1. Find the exact measurement of the angle (no more than 180^o) formed by a clock's hands at each of the following times.
   (a) 4:30
   (b) 7:20
   (c) 1:45
   (d) 6:45
   (e) 10:25
   (f) 5:35
2. Find the exact measurement of the angle (no more than 180^o) formed by a clock's hands at each of the following times.
   (a) 9:06
   (b) 7:18
   (c) 3:24
   (d) 1:59
3. Name a time of day when the hands of a clock will form the following angles:
   (a) 90^o
   (b) 120^o
   (c) 105^o
   (d) 15^o

Math 118 - Dr. Miller - Solutions to HW #13: Clock Angles

1. The position of the hour hand between the clock's numbers is crucial in this type of problem, as well as whether we want to include how far it has already traveled between the numbers or how far it still has to go to reach the next number.

   (a) At 4:30, the hour hand is exactly halfway between 4 and 5. The total angle sweeps from this halfway point to the 5 and on to the 6, for a total of $15^o + 30^o = 45^o$.

   (b) At 7:20, the hour hand is exactly 1/3 of the way from the 7 to the 8. The total angle counts three 30^o angles from the 4 to the 7 and then another 1/3, or 10^o beyond. The measurement is 100^o.

   (c) At 1:45, the hour hand is exactly 3/4 of the way from the 1 to the 2. The total angle counts four 30^o angles from the 9 to the 1 and then another 3/4, or 22.5^o beyond. That's 142.5^o.

   (d) At 6:45, the hour hand is 3/4 of the way from the 6 to the 7. However, the angle we want extends through the two 30^o sections from 9 back to 7, and then needs only an extra *one* fourth of the section from 7 to 6. That's 67.5^o.

   (e) At 10:25, the hour hand is 25/60 of the way from the 10 to the 11. We want that piece plus the five full 30^o angles between the 5 and and the 10. That's $150 + 12.5 = 162.5^o$.

   (f) At 5:35, the hour hand is 35/60 of the way from the 5 to the 6. We don't want that angle, though, but the amount that it still has to go to reach the 6, plus the full 30^o angle between the 6 and the 7. That's a total of $30 + 17.5 = 47.5^o$.

2. Now we have to pay attention to both the position of the hour hand between numbers on the clock face AND the position of the minute hand. Notice that we have to use different types of fractions to deal with each of these.

   (a) At 9:06, the hour hand is 1/10 of the way from 9 to 10 and the minute hand is 1/5 of the way from the 1 to the 2. Our angle counts three whole 30^o sections between the 10 and the 1, the fifth of one (6^o) created by the minute hand, and 9/10 of the way from the 10 back to the 9 (that's 27^o more). The total is 123^o.

   (b) At 7:18, we want the three full 30^o angles between the 4 and the 7, plus the section traveled by the hour hand from the 7 onward toward the 8 (that's 18/60 of 30^o) plus the section still to be traveled by the minute hand, moving toward the 4 (that's 2/5 of 30^o). The total is $90 + 9 + 12 = 111^o$.

   (c) At 3:24, we want just the part of the section still needed by the hour hand as it moves toward the 4 (so 36/60 of 30^o) plus the angle covered by the minute hand in moving 4/5 of the distance between the 4 and the 5 (so 4/5 of 30^o). That's a total of $18 + 24 = 42^o$.

   (d) At 1:59, we want the full 30^o angle between the 12 and the 1, plus the small section still to be traveled by the minute hand as it moves toward the 12 (it has 1/5 of 30^o to go) plus the almost complete sectino traveled by the hour hand from 1 to 2 (it has gone 59/60 of 30^o). The total is $30 + 6 + 29.5 = 65.5^o$.

3. (a) 3:00 and 9:00 are easy examples.

   (b) 4:00 and 8:00 are easy.

   (c) Think of 105^o as $90^o + 15^o$. Now make a time when an extra 15^o angle will be tacked onto a right angle, usually at half past an hour. 2:30 and 9:30 are good choices.

   (d) We need the hour and minute hands to be only "half a step" apart. 5:30 and 6:30 are good choices.