rise in test scores. Especially helpful to our students was the color-coding of manipulatives to highlight place value concepts. Many of our upper-grade teachers expressed amazement that fourth and fifth grade students who had struggled with division when taught the traditional way, now comprehended these operations because they finally understood place value. Because these materials helped my elementary students and pre-service teachers (now that I prepare elementary teachers to teach mathematics) understand division better, we thought they would be beneficial to the middle school students with learning disabilities in mathematics that the first author teaches. This article describes how to perform division with skittles and base ten blocks (substituted for the less common Montessori beads), a technique that concretely allows students to understand the operation of division.

Literature Review Until recently, Montessori education in the United States has largely been a private rather than public school endeavor with preparation of teachers generally occurring at private centers. Because universities conduct most education research studies on public education issues, little has been written about Montessori mathematics in the professional literature. However, in the past fifteen years, many public school districts have opened Montessori magnet schools and the American Montessori Society has encouraged its members to conduct and publish research to support its teaching practices. A recent article by Pickering (2004) addressed how Montessori techniques help at-risk children learn. Montessori mathematics materials assist children in focusing their attention through work with concrete manipulatives and in emphasizing order through organized layouts of numeral cards and quantities in place value positions.

In the past, authors have suggested several different approaches to solving long division problems. Rivera and Smith (1988) conducted a small study with eight middle school students with learning disabilities in mathematics using the traditional long division algorithm. They advocated a "demonstration" strategy in which the teacher demonstrated solving a problem with the traditional algorithm, left this as an example that children followed in solving subsequent problems, and used key words ("go into," "place dot," "divide," "multiply," "subtract," "check," "bring down," "repeat," and "put up remainder"). Other authors (e.g. Duffin, 2000; Ng, 1999), however, argue that although students may be successful in finding the correct answer to the problem by following a step-by-step procedure, their understanding of the meaning of long division is not enhanced.

Marilyn Burns (1999) presented a paper-and-pencil method of estimating "friendly" numbers of times (such as 100 or

10) the divisor might fit into the dividend. These partial quotients were recorded along the right side of the long division problem as it was solved and were added at the end to determine the quotient. This is an improvement on the traditional algorithm because the student looks at the whole dividend and estimates the answer, rather than just treating it one digit at a time.

Some authors focused on partitioning sets of materials to make sense of long division. Bidwell (1987) showed how arrays of dots might be successfully employed in helping students understand long division as the inverse of multiplication. Van de Walle and Thompson (1985) partitioned small items such as beans, cubes, or candies as a way of understanding the process. They suggested grouping the counters into cups and designating a cup of five equal to a counter of another color, to initiate place value trading. Sweeney-Starke and Episcopo, (1996) elaborated on this excellent chip-trading technique. Finally, Fast and Hankes (1997) and Zollman, Porzio, and LaBerge (1997), used base ten blocks to model the long division process, allowing students to see the concrete meaning of each step as the materials were distributed and connecting the manipulatives to the paper-and-pencil algorithm. Our Montessori-based technique is similar, but addresses more components of place value and uses skittles (place value color-coded people) to represent the divisor displaying more aspects of the problem concurrently.

In this article, we review national standards that may be applied to this technique of solving long division problems, describe the materials and process we used, and present the results of our successful study comparing the performance of middle school students with learning disabilities reviewing division through these manipulatives versus typical worksheet and computer practice activities.

Advantages of Division with Skittles

- The materials are concrete, colorful, and engaging.
- The materials and process help students understand different aspects of place value.
- Division can be carried out without initial guessing of the partial quotients, therefore ensuring student success.
- The work with manipulatives follows closely both forms of the paper and pencil algorithms: division by repeated subtractions and division by subtraction of multiplied divisors.

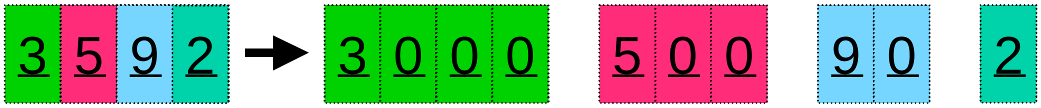
Place Value Numeral Cards

One of the trickiest parts of long division is understanding and keeping track of place value. In division with skittles, the manipulatives and color-coded numeral cards highlight concepts of place value. As in all Montessori place value materials, the ones place of each family (ones, one-thousands, one-millions) is color-coded green. Similarly, the tens place is always blue (tens, ten-thousands ten-millions) and the hundreds place is red (hundreds, hundred-thousands, hundred-millions). Before work begins, students should arrange the color-coded numeral cards into a "big layout", as in Figure 1, so that cards can easily be chosen to form the dividend. The numeral cards are carefully designed to stack on top of each other creating a multi-digit numeral that can then be taken apart or expanded to show its component parts. See Figure 2.

Figure 1. The big layout of numeral cards

9000	900	90	9
8000	800	80	8
7000	700	70	7
6000	600	60	6
5000	500	50	5
4000	400	40	4
3000	300	30	3
2000	200	20	2
1000	100	10	1
	000	00	0

Figure 2. Stacked cards (left) expanded to show place values (right).

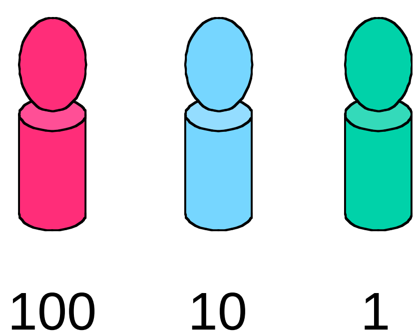


Numeral cards can be made by color-photocopying Figure 1 onto cardstock, cutting the cards apart (being careful not to separate the zeros from the other digits), and mounting them on mat board rectangles (do not leave a border on the right or left edges). Each group of students will need a complete set of numeral cards.

Skittles

A concrete way of thinking of division problems is to imagine an amount (money, supplies, cookies) being given out equally to a number of people. Skittles are small bowling pin-like pieces that can be used to represent these people. However, representing a large number such as 241 would take a lot of skittles if each skittle could stand only for one person. That is where place value color designations can help. Skittles of different colors are given the names of ranks of the Roman army and signify different numbers of people. Green skittles represent single foot soldiers. Blue skittles, called decurions, represent ten people ("deca" means "ten"). In the Roman army, decurions supervised and collected supplies for ten soldiers. Finally, red skittles, called centurions ("cent" means "hundred"), represent one hundred people because centurions in the Roman army were in charge of one hundred soldiers. See Figure 3.

Figure 3. Skittles of different place value denominations: red centurion (hundreds place), blue decurion (tens place), and green foot soldier (ones place).



The skittles we used were wooden “people” purchased unfinished at a craft store. They were painted the place value colors with acrylic craft paint. Wooden clothespins also make good skittles. Any simple piece that resembles a person would work. Plastic pawns or game pieces of the correct colors or white ones mounted on colored mat board bases can also be used.

Group Work

Students can best learn how to use the long division manipulatives if they work cooperatively in groups to share strategies and information. Table 1 shows jobs for cooperative groups of three students. Jobs can be reassigned to accommodate larger groups. The important idea is that each student has a contribution to solving the problem. Jobs should be rotated for each new problem so that every student has experience with every aspect of the solution.

Preliminary Work

Before students work with the materials to solve long division problems, they should become acquainted with the materials. Begin by having students assemble the big layout of numeral cards in their correct place value positions. Review the color-coding scheme. Ask students to choose the numeral cards for a four-digit number, stacking them so that the thousands-place numeral card is on the bottom followed by the hundreds-place card, tens-place card and ones-place card. The right edge of all numeral cards should be aligned so that the four-digit numeral shows in the different colors. Then, have them expand the number by pulling the cards apart and placing them in their place value positions. The students may see now see what each digit represents. They should obtain the quantity in base ten blocks for each digit. Students should use this technique to compare two four-digit numbers, such as 4,863 and 4,638 with an inequality symbol. Students should also be introduced to skittles and their place value denominations.

In the next section, we describe a long division problem with a three-digit divisor to show the power of the materials in portraying problems with large divisors. However, students should begin with problems using one-digit divisors and gradually move to more complex divisors, such as "12". In fact, solving the same problem using a divisor of twelve green foot soldier skittles compared to using one blue decurion with two green foot soldiers will help students see the advantage of using place value. The latter technique is better because fewer trades and less distribution of materials take place.

Division by Repeated Subtractions

Students begin work by obtaining numeral cards and base ten blocks for the dividend. For example, let’s consider the problem 2,952 123 = ?. The numeral cards and base ten blocks for the dividend, two thousand-cubes, nine hundred-flats (or “rafts”), five ten-rods (or “longs”), and two units, are shown in Figure 4. The next step is to form the divisor with skittles. The divisor for this problem consists of three foot soldiers, two decurions, and one centurion. See Figure 5. Spread out the skittles so there is ample room for distributing base ten blocks.

Figure 4. Numeral cards and base ten blocks representing the dividend.

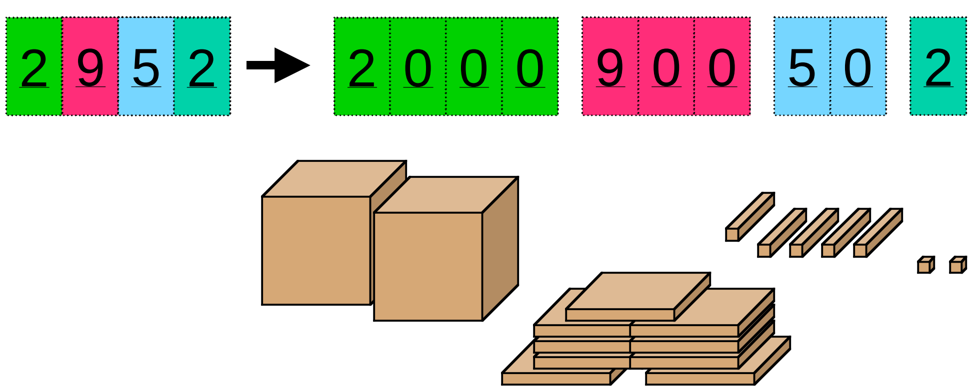
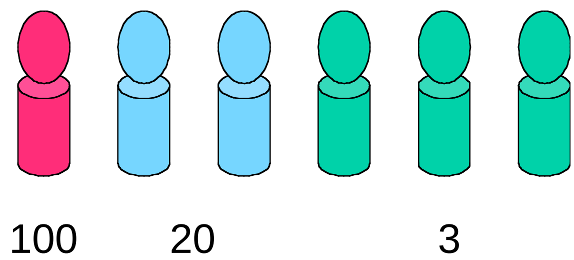
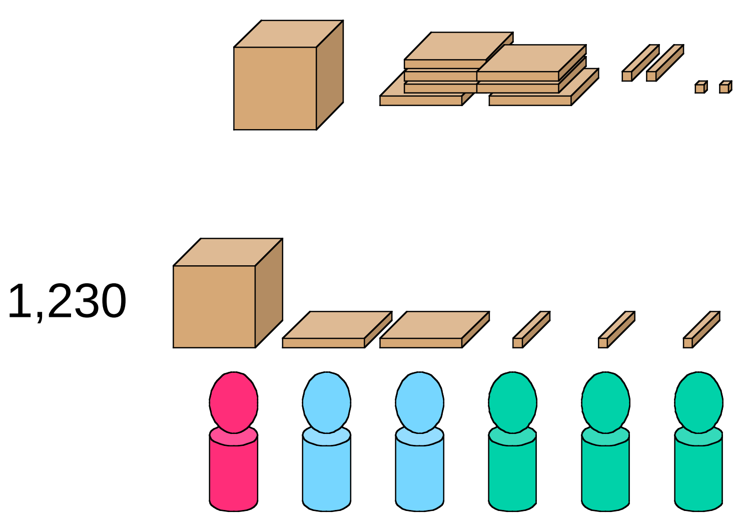


Figure 5. Skittles representing a divisor of 123.



As in paper and pencil division, the student should always begin with the highest magnitude place value of the dividend. For our example problem, this is the thousands place. There are two thousand-cubes in the dividend. Take one cube and give it to the red centurion (the highest place value of the divisor). Ask students, “If I give 1000 to a centurion who collects for 100 people, what is each person really getting?” The response should be that each person gets 10. Explain that in division, everything is *always* fair and even. Each *person* (but not necessarily each skittle), in the end, must get the exact same amount. Now point to one of the blue decurions. “If the centurion, who collects for 100 people, received a thousand-cube, what should I give to this decurion who collects for ten people, so that every person will get the same amount – each will get ten?” The response should be that the decurion gets a hundred-flat because a hundred divided among ten people is ten. So give each of the decurions a hundred-flat. Then move to one of the green foot soldiers. “If we have been giving out base ten blocks so that every person gets ten, what should we give a foot soldier?” The response should be ten. Give each foot soldier a ten-rod. See Figure 6 for distribution so far.

Figure 6. First distribution of base ten blocks.



The above operation can be recorded on paper as division by repeated subtractions as shown in Figure 7. Determine the amount that has been distributed thus far and subtract it from the dividend: subtract 1,230 to obtain 1,722. Then continue distributing base ten blocks. The student will encounter difficulty because there is no ten-rod for the last foot soldier. Ask students how they might obtain more ten-rods. The response should be that a hundred-flat could be exchanged for ten ten-rods. Have the student who first arranged the base ten blocks act as banker and accomplish the trade. See Figure 8. Then continue distributing. The same amount of blocks can be distributed to the skittles as was previously distributed. See Figure 9. These base ten blocks can be recorded as another subtraction. Because all of the thousand-cubes have been distributed, it is time to mark a digit in the quotient. The student should look at what has been given to one foot soldier in order to determine the digit. Each foot soldier received two ten-rods. Therefore, the digit that should be recorded is a "2" in the tens place.
See figure 10.

Figure 7. The beginning of solving the problem by repeated subtraction.

$$\begin{array}{r|r} 123 & 2\,9\,5\,2 \\ & -\,1\,2\,3\,0 \\ \cline{2-2} & 1\,7\,2\,2 \end{array}$$

Figure 8. Exchanging a hundred-flat for ten ten-rods.

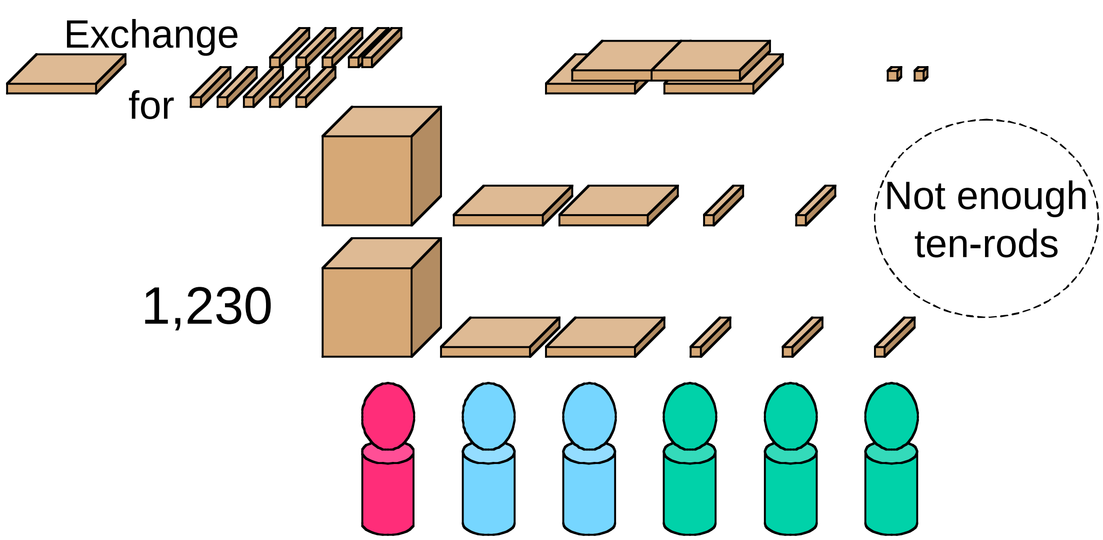


Figure 9. Distribution on additional base ten blocks.

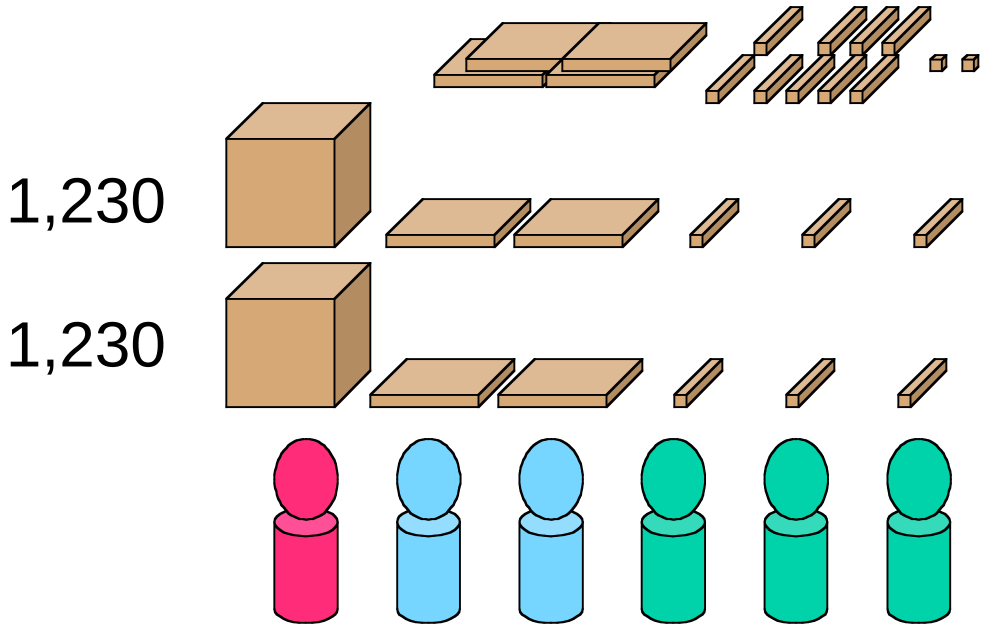


Figure 10. The paper and pencil version of this problem thus far.

$$
\begin{array}{r|l}
123 & \ \ 2\,9\,5\,2 \\
 & \underline{-\,1\,2\,3\,0} \\
 & \ \ 1\,7\,2\,2
\end{array}
$$

Now the student can start distribution of base ten blocks again, this time giving a hundred-flat to the red centurion. Ask the student, "If you give a hundred-flat to a centurion who collects for a hundred people, what will you give to a decurion who collects for ten people?" The response should be "ten," because in both those cases, each individual will receive one. Each of the decurions will receive a ten-rod and each foot soldier will receive a unit. It will become necessary to exchange a ten-rod for ten units because there are not enough units to give each foot soldier one. See Figure 11. Complete the trade and continue giving each foot soldier one. The base ten blocks can be distributed in a similar manner for a total of four times until the base ten blocks of the dividend are exhausted. See Figure 12.

Figure 11. Trading of a ten rod for ten ones.

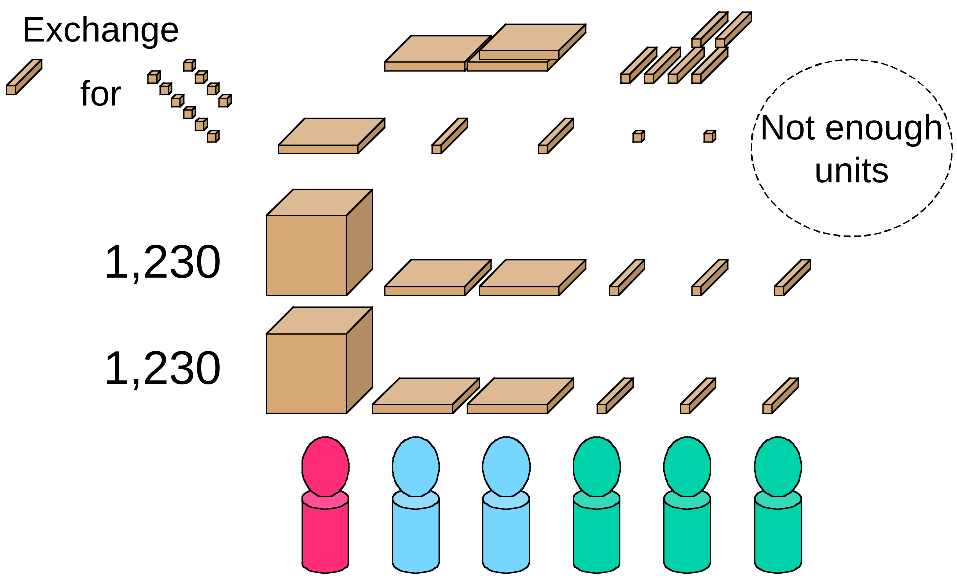
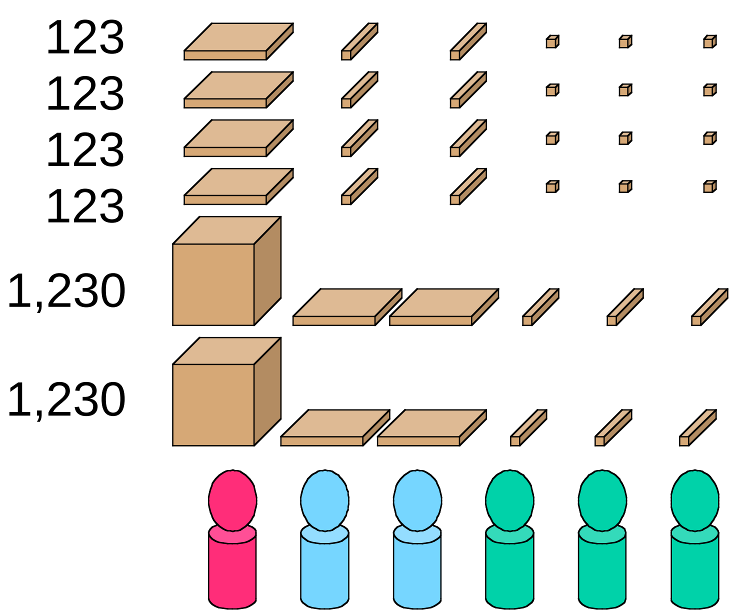


Figure 12. Final distribution of base ten blocks.



As before, the operation can be recorded as a series of four equal subtractions. To record a digit in the quotient, look at what one foot soldier received. Each foot soldier received four units, so place the digit “4” in the ones place. As a final check, tell students, “The answer in division is what one person gets." Ask students to determine which skittle represents one person. The response should be one foot soldier. Ask students, "What did one foot soldier get?" The response should be "24." Check to see that this is the same number as the quotient. See Figure 13.

Figure 13. Division by repeated subtractions.

$$
\begin{array}{r}
24 \\
123\,\overline{)\;2952} \\
\underline{-1230} \\
1722 \\
\underline{-1230} \\
492 \\
\underline{-123} \\
369 \\
\underline{-123} \\
246 \\
\underline{-123} \\
123 \\
\underline{-123} \\
0
\end{array}
$$

Conventional Division

The above process can also be viewed and recorded in a different way to correspond more closely to the division model of subtraction of multiplied divisors. Instead of subtracting after each distribution, one could subtract after each *type* of distribution. In this problem, we had two types of distributions – distributions in which centurions received thousand-cubes, and distributions in which centurions received hundred-flats. Instead of recording a series of small subtractions, we could record two main subtractions. For the first main distribution, count the value of all of the base ten blocks that were given out: 2,460. Subtract this. Record a “2” in the tens place of the quotient because each foot soldier received 2 ten-rods. Note that the divisor, 123, times 20 equals 2,460. Now add the value of all of the base ten blocks in the second distribution: 492. Subtract 492 from the remaining dividend. Record a “4” in the ones place of the quotient because each foot soldier received 4 units. Again, note that 123 x 4 = 492. See Figures 14 and 15.

Figure 14. Conventional division.

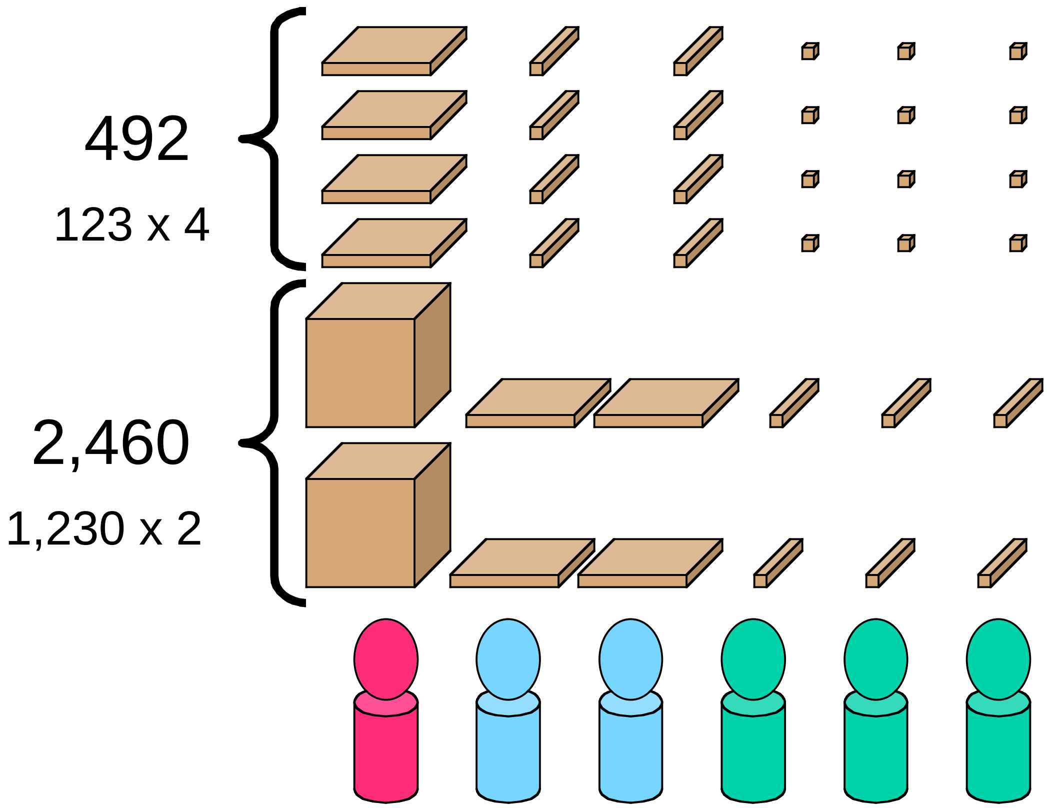


Figure 15. Conventional division.

$$\begin{array}{r} 2\,4 \\ 123\,\overline{\big)\,2\,9\,5\,2} \\ -\,2\,4\,6\,0 \\ \hline 4\,9\,2 \\ -\,4\,9\,2 \\ \hline 0 \end{array}$$

Our Study Involving Students with Learning Disabilities in Mathematics

Twenty-seven students enrolled in five small classes of a special education program for students with mathematics learning disabilities in a rural middle school in central New York State taught by the first author participated in the study. These five small heterogeneous classes contained only these students with learning disabilities in mathematics; classes were small so that students would receive individualized attention. Students qualified for these special education services by low achievement scores on the mathematics section of the Kaufman Test of Educational Achievement (K-TEA) (Kaufman & Kaufman, 1998). The five classes were distributed into two almost-equal groups with similar numbers of seventh and eighth grade students and similar numbers of males and females (Unfortunately, student schedules did not permit students to be individually randomly assigned to the two groups). Then the two groups were randomly assigned the condition (by blindly selecting the condition written on a piece of paper from a container) as control group (N=13; 6 female, 7 male; 6 seventh grade, 7 eighth grade; 13 Euro-American) and experimental group (N = 14; 5 female, 9 male; 6 seventh grade, 8 eighth grade; 13 Euro-American, 1 African-American).

A thirty-minute pretest on several concepts important to division was administered at the start of the study to all students. Table 2 shows the test questions and Table 3 gives the pretest, posttest and gain scores. An identical test was used at the end of the study to determine growth in understanding of division as a result of comparison treatments. Each group received ten 30-minute lessons taught by the same instructor, the first author, Peggy. Peggy had been teaching long division with the control condition methods for several years and was confident that she had been supplying students with appropriate instruction and practice, however, she was curious to see how division with skittles, numeral cards, and base ten blocks might affect student learning. Peggy was able to teach both the control and experimental conditions with equal enthusiasm. Both conditions addressed all concepts covered by the pretest/posttest.

Experimental Group Procedures

The experimental group represented simple division facts as arrays with colored counters. Then they practiced forming and expanding numbers with the numeral cards and choosing corresponding base ten block quantities. They compared quantities using inequality symbols.

Students first used the numeral cards, base ten blocks, and skittles to solve division problems with one-digit divisors, without and with remainders, then worked on problems with two- and three-digit divisors, again without and with remainders. All the initial division problems solved with numeral cards and skittles were presented as story problems; as students became more proficient in the technique, they were asked to create their own story problems and solve them using the materials.

As students worked problems with larger dividends, they switched to a more abstract representation of the quantities involved. Instead of using base ten blocks, they used color-coded counters. The counters were flat glass marbles purchased at a craft store (these are often put in a vase to anchor an arrangement of flowers) in the following colors and denominations: light green (ones), blue (tens), red (hundreds), and dark green (thousands). Students also created and solved their own story problems with these manipulatives and recorded their work as paper and pencil algorithms.

Control Group Procedures

Students participating in the control group reviewed division the way the teacher had been doing it for several years. They started by finding clue words for mathematical operations in problems projected on an overhead transparency and completing a worksheet that focused on the same skill. Students listed the division facts and quotients. They practiced fact problems shown on an overhead. Then they practiced problems on a fun Internet site.

The teacher explained the meaning behind all the steps in long division as students took notes. Students practiced the steps for computing long division by solving problems. The acronym, D, M, S, C, B (representing Division-Dad, Multiplication-Mother, Subtract-Sister, Check-Cousin, Bring down-Brother) was used to help students remember the algorithm. Students practiced solving problems on paper using the algorithm and completed a worksheet of word problems that required long division. They practiced division word problems on an Internet site, AAA Math (Banfill, 2002), reviewed place value, and practiced more problems. Students composed a story problem for a given division problem, then solved it. They used paper to record their work step by step. Finally, students practiced solving long division problems using interesting mathematics software.

Pretest Scores

The control group students scored somewhat higher than the experimental group in every test section of the pretest. This is shown clearly in Table 2. The mean score of all sections on the pretest for students in the control group was 58.9% (or, 58.9 points out of the 100 points of the test), whereas, the mean score on all sections for students in the experimental group was only 45.9%. This difference in pretest scores was unavoidable because of scheduling difficulties; the students' classes could not be rearranged to mix the students further. Both the control and experimental group pretest scores paralleled each other in general performance across the different test sections. Students of both groups performed most poorly on sections which dealt with student understanding of the meaning of division. Students also scored poorly in the word problem section (Part B-2). Students of both groups performed best in determining place value (Part C-3).

Table 3. Mean pretest, posttest, and gain scores on the division assessment. Standard Deviations are shown in parentheses.

Posttest Scores

The average posttest scores for all test sections for both groups were approximately the same (63.4 % for the control group and 63.0% for the experimental group), indicating that although the experimental group had performed poorer on the pretest, these students had caught up with their control group peers by the time of the posttest. The experimental group scored somewhat higher on the posttest in some test sections (Parts A-1, A-2, C-1, C-3). These sections address determining the correct operation for word problems, simple division facts, understanding the meaning of the answer in division, and place value. The control group performed somewhat better than the experimental group in setting up and solving division word problems (Part B-2) and solving a long division problem (Part B-1), however, the control group started with an advantage in each of these areas.

Gain Scores

The experimental group made larger gains than the control group overall (17.1% gain of experimental group versus 4.5% gain of control group). There was a significant difference found between the gain scores of the two groups ($F = 0.233$, $df = 1/126$, $p = 0.63$). The experimental group made greater gains than the control group in every section except the section that addressed computation of a long division problem (Part B-1) where the difference between the scores was very slight. The test sections in which the experimental group made the largest gains were B-2 and C-1. These sections addressed setting up and solving word problems, and explaining the meaning of the answer in division.

Teacher's Observations

Although all the students in both the experimental and the control group stayed on task during the lessons, their attitudes towards doing mathematics differed markedly. The teacher kept brief notes on student comments and behaviors during the lessons.

Peggy had not seen the students so excited to do math as with the experimental group during this study. A boy excitedly queried at the beginning of class, “Are we doing division today?” One student commented, “I can do division using these marbles, I never could before.” Another remarked, “I like math using these materials, it makes it easier." A girl asked if the first author would teach her math all the time because she learned division by using the manipulatives and she wanted to learn more. She expanded on her liking for the manipulatives, “I liked putting them together to get the answer, it helped me to figure out the answer by separating them.” A boy added, “It helped me, lining the skittles up and dividing the marbles. It helped me to understand how to divide stuff.” Another student's response was, “It helped me figure out division problems because I had things to separate, I could see the amount to separate. I liked separating them.” Another explained that he liked using the flat marbles because he learned how to divide without using a calculator. Obviously, the long division manipulatives were the motivation propelling these students to explore and understand math.

The teacher observed that students in the control group stayed on task during the lessons but lacked the excitement of the experimental group. One student commented, “Do we have to do division today? I’m sick of doing division.” Yes, they were motivated by the use of the computers for generating practice problems at the beginning of the study but by the third computer session, a student complained, “Do we have to get on the computers today?” while another whined, “Do we have to do division again?”

Before the posttest, the teacher, Peggy, asked students how they had found the long division lessons. A few control group students maintained their liking for computer practice: “I liked it when we did division using the computer, it was better than just using paper and pencil,” and, “ I liked using the computer because I didn’t have to write as much.” Another student summed the control group's sentiments (students nodded agreement) when he stated that he liked learning division a little but he eventually became bored with it.

Conclusion

This investigation has shown that the use of numeral cards, skittles and base ten blocks or colored counters can help middle school students memorize the algorithm of division, along with giving them a concrete understanding of the meaning of the steps in long division. As Kenyon (2000) indicates in her article on students with learning disabilities in mathematics, such students experience a large variety of different mathematical difficulties. Using real-life story problem examples, manipulative materials, making connections between manipulatives and paper-and-pencil work, and paying attention to place value, can help students with mathematical learning disabilities succeed in mathematics. In our study, which compared two groups of students with learning disabilities in mathematics, students using the division materials (skittles, numeral cards, base ten blocks and colored counters) and activities we described here displayed higher motivation. They performed as well as those using more traditional methods and computer practice. Additionally, because our experimental group scored lower on the pretest, they exhibited higher gain scores during the experiment. Your students will enjoy and learn the meaning of long division with this technique, too.

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