

ESSENTIAL K-3 TEACHERS GUIDE

TEACHING NUMBER SENSE



WE are TEACHERS

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Dear Educators,

We know that helping kids gain number sense in the early grades is a priority, but we've also discovered it can be a tough concept to teach. That's why we put together this guide designed to give you all the tools you need. We've included articles about how and why number sense matters, and we'll tell you how to know whether or not your kids have "got it." We've asked expert teachers and math professionals to weigh in, and we've gathered up a list of picture books you can use to reinforce your teaching. Plus, we've included activities and printables that you can use in your classroom to help kids practice these essential skills. This guide to Teaching Number Sense, sponsored by Didax and designed especially for primary teachers, will show you how you can make number sense an integral part of your math curriculum. [Click here](#) to download additional copies.



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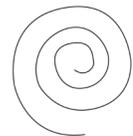
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WHAT EVERY TEACHER NEEDS TO KNOW ABOUT HOW KIDS LEARN NUMBER CONCEPTS



We've all seen the eager new student who puffs up with pride and shouts out, "I know! Two plus two is four!" But we also know that more often than not, those students have memorized a fact. They probably don't know how addition works, and they may not even fully understand the concept of "2" or "4." They have simply memorized a few words. It is essential for young learners to have a solid understanding of math concepts—to not just perform operations by memory. When children grasp these concepts during their early years, they will have a stronger foundation for tackling more complex mathematical chal-

lenges (as is required by the Common Core standards) as they get older.

The Developmental Stages of Mathematical Thinking

To build mathematical fluency in young children, we need to recognize the different stages students go through as they develop number sense, spot their misperceptions about counting, and provide plenty of practice so that numbers become meaningful to them. When we identify where students are in the process of developing math concepts by asking them the right questions and using formative assessments, we are better able



"It is essential for young learners to have a solid understanding of math concepts."

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to provide learning experiences that are more relevant to our students' specific needs.

Maggie McGinty, preK–8 mathematics coordinator for Northborough/Southborough Public Schools in Massachusetts, says that the key is to look at what students already know. We can learn a lot by figuring out what strategies students are using to

Concepts (Math Perspectives) and activities from *Developing Number Concepts* (Math Perspectives) to build students' early math skills and knowledge.

"It's so targeted," Morris says. "Teachers say, 'Here's what I can do to move my students' understanding.'" Giving our students a strong foundation in number concepts—and

includes all of the items in that group. When a young child is asked to put away five cubes, he or she might just pick up the fifth one counted.

To help children practice counting and understand quantities, practice with a wide variety of manipulatives is needed. Clothespins, buttons and beans, for example, can help teachers easily see whether students have counted out the correct number.

2. One-to-one correspondence: Are they just saying numbers or actually counting?

Most children love to count and will mimic the practice of pointing to the items they are counting. But they might not fully understand that each number in the sequence must match each object they are pointing to or touching. Even when they develop this skill up to 10, they might lose track with higher numbers.

Using dot cards, on which students cover each dot as they count, is one activity for reinforcing this skill. Having children count each item as they place it in a container is another way for them to practice.

Kindergarten teacher Tiffani Mugarussa, who teaches at John Reed Elementary, north of San Francisco, practices counting and number sense with her students using a variety of picture books. The rhymes in books can help students remember the counting sequence, but they also provide real examples that kids will remember.

3. Conservation of number: Is it the same number even if I'm counting different things?

This concept involves understanding that just because the size or placement of items might change doesn't mean the number of items has changed. For example, a child might think that eight trucks

is a greater number than eight marbles just because the trucks are much larger. Or they might think that the number of items has changed if one is covered or hidden. This demonstrates how young children believe strongly in what they perceive to be true. This is another area where multiple experiences with counting different sizes of objects and rearranging them can help students develop this concept.

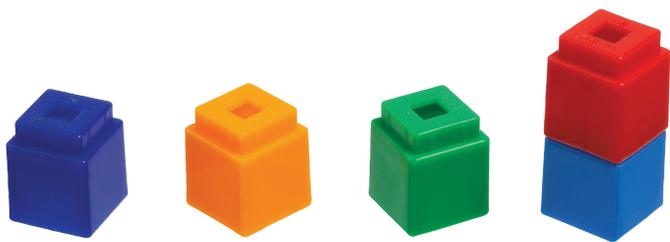
There are a variety of activities that can help students better understand conservation. One idea is to take a group of objects and spread them out, then take the same number of objects and clump them together. When the students count them, they will see there is the same number of items in each group.

4. Number sense and relationships: Which is more, 4 or 5? How much more is one more?

Knowing how numbers relate to one another means children can understand whether a number is more or less than another one, or how many more objects they will need to reach a certain amount. Number sense means beginning to form mental images of quantities and not skipping over numbers when counting.

Specific Activities Target Specific Skills

An activity from Kathy Richardson's *Developing Number Concepts* (Math Perspectives) called Towers, Towers gives children practice in comparing amounts and determining how many more or less there are. Use towers of different heights made of connecting cubes and then ask children how many cubes are in each tower. When children are still learning number relationships, they might still point to each cube while they count or only recognize



“If students reveal an understanding of number, relationships and patterns, they often have a strategy to figure out simple problems and computation.”

solve math questions.

“If students reveal an understanding of number, relationships and patterns, they often have a strategy to figure out simple problems and computation,” McGinty says. “If that is mastered, it can allow students to enter into more complex math problems.”

Primary students experience more success with new math concepts when we can give them specific strategies and techniques, says Kathleen M. Morris, an administrative coordinator for elementary/Title I mathematics for Prince William County (Va.) Public Schools. For several years, the district's Title I schools with math resource teachers have been using *Assessing Math*

assessing their understanding of those concepts along the way—is what can help them experience success.

How Primary Students Learn Number Concepts

There are four key ideas around numbers that children need to understand. As they progress in their learning, comprehension in each of these areas deepens.

1. Inclusion: What does a group of five really look like?

When children are first learning to count—say crackers or building blocks, for example—they might not understand that the last item they point to

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that one is shorter than the other. But eventually they will be able to recognize how many additional cubes are in the taller tower or how many to “take away” to make the towers the same height.

Give students a reason for counting and working with numbers to help them develop consistency and confidence in their math skills. Instructions such as “Make sure you and your partner each have five cards” and “Can you tell if there are too many children in

tice writing the numerals at the same time they are counting that number of objects.

When we use these activities in class, we address gaps in our students’ understanding identified by pre-tests. McGinty says that if our students are able to explain their own thinking, they should be better equipped to “see where they might have gone wrong and correct on their own.” And they might even be able to share their ideas with another student.

It’s easier to get a clear pic-



“Give students a reason for counting and working with numbers to help them develop consistency and confidence in their math skills.”

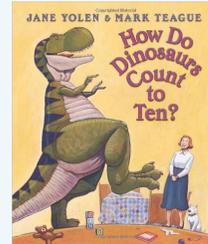
the writing center?” gives students an opportunity to practice counting and apply what they’re learning.

One task that doesn’t encourage the development of these concepts, however, is practice writing numbers. Just because a child can print a 5 or a 9 correctly doesn’t mean he or she can recognize that amount of objects or knows which one is more or less. Teaching what the symbols stand for should always be taught in connection with the quantities they represent. Number cards, for example, with dots on one side and the corresponding numeral on the other, are great materials for reinforcing what the symbols mean. Students can also prac-

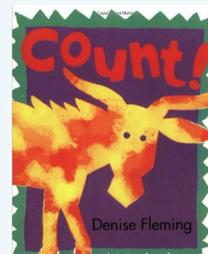
ture of what our students know when we use assessments in combination with activities and conversations. But those assessments can also reveal some other surprises, Morris says.

“Sometimes teachers think ‘I thought he knew that,’ only to find out that the student has not mastered the skill” and needs more practice, she says. But then other times the teachers react with “Wow, I didn’t know this student could do that!” Specific teaching strategies coupled with frequent, strategic assessment can help us give more of our students a complete understanding of number that they can carry with them as they move on to more complex mathematical concepts in the older grades. ■

6 Picture Books That Teach Number Sense



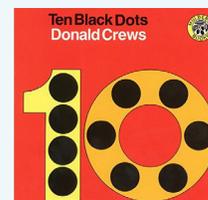
How Do Dinosaurs Count to 10?
by Jane Yolen
and Mark Teague



Count!
by Denise Fleming



The Icky Bug Counting Book
by Jerry Pallotta
and Ralph Masiello



Ten Black Dots
by Donald Crews



Ten Little Ladybugs
by Melanie Gerth
illus. by Laura Huliska-Beith



Each Orange Had 8 Slices
by Paul Giganti Jr.
illus. by Donald Crews

WHAT DO THEY KNOW?

ASSESSING KIDS' NUMBER SENSE



Getting the right answer is not enough. To truly learn math, find meaning and advance, students need to understand some critical ideas. Each learning phase must be in place before they move on to the next.

Just how do you know if students have met these milestones? Checking – and often.

Evaluate Learning Frequently for a More Accurate Picture of What Students Know

Formative assessments are the key to tracking whether a student has a foundational understanding and is not just imitating procedures or going through the motions, according to Kathy Richardson, author of *How Children Learn Number Concepts* (Math Perspectives).

“When children are taught mathematical concepts or procedures before they reach certain levels of thinking, they do not see the underlying logic of the mathematics they are working with,” writes

Richardson. “All they can do is memorize processes and procedures.”

If teachers are only looking for correct answers, they miss the information needed to determine what children know and still need to learn. Once a teacher has an insight into students’ thinking processes and depth of knowledge, she can be assured students have the concept down and move on. If there is a gap in understanding, teachers can give tailored instruction at an appropriate level until the concept is mastered.

Number concepts are a foundation that children must have in order to achieve high standards in mathematics as a whole, writes Richardson. Then children know that they use numbers to describe quantities and relationships, and that numbers are useful tools for getting information about the world.

Children develop a true sense of number by working with real things rather than

“Her research is all about **working one-on-one and in small groups** with kids to see what it is they understand, how they understand it, and **how we can help them develop their understanding in math.**”

—Kathleen Morris

symbols. As they develop their understanding, they begin with models, moving them to aid their thinking, according to Richardson. They advance from thinking about numbers when they can move objects, to thinking about the relationships when the model is present but not touched, to thinking about relationships without a physical model.

Frequent assessments are vital because each stage of learning is much more complex than is generally recognized.

For instance, counting includes major ideas such as one-to-one correspondence, keeping track, remembering how many, and knowing one more and one less.

Formative Assessment Is Always Happening in the Classroom

Kathleen Morris, administrative coordinator for elementary/ Title I mathematics in the Office of Student Learning for Prince William County Public Schools in Manassas, Virginia, says formative assessment is

WHAT DO THEY KNOW? ASSESSING KIDS' NUMBER SENSE

happening all the time in the classroom. It occurs as students answer questions aloud through a number talk or as teachers observe students completing tasks that require higher-level thinking. “Whenever a teacher is tuned in to anything a student is saying or doing—whether they are marking on a checklist or making a mental note that a student is having a little trouble with ‘x’ and I need to make sure to pull them to work on this—it’s just going on every moment,” said Morris.

Prince William teachers use the assessments developed by Richardson, and the approach resonated with Morris. “It got to the heart of what it is that we need to know about what young children understand about number, particularly,” said Morris. “Her research is all about working one-on-one and in small groups with kids to see what they understand, how they understand it, and

more coherent understanding of mathematics, particularly number sense, says Morris.

The frequent assessments are important because this frequency takes students’ math understanding beyond memorization to higher-level math, said Morris.

“The purpose of assessment is to provide teachers with information about the students’ understanding or lack of understanding and about where there are gaps. We often have illusions of learning and think students understand, but there is something missing. I think of them as potholes that if you don’t fill them, they just keep getting bigger and bigger.”

Individualization Is Easier With the Right Information

Teachers in Sudbury Public Schools in Sudbury, Massachusetts, also use

The post-assessment data from kindergarten travels with students to first grade so teachers know where to start and help with the transition.

“What’s great about the assessment is the interview. The process helps teachers to understand the students’ thinking,” said McGinty. “It’s helpful to see if they use their fingers or tap their foot—subtle movements—so we can bring the tools that they need to be fluent.”

The formative assessment tells teachers which students have mastered a topic or strategy so they can individualize those students’ education.

Richardson notes that a deep understanding of number concepts and relationships does not develop quickly. Children need many ongoing opportunities to master number sense, and educators should provide a variety of complex experiences geared to the level they need.

Tips for Improving Students’ Math Progress



Use assessments with fidelity. There is a script provided for a specific purpose to find out if a student knows something regardless of how the question is asked or materials are presented.

Be open to listening. Pay attention to what your students are saying and watch what they are doing. Be committed to wanting to know where the gaps are.

Partner with parents. Teachers work with student on rich tasks that get them to think rather than just do. Explain to parents that kids need to develop a deeper understanding of math, and share suggestions for activities to think, reason and problem solve at home. Encourage families to play dice games, bring math into the everyday conversation. Work with families to dispel the idea there is a math gene. Everybody can do math.

Source:
Kathleen Morris



“The process helps teachers to understand the students’ thinking.”

how we can help them develop their understanding in math.”

A huge aspect of their program is determining where students’ learning gaps are, where their understanding falls apart, and what to do about that so they have a

Richardson’s formative assessment tool. Maggie McGinty, who served as Sudbury’s Mathematics Director, said teachers use the data to make groups within each class and work on tasks tailored to their level to reach math fluency.

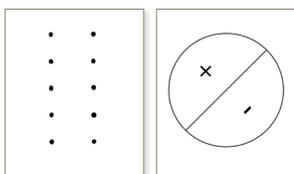
Adds Richardson: “Once teachers have identified what children really know and what they need to learn, they will be able to give children a solid foundation on which to build, ensuring success for all students.” ■

NUMBER SENSE ACTIVITIES YOU CAN USE NOW



Number sense matters. We've seen the evidence. How do we make sure we're teaching it to our kids? We have six easy-to-implement activities from the *Developing Number Concept* series (Math Perspectives). All of the materials you need to teach these lessons and play these games (except manipulatives) are included at the end of this report.

1. Roll and Count



MATERIALS NEEDED:

- Plus or Minus Spinner, P. 13
- Working-space paper, P. 12
- 1 die or number cube
- Cubes or counters
- Xylophone or chime (for Listen and Count only)

TARGET GRADES:

Kindergarten and first

SOURCE:

Page 26 *Developing Number Concepts: Addition and Subtraction*

DIRECTIONS:

Working in a small group with the teacher, children take turns spinning the spinner and rolling the die. They use the spinner to decide if they should add or subtract. The die tells how many to add or subtract. Children place counters on their working-space paper to correspond with the preprinted dots. If the die instructs children to subtract more counters than there are on the play space, they say "Not enough" and forfeit their turn. Play continues indefinitely. Children may use more than one working-space paper if they exceed the original 10 dots. Teachers can begin the activity with the words "get more" and "take away," moving to "add" and "subtract" as the children become more comfortable with the activity.

VARIATIONS:

Listen and Count. This game works the same way, except instead of rolling a die, children listen to the teacher ring a bell a

certain number of times. They count the chimes and add or subtract accordingly.

LEARNING OBJECTIVES:

With both games, children are becoming comfortable with plus and minus signs. They are also becoming more flexible with number sense as they add and remove counters from their working space.

SOURCE:

Page 78 *Developing Number Concepts: Place Value, Multiplication and Division*

DIRECTIONS:

Ask students to place cubes along the line on the task card. Once students have covered the entire line, ask them to group their cubes into tens and ones. Ask students to record their numbers on the Tens and Ones recording sheet.

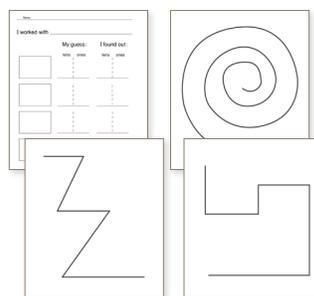
VARIATIONS:

Create your own lines for children to use or have the children create their own lines. You can also ask students to create closed shapes, fill the shape with cubes and then group those cubes into tens and ones on the recording sheet. Older students can create estimates before they cover the lines and then compare answers from two different line task cards to decide which answer is greater.

LEARNING OBJECTIVES:

Students are working toward becoming more comfortable with a base 10 number system. Practice arranging numbers by tens and ones prepares them

2. Lots of Lines



MATERIALS NEEDED:

- Unifix cubes
- Tens and Ones recording sheet, P. 14
- Lots of Lines task cards, P. 15-17

TARGET GRADES:

Second and third

NUMBER SENSE ACTIVITIES YOU CAN USE NOW

for the regrouping that will be required as they begin adding and subtracting two-digit numbers. Assess student comfort with numbers to 10 before asking them to complete this activity.

3. Zib



MATERIALS NEEDED:

- Place-value mat, P. 18
- Unifix Cubes or other connecting cubes
- Small cups

TARGET GRADES:

Second and third (Some first graders may be ready for this activity but will need help and may not understand all concepts.)

SOURCE:

Page 15 *Developing Number Concepts: Place Value, Multiplication and Division*

DIRECTIONS:

Choose a number to be “zib.” It’s best to start with numbers under five. For example, decide with your students that “four” will now be called “zib.” Designate the left side of the place-value mat as the zib side. Tell your students, “We will use our place-value boards to help us organize our counting. The darker side of the board will be where we put the zibs and the white side will be where we put loose cubes when we don’t have enough for a zib. When I say, ‘Plus one,’ I want you to put one cube on the white side of your board.”

Instruct the students to add cubes to the board one at a time by saying “Plus one.” After you have added cubes, ask the children to tell you how many cubes are on the board. For example, after you’ve said

“Plus one” three times, when you ask the children how many they have, they will say “0 zibs and 3.”

Once students have added four cubes to their mats, teach them how to snap the four cubes together to create a single zib. Tell the children to move the newly created zib to the darker, zib side of the board.

Continue asking students to increase the number of cubes by saying, “Plus one.” Ask the children to report the number of cubes in “zibs” and “extras” after each addition. For example: “We have 2 zibs and 2” (which is actually 10 cubes). Continue to create zibs for as long as you have enough cubes to do so.

VARIATIONS:

- Continue the game, but when there are four zibs on the zib side, move those four zibs into a cup, place them in a third section and call them a “big zib.” Only take this step with children who are comfortable building zibs.
- Play the same game, but create new names for different numbers, for example, three could be “zap.”
- Play the same game, but say “Minus one” instead of “Plus one,” deconstructing zibs as you reduce the number of cubes.

LEARNING OBJECTIVES:

These activities introduce children to the concept of regrouping but use smaller, more manageable numbers. It is important that children have a solid understanding of the value of numbers 1 through 10 before using this activity with them. Assess children and be certain that they are confident with manipulating numbers 1 through 10 before beginning this activity.

4. The Tub Game

MATERIALS NEEDED:

- Counters, sorted by color

- Small plastic tubs—old margarine tubs, opaque Tupperware

TARGET GRADES:

Kindergarten and first

SOURCE:

Page 58 *Developing Number Concepts: Addition and Subtraction*

DIRECTIONS:

Work with four or five children in a small group. Each student has his or her own tub and a set number of counters. Five counters is a good number to start with. Ask students to hide some of their counters under the tub and place the rest of their counters on top. Students take turns describing how many of their own cubes are on top and then lifting the tub to see how many are hidden. Repeat with all number combinations.

VARIATIONS:

Once students are more comfortable with all of the combinations for a given number, ask children to predict how many counters are hiding before they lift the tub to count them. Also invite children to work with higher numbers.

LEARNING OBJECTIVES:

Consider grouping together students who are struggling with the same number for instruction and review.

5. How Many Cups

How many? (cup, row, group, stack)	How many in each?	How many altogether?

MATERIALS NEEDED:

- Counters
- 16-oz. plastic or paper cups (at least 10)
- 1 die or number cube
- How Many Groups worksheets, P. 19

TARGET GRADES:

Second and third

SOURCE:

Page 163 *Developing Number Concepts: Place Value, Multiplication and Division*

DIRECTIONS:

The child rolls a die to determine the number of cups to use. After taking that number of cups, the child fills each cup with any number of counters as long as each cup holds the same number. In other words, the number of groups is determined by the roll and the number of counters in each group is determined by the child. The child determines how many counters there are all together and then records the information on the How Many Groups worksheet.

For example, the child might think, “I rolled a two. I have to get two cups. I want to put five cubes in each cup. Now I have 10 cubes all together.” On her next turn, when she rolls a four, she will take four cups and perhaps decide to put one cube in each cup. The important thing is that she practice distributing cubes equally and recording her results.

VARIATIONS:

To make the task more abstract, ask students to create groups without cups. Or ask them to put the groups in rows to create arrays. Students can also build towers of equal height. The roll of the dice determines the number of towers. Students determine how high (within reason) to build those towers.

LEARNING OBJECTIVES:

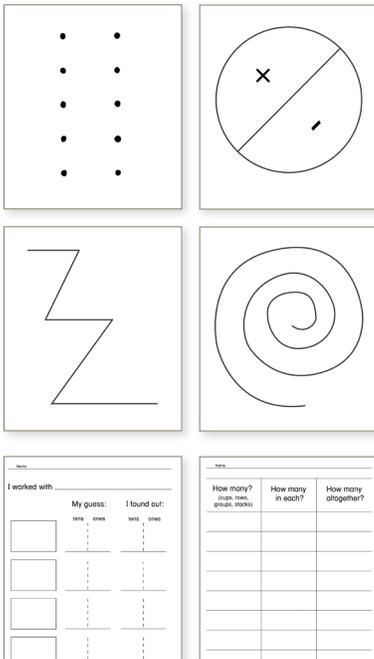
This activity introduces children to the concept of division and asks them to expand their understanding of numbers to include breaking numbers apart equally. Teachers can assess children as they are learning to shape instruction. Note that young second graders may not be developmentally ready to perform these tasks. ■

RESOURCES FOR TEACHING NUMBER SENSE

Take advantage of these great teaching resources. Enhance your math program with books, tools and curriculum from Didax.

Free Printables:

(Use with activities on Page 8)



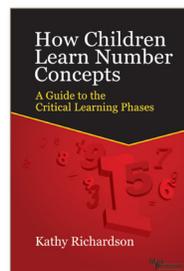
PreK Math: Curriculum and Assessment

Kathy Richardson offers resources to support preK math educators including [assessment](#) and [instruction](#).

Professional Development: Math Perspectives

[Math Perspectives](#) offers a wide range of professional development courses to provide teachers with a knowledge base that allows them to continuously improve the teaching and learning of mathematics in their classrooms.

Book: *How Children Learn Number Concepts* by Kathy Richardson



A “must-have” book for every educator wanting to know how young children develop an understanding of number concepts. www.didax.com/211348

Assessment Tool: Online Formative K-2 Math Assessment



AMC Anywhere provides easy-to-administer assessments that will inform teacher

instruction. More information is available at www.didax.com/kathyrichardson/AMCanywhere/

Video: How to Assess Number Sense



Curriculum: K-2 Curriculum Resources for Number Concepts

Developing Number Concepts by Kathy Richardson is a complete curriculum for teachers to ensure children develop the foundational understanding they need. <http://www.didax.com/kathyrichardson/DNC/>

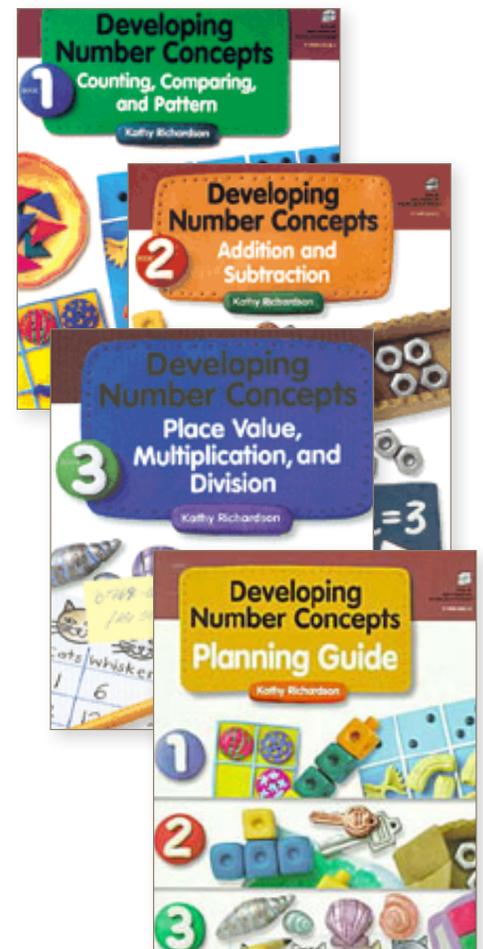


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