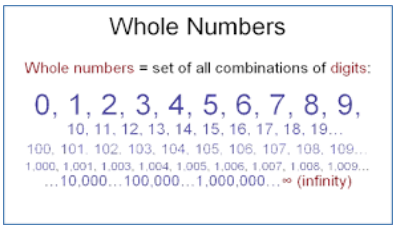
GRADE 7 MATHS TERM 1: WHOLE NUMBERS

WHOLE NUMBERS



All the positive numbers 1; 2; 3; 4; … are called the set of natural numbers. If we include the 0 in the set of natural numbers, we get the set of counting numbers or whole numbers. We use numbers to add, subtract, multiply and divide. We can also write numbers in a particular order, e.g. from smallest to largest or from largest to smallest.

When we need to estimate, we can round off numbers to the nearest 5, 10, 100 or 1000.

Rounding off is often used to make calculation easier.

When rounding off to the nearest 10, look at the unit digit.

When rounding off to the nearest 100, look at the tens digit.

When rounding off to the nearest 1000, look at the hundreds digit.

Whole numbers – or counting numbers are the numbers, 0; 1; 2; 3; 4; … and are represented by the symbol N_0.

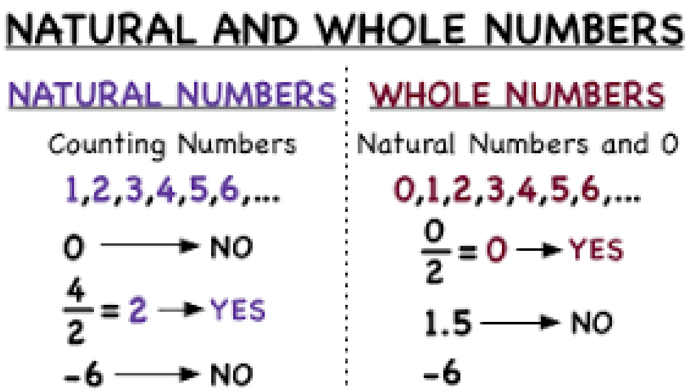
Natural numbers – are whole numbers greater than or equal to 1: (1; 2; 3; 4; …) and are represented by the symbol N.

If the digit you are looking at is a 5 or larger, increase the required digit by one and make the last digit(s) zero. This is called rounding up.

If it is a 4 or less, leave the required digit as it is and make the last digit(s) zero. This is called rounding down. For example: 465 784 rounded off to the nearest 10 is 465 780.

465 784 rounded to the nearest 100 is 465 800.

465 784 rounded to the nearest 1000, is 466 000.



CALCULATIONS WITH WHOLE NUMBERS

Addition means: Find the sum of…

You can use columns, you can write the numbers directly underneath each other in columns and then add them together.

Another way of adding is grouping the hundreds parts, the tens parts and the units separately.

0 is called the identity element for addition, as a number remains unchanged when 0 is added, for example: 24 + 0 = 24.

Subtraction means: To take away…

Write the number directly underneath each other in columns and then subtract them.

Multiplication: 1 is called the identity element for multiplication as any number remains unchanged when multiplied by 1. When you multiply large numbers together, you use long multiplication. By using this method, you break a difficult product into the sum of simple products.

Division: When you divide large numbers, you use a method called long division.

Multiples and factors

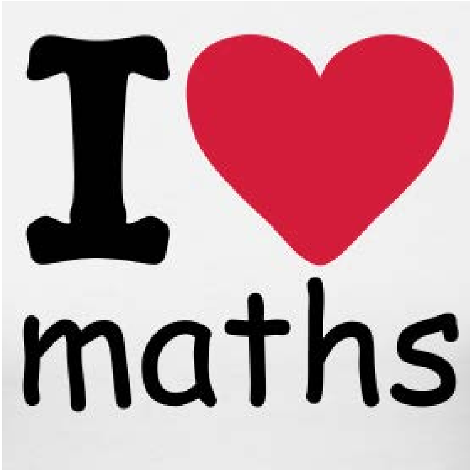
A **factor** is a number that divides exactly into a whole number and leaves no remainder.

Prime numbers are numbers that have only two factors, themselves and 1. The number 2 is the first prime number. We say that 2 ×1 = 2. The number 2 is the only even prime number as all other numbers have more than two factors. The numbers 2,3,5,7 and 11 are examples of prime numbers because they have only two factors, the number itself and 1.

Numbers with more than two factors are called **composite numbers**. The numbers 4,6,8, and 10 are examples of composite numbers because they have more than two different factors.

The number 1 is neither a prime number nor a composite number.

TERMINOLOGY



1. Ordering numbers

Smallest to biggest – Ascending

Biggest to smallest – Descending

2. Comparing Numbers

Use symbols: < means “less than”

: > means “greater than”

3. Round off numbers to the nearest 5, 10, 100 or 1000

Rounding off to the nearest 5:

1; 2 – “Move back to number ending in 0”

3; 4 – “Move forward to the number ending in 5”

6; 7 - “Move back to number ending in 5”

8; 9 - “Move forward to the number ending in 0”

Round off a number to the nearest 10:

Underline the Tens digit - 5**8**6

Look at the digit to the RIGHT of the Tens digit - 58**6**

If this digit is 0, 1, 2, 3, or 4, the Tens stay the same

If this digit is 5, 6, 7, 8 or 9, round up

586 rounded to the nearest 10 is 590.

We use the same method to round off to 100 and 1000.

4. Representing numbers.

Place value table:

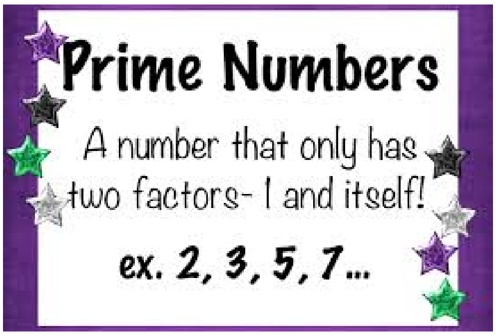
HM	TM	M	HTH	TTH	TH	H	T	U

5. Factors

Factors are those numbers that can fit equally into another number without any remainders. F_{10} = {1; 2; 5; 10}

6. Multiples

Multiplication Tables. M_7 = {7; 14; 21; 28 ….}



7. Prime numbers

A prime number has only 2 factors: 1 and itself.

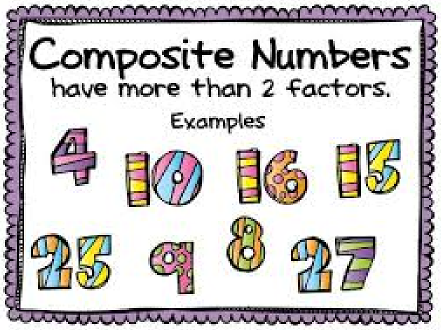
e.g. F_3 = {1; 3}

3 is therefore a prime number.

8. Composite numbers

A composite number has more than 2 factors.

e.g. F_{20} = {1; 2; 4; 5; 10; 20}

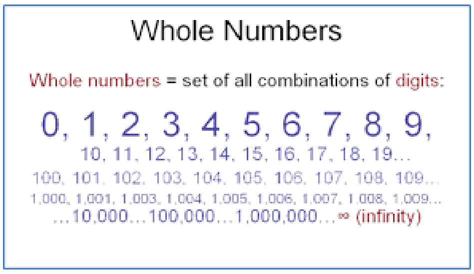


9. Inverse Operations

Multiplication and division are inverse operations. They can be used to check answers.

Addition and subtraction are inverse operations. They can be used to check answers.

PROPERTIES OF WHOLE NUMBERS



Adding numbers is called finding the sum, and subtracting numbers is called finding the difference.

Multiplying numbers is called finding the product. When you add or multiply numbers, the order of the numbers does not matter, for example: 4+5 =5+4 and 4X5 = 5X4. This is called the commutative property of addition and multiplication.

The order in which you add or multiply numbers also does not matter, for example: (4+5) +6 = 4+(5+6) and (4x5) x6 = 4x(5x6). This is called the associative property of addition and multiplication.

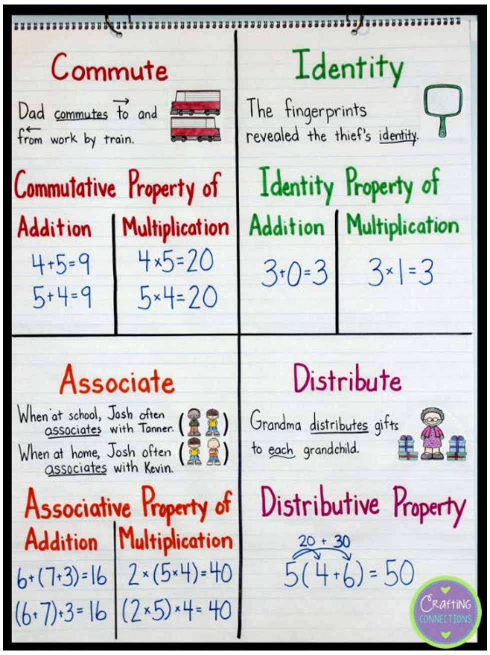
When numbers in brackets are multiplied by a number in front of the brackets, each number inside the brackets is affected. The property of numbers works for addition and subtraction, for example:

4(5+6) =(4x5) + (4x6) or 6(5-4) = (6x5) – (6x4) This property is called the distributive property of multiplication.

Addition and subtraction are called inverse operations. If you add and subtract the same amount from a number, you end up back where you started. These operations have an inverse effect on each other, for example: 856 + 12 -12 = 856.

Multiplication and division are called inverse operations. If you multiply and divide a number by the same amount, you end up back where you started as the operations have an inverse effect on each other, for example:

524 x 12 ÷12 = 524.



10. Properties of Numbers

A. COMMUTATIVE PROPERTY

The commutative property changes the order of numbers.

2 + 4 = 4 + 2

9 x 6 = 6 x 9

Does not work for Subtraction and Division.

B. THE ASSOCIATIVE PROPERTY

The associative property changes the way numbers are grouped, but still keeps the numbers in the given order.

Study the examples given below:

(20 + 10) + 15 = 20 +(10+15)

(5 x 20) x 4 = 5 x (20x4)

C. DISTRIBUTIVE PROPERTY

The distributive property means multiplication is applied to addition and subtraction to make the calculation easier to work out.

Study the following examples:

2 x 5 + 2 x 6 – 2 x 7

= 2 x (5 + 6 - 7)

= 2 x 4

= 8

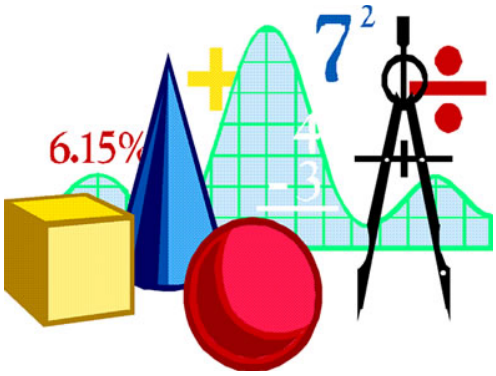
123 x 7

= (100 + 20 + 3) x 7

= (100 x 7)+(20 x 7)+(3 x 7)

= 700 + 140 + 21

= 861



12(6 + 7)

= 12 X 6 + 12 X 7

= 72 + 84

= 156

11. When multiplying or dividing by multiples of 10, 100, 1000

54 x 300

= 54 x 3 x 100 (“Break down” the one number)

= 162 x 100

= 16200.

12. Identity element for addition

Is 0.

Whether you add or subtract 0 from a number, the number never changes

13. Identity element for multiplication

Is 1

Whether you multiply or divide a number by 1, the number never changes

Exercise 1

1. Why is the number 1 neither a prime nor a composite number?
2. Write down the following sets of numbers:
   a) Odd numbers between 100 and 120.
   b) Even numbers from 364 to 372.
   c) Prime numbers greater than 5 but smaller than 27.
   d) The first 5 multiples of 12.
   e) Multiples of 8 from 48 to 80.
   f) The first 10 composite numbers.
   g) The factors of 144.
   h) The first 5 counting numbers.
   i) The first 5 natural numbers.
   j) The prime factors of 30.

3. Write down ALL the factors of the following numbers:
   a) 50
   b) 25
   c) 48

4. Write down the first 6 multiples of the following:
   a) 10
   b) 25
   c) 125

5. Solve the problems below by first rounding off each number to the nearest 10 000.
   a) 171 643 + 16 124
   b) 399 106 + 71 257 + 9 199

6. Repeat the method used above, but this time, round off to the nearest whole number.
   a) 128,69 – 99,6
   b) 34,9056 ÷ 5,4

7. Use the same method again but, round off these numbers to the nearest 100:
   a) 9 876 543 – 210 369
   b) 12 413 x 125

8. Use the same method again but, round off these numbers to the nearest 10:
   a) 8 342 x 29
   b) 211 x 43

Exercise 2.

1. Use your knowledge of the Distributive Property to solve the following problems:
   a) 29 x 4 + 29 x 7 + 29 x 10
   b) 58 x 19
   c) 795 x 6
   d) 124 x 50
   e) 450 ÷ 10 + 680 ÷ 10
   f) 1 250 ÷ 50 – 600 ÷ 10
   g) 25 x 5 + 25 x 6 + 25 x 8
   h) 50 x 369

2. Which of these are true and which are false? If true, state which property you have used.
   If false, write the correct method and/or answer.
   a) 8 + 7 = 7 + 8
   b) 10 x (7 +3 +4) = (10 x 7) + (10 x 3) + (10 x 4)
   c) 5 + (10 – 3) = (5 + 10) – 3
   d) (2 500 ÷ 20) ÷ 5 = 2 500 ÷ (20 ÷ 5)

3. Use any method to solve each of the following:
   a) A grocer buys 480 trays of oranges. He has to share the oranges equally between 20 clients. How many trays will each client receive?
   b) Mr Padaychee owns a clothing shop. He has 25 designer tops which he sells for R69 each. Assuming Mr Padaychee sells all of the tops, how much money will he receive?
   c) Julia orders 32 tablecloths for the school. Each tablecloth costs R122. How much does she have to pay?

d) The Foundation Phase children entered a competition and won and amount of R34 545. The teachers decided to share the prize money between 350 children who entered. How much money would each child get?

e) Angel uses her money to purchase the following items:

Apples	R9,00
Potatoes	R35,00
Bread	R6,00
Milk	R4,00
Chocolate	R5,00

How much money does she have to pay?

If she pays with R100, how much change will she get?

DOING THE FOUR OPERATIONS – A REMINDER:

VERTICAL METHOD

ADDITION

```
  H T U
  3 8 4
  5 2 7
+ 4 7 2
1 3 8 3
```

SUBTRACTION

```
 7000
- 549
 6451
```

MULTIPLICATION

159 x 27 =

```
   1 5 9
 X   2 7
 -------
 1 1 1 3
 3 1 8 0
 -------
 4 2 9 3
 -------
```

DIVISION - (LONG DIVISION)

```
     576
9 5 184
  4 500
  -----
    684
    630
    ---
     54
     54
     --
```

Exercise 3.

1. Use the VERTICAL METHOD:
   a) 4 260+5 721+842+393
   b) 33 333+55 555 - 77 777
   c) 100 403+859 782
   d) 592 710 – 361 204
   e) 240 040 - 67 952

2. Use the VERTICAL METHOD:
   a) 246 x 37
   b) 378 x 24
   c) 624 x 38
   d) 732 x 26

3. Use the VERTICAL METHOD:
   a) 350 ÷ 14
   b) 1 512 ÷ 28
   c) 5 168 ÷ 17

d) 2 106 ÷ 39

e) 9 936 ÷ 48

ADDITION HORIZONTAL METHOD

384 + 527 + 472 = (300+500+400) + (80+20+70) + (4+7+2)

= 1200+170+13

= <u>1 383</u>

HORIZONTAL MULTIPLICATION

159 x 27 = (159x20) + (159x7)

= 20 x (100+50+9) + 7 x (100+50+9)

= (20x100) + (20x50) + (20x9) + (7x100) + (7x50) + (7x9)

= 2 000 + 1 000 + 180 + 700 + 350 + 63

= 3 180 + 1 113

= <u>4 293</u>

235 x 34 =(235 x 30) + (235 x 4)

= 30 x (200+30+5) + 4 x (200+30+5)

= (30 x 200) + (30 x 30) + (30 x 5) + (4 x 200) + (4 x 30) + (4x5)

= 6000 + 900 + 150 + 800 + 120 + 20

= <u>7990</u>

Exercise 4

1. **<u>Use the HORIZONTAL METHOD.</u>**

   a) 456 +350 + 239

   b) 648 + 352 + 371

2. **<u>Use the HORIZONTAL METHOD.</u>**

   a) 1 226 x 82

   b) 3 437 x 24

FOLLOWING A SPECIFIC ORDER OF OPERATIONS

BODMAS

The order of operations for BODMAS is as follows:

B	-	Brackets
O	-	Of
D	-	Division
M	-	Multiplication
A	-	Addition
S	-	Subtraction

Study the calculation below:

16 + 4 x 5 =

If you do the calculations from left to right, the answer is 100

(i.e. (16+ 4) x 5 = 100)

If you do the calculation according to BODMAS, the answer is 36.

(i.e. 16 + (4 x 5) = 36)

Both these answers seem correct.

HOW IS THIS POSSIBLE?

When we read words or numbers, we read from left to right. However, mathematicians decided many years ago that a method should be used that orders operations in a specific way. This is how BODMAS came about.

Remember, when doing BODMAS, that when multiplication and division are carried out, you may start with whichever one of these operations comes first, in order from left to right.

The same principle applies to addition and subtraction. You may start with whichever one comes first, in order from left to right.

Example:

235 + 80 x 5 ÷ 10 – 215

= 235 + 400 ÷ 10 – 215 (multiplication first)

= 235 + 40 – 215 (division)

= 275 – 215 (addition)

= 60 (subtraction)

Exercise 5.

Complete the sums below using BODMAS/BOMDAS.

No calculators are allowed. Show all working out.

a) 30 X 12 ÷ 4 + ¾ 0f 20

b) (17 x 25) x (24 ÷ 2) – 10 x 10

c) (24 – 14) x 25 ÷ 5

d) (1 420 + 780) – (509 + 351)

e) ($\frac{7}{10}$ of 500) x $\frac{48 \div 12 \, x \, 2}{100 \div 25}$

f) 17 x (4 + 9) ÷ (23 – 10)

g) 235 + 80 x 50 ÷ 10 – (215 + ¾ of 8)

h) (65 – 15) x 3 + 21 ÷ 7

i) 200 ÷ 20 + 285 ÷ 95 – 8 + 4

j) 71 x $\frac{10 \, x \, 10{,}4}{4 \, x \, 13}$

k) 790 + 1 000 ÷ 125 – 50 ÷ 10

l) 12 000 - $\frac{10 \, x \, 100}{\frac{1}{2} of \; 250}$ + ½ of 100 + (200 ÷ 10 x 2)

m) 2 + $\frac{93 \, x \, 28}{2\,604 \div 12}$ + $\frac{2}{9}$ of 108

n) 50 + $\frac{3}{5}$of 75 – 32

o) 470 + 692 x 10 ÷ 20 – 630

p) $\frac{3}{8}$ of 4 000 + ½ of 250 – 10 ÷ 5

q) 150 + ¾ of 24 + 27 ÷ 9 + $\frac{35 \div 7 + 40}{9 \, x \, 5}$

r) 163 – 25 x (26 – 20)

s) 46 – 32 + 27 x (104 ÷ 8)

t) $\frac{144}{12}$ + $\frac{1}{8}$ of 24 - $\frac{35 \, x \, 2 + 10}{108 \div 12 + 11}$

u) 8 967 - $\frac{240 \div 5 \text{ x } 3}{48 \div 4}$ + $\frac{10}{12}$ of 48 + (9 x 6)

MULTIPLES AND FACTORS

A multiple is the product of two natural numbers. For example: 8 × 3 = 24. The number 24 is a multiple of 8 and 3 because 8 × 3 =24. The number 24 is also a multiple of 12 and 2 because 12 ×2 = 24.

Every natural number has an infinite number of multiples.

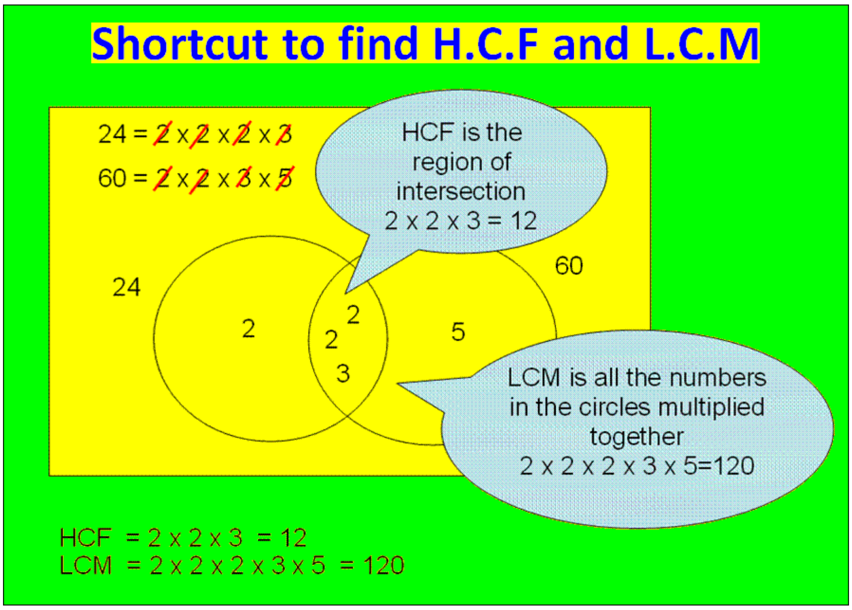
HCF and LCM are numbers that share the same factors. These are called common factors and you can find the highest common factor, HCF, of two or more numbers.

You can also find the lowest common multiple, LCM, of two or more numbers. For example: The multiples of 12 are 12 ; 24; 36; 48; 60; 72; 84;… and the multiples of 15 are 15; 30; 45; 60; 75; 90;… which means that the LCM of 12 and 15 is 60.

Ratio: A comparison between two numbers or two quantities that are measured in the same units. For example: The ratio of the original price of the coat for the sale price is R300: R210. We simplify this to 10 : 7.

Profit and loss: Profit is the difference between the selling price and the cost price of an article. If the selling price is less than the cost price, the difference is called the loss.

Profit or loss can be calculated as SP (selling price) – CP (cost price)



Highest Common Factor and Lowest Common Multiple

This section introduces the idea of the *highest common factor* and *lowest common multiple* of a pair of numbers.

> The **highest common factor (HCF)** of two whole numbers is the largest whole number which is a factor of both.

HCF Example

Consider the numbers 12 and 15:

The factors of 12 are : **1, 2, 3, 4, 6, 12**.

The factors of 15 are : **1, 3, 5, 15**.

1 and **3** are the only **common factors** (numbers which are factors of both 12 and 15).

Therefore, the **highest common factor** of 12 and 15 is **3**.

> The **lowest common multiple (LCM)** of two whole numbers is the smallest whole number which is a multiple of both.

LCM Example

Consider the numbers 12 and 15 again:
The multiples of 12 are : **12, 24, 36, 48, 60, 72, 84, ...**
The multiples of 15 are : **15, 30, 45, 60, 75, 90, ...**
60 is a **common multiple** (a multiple of both 12 and 15), and there are no lower common multiples.
Therefore, the **lowest common multiple** of 12 and 15 is **60**.

> Although the methods above work well for small numbers, they are more difficult to follow with bigger numbers. Another way to find the highest common factor and lowest common multiple of a pair of numbers is to use the prime factorizations of the two numbers.

Finding HCF & LCM with prime factorizations

We want to find the HCF and LCM of the numbers 60 and 72.

Start by writing each number as a product of its prime factors.

60 = 2 * 2 * 3 * 5
72 = 2 * 2 * 2 * 3 * 3

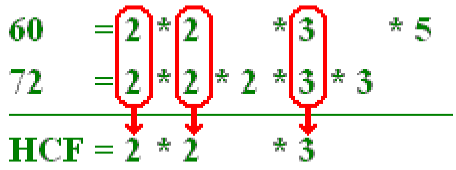
To make the next stage easier, we need to write these so that each new prime factor begins in the same place:

60 = 2 * 2   * 3   * 5

72 = 2 * 2 * 2 * 3 * 3

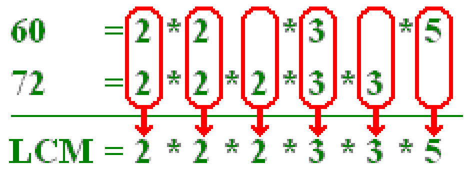
All the "2"s are now above each other, as are the "3"s etc. This allows us to match up the prime factors.

The highest common factor is found by multiplying all the factors which appear in both lists:



So the HCF of 60 and 72 is **2 × 2 × 3** which is **12**.

The lowest common multiple is found by multiplying all the factors which appear in either list:



So the LCM of 60 and 72 is **2 × 2 × 2 × 3 × 3 × 5** which is **360**.

Exercise 6

Question 1

Find the HCF of:
(a) 6 and 9 :
(b) 14 and 18 :
(c) 30 and 24 :
(d) 15 and 10 :

Question 2

Find the LCM of:
(a) 5 and 3 :
(b) 9 and 6 :
(c) 8 and 10 :
(d) 12 and 9 :

(e) 15 and 20 :
(f) 6 and 11 :

Question 4

Find the LCM of:
(a) 28 and 30 :
(b) 16 and 24 :
(c) 20 and 25 :
(d) 60 and 50 :
(e) 12 and 18 :
(f) 21 and 35 :

Question 5

Two lighthouses can be seen from the top of a hill.
The first flashes once every 8 seconds, and the other flashes once every 15 seconds.

If they flash at the same time, how long will it be until they flash at the same time again?

Exercise 3
(Revision of LCM & HCF)

Find the Lowest Common Multiple (LCM)

Find the LCM of 2, 4 and 5

M2= [2;4;6;8;10;12;14;16;18;20;22;24;....] M4= [4;8;12;16;20;24;.....]

M5= [5;10;15;20;25;30;.....]

LCM = 20

Find the LCM for a) 6;12 b) 5;7 c) 4; 8;16

A factor of a number is another number that divides into it without a remainder.
A prime factor is a factor that is also a prime number.
2 is a factor of 6 because 2 can divide into 6 without a remainder.

F_6 = [1;2;3;6]

The prime factors of 6 are 2 and 3 because 2 and 3 are factors as well as prime numbers.

F_{120} = [1;2;3;4;5;6;8;10;12;15;20;24;30;40;60;120]

Prime Factors of 120 are 2;3;5

Highest Common Factor (HCF)

The HCF of two or more numbers is the highest number that you can divide into those numbers without a remainder.

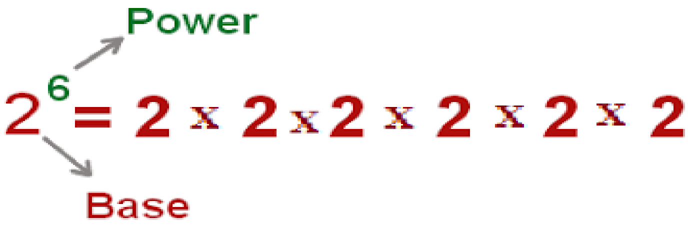
Find the HCF of 24; 36 and 48

24 = 2x2x2x3

36 = 2x2x3x3

48 = 2x2x2x2x3

HCF = 2x2x3 = 12



EXPONENTS

A number that shows how many times a base is used as a factor.

You can use an exponent to show repeated multiplication. If you multiply a number by itself the answer is a square number and if you multiply a number by itself two times the answer is a cube number.

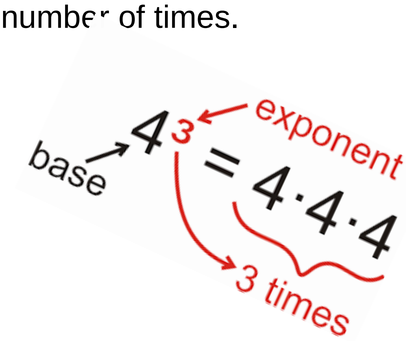
Square number: A number multiplied by itself, for example $a^2 = a \times a$

Cube number: The product of a number multiplied by itself twice, for example: $a^3 = a \times a \times a$.

Power: A base together with an exponent is called a power. Any number can be represented in exponential form without needing to calculate the actual value of the number. An expression to show that a number is multiplied by itself a number of times.

The first twelve square numbers are:

1 = 1 × 1	4 = 2 × 2	9 = 3 × 3
16 = 4 × 4	25 = 5 × 5	36 = 6 × 6 or
49 = 7 × 7	64 = 8 × 8	81 = 9 × 9
100 = 10 × 10	121 = 11 × 11	144 = 12 × 12



The first cube numbers are:

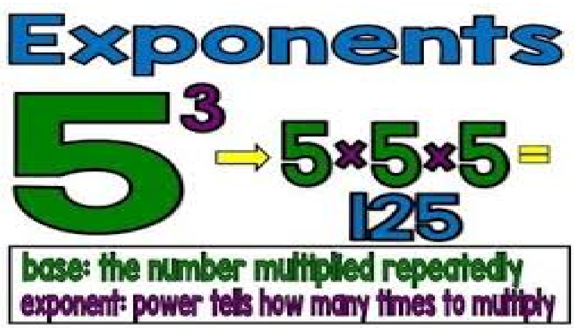
1 = 1×1×1

8 = 2×2×2

27 = 3×3×3

64 = 4×4×4

125 = 5 ×5×5



A square root is the inverse operation of squaring a number. For example: $3^2 = 9$ and $\sqrt{9} = 3$. Note that $\sqrt{9}$ can be written as $\sqrt{3 X 3}$ or $\sqrt{3}$ X $\sqrt{3}$.

A cube root is the inverse operation of finding the cube of a number. For example: $2^3=8$ and $\sqrt[3]{8} = 2$. Note that $\sqrt[3]{8}$ can be written as $\sqrt[3]{2 X 2 X 2}$ or

$\sqrt[3]{2}$ X $\sqrt[3]{2}$ X$\sqrt[3]{2}$

Exponents

You can use an exponent to show repeated multiplication.
If you multiply a number by itself the answer is a square number
If you multiply a number by itself two times the answer is a cube number

Example
$3 \times 3 = 3^2 = 9$ is a square number

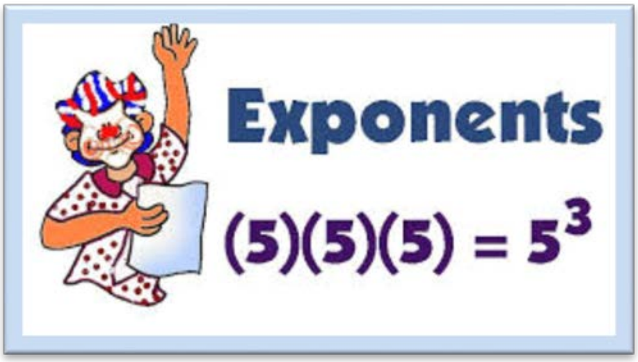
$3 \times 3 \times 3 = 3^3 = 27$ is a cube number

When you write the number of the times that the natural number appears in repeated multiplication in the shortened form, we call this exponential form.

This means that in 3^2 the number 3 is read as 3 to the power of 2 where 3 is the base number and 2 is the exponent.

A base together with an exponent is called a power.

Exercise 1



1. Write the following in exponential form
   a) 2x2x2x2

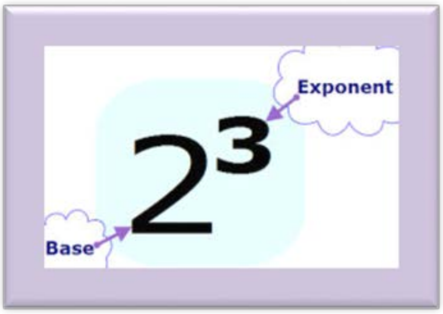
2. Write out the squares of these numbers
   a) 1
   b) 5
   c) 8
   d) 30

3. Write out the cubes of these numbers
   a) 1
   b) 2
   c) 3
   d) 4

Square roots

Finding the square root of a number is the inverse operation of squaring a number. To find the square root ask yourself which number multiplied by itself will give you this number.

Example: 4 is the square root of 16 because $4 \times 4 = 16$.



We use the square root sign $\sqrt{16}$ = 4

10 x 10 = 100

$\sqrt{100}$ = 10

Cube Root

Finding the cube root of a number is the inverse operation of cubing a number. To find the cube root ask yourself which number you must multiply three times to give you this number.

The cube root of 8 is 2 because 2 x 2 x 2 = 8

We use the cube root sign to write $\sqrt[3]{8}$ = 2

5 is the cube root of 125 because 5^3 = 5 x 5 x 5 = 125 therefor $\sqrt[3]{125}$ = 5

Compare and represent numbers in exponential form

We use the exponential form to write large numbers in a shorter way.

Example: 10 000 000 = 10 x 10 x 10 x 10 x 10 x 10 x 10 = 10 to the power of 7, 10 is the base of the power and 7 is the exponent or index.

The exponent shows the number of factors that are multiplied.

Exercise 2

A. Write down "b" the base of the power and "a" the exponent form.

Example 4^2 = a) 2

b) 4

1) 2^4

2) 5^3

3) 6^9

4) 17^{10}

5) 25^{12}

6) 8^0

B. Replace each * with < or = or >

1) 2^1 * 2

2) 3*3^2

3)2^3 * 2^2

4)5^1*5^2

5) 10^2 * 10^5

6) 10^2*9^2

7) 25^1 * 25

8) 25^4*30^4

9) 2^5 * 5^2

10)5^2*4^4

Exercise 3

1. Draw a table in your book and complete it as follows:

Number	Number x itself	Exponential notation	Answer	Square root	Answer
Eg. 5	5x5	5^2	25	$\sqrt{25}$	5

Complete from 1 to 13

2. Next, draw another table in your book and complete it as follows:

Number	Number x itself x itself	Exponential notation	Answer	Cube root	Answer
Eg: 4	4x4x4	4^3	64	$\sqrt[3]{64}$	4

Complete from 1 to 7

Use prime factors to write numbers in exponential form (Ladder method)

Write in exponential form:

a) 72

2	72
2	36
2	18
3	9
3	3
	1

72 = 2x2x2x3x3

= 2^3 x 3^2

Exercise 4

Write in exponential form using only prime numbers as bases. (Ladder Method)

1) 32
2) 125
3) 256
4) 200

5) 275
6) 588

Use the exponential form to calculate square roots and cube roots

You can use the exponential form of numbers to calculate the square root and cube root. First write the number as the product of the prime factors and then calculate the root.

Worked Example

Calculate :

a. $\sqrt{81}$

$= \sqrt{3^4}$

$= 3^2$

$= 9$

3	81
3	27
3	9
3	3
	1

b. $\sqrt[3]{1728}$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$\sqrt[3]{2^6 x 3^3}$

$2^2 \times 3$

12

Exercise 5

1) $\sqrt{25}$
2) $\sqrt{16}$
3) $\sqrt{729}$
4) $\sqrt{256}$
5) $\sqrt{3025}$
6) $\sqrt[3]{27}$
7) $\sqrt[3]{125}$
8) $\sqrt[3]{512}$

Calculations with exponents

The square root sign works like a bracket. Simplify any addition or subtraction underneath the sign where possible.

Example $\sqrt{16+9} = \sqrt{25} = 5$

$\sqrt{169 - 144} = \sqrt{25} = 5$

The square root of a number multiplied by itself is equal to the number.

Example $(\sqrt{20})^2 = 20$

The cube root of a number that is then cubed is equal to the number.

Example $(\sqrt[3]{20})^3 = 20$

Calculations using numbers in exponential form

Calculations with ordinary numbers follow the "Laws of operations" such as the laws for addition, subtraction, division and multiplication. Calculations using numbers with exponents or roots also follow rules.

Example:

$(7+3)^2 = (10)^2 = 10^2 = 100$

$(7 - 3)^3 = (4)^3 = 4^3 = 64$

a) $\sqrt{16+9}$
$=\sqrt{25}$
$= 5$
$=12$

b) $\sqrt{16 \times 9}$
$= \sqrt{144}$
$= 12$

or $\sqrt{16 \times 9}$
$=\sqrt{16} \times \sqrt{9}$
$= 4 \times 3$
$=12$

c) $\sqrt{49} - \sqrt[3]{8}$
$=\sqrt{7 \times 7} - \sqrt[3]{2 \times 2 \times 2}$
$= 7 - 2$
$= 5$

Exercise 6

1) Find the value of:

a) $\sqrt{9 \times 4}$
b) $\sqrt{9} \times \sqrt{4}$
c) $\sqrt{16 \div 4}$
d) $\sqrt{16} \div \sqrt{4}$
e) $\sqrt{100} \times \sqrt{4}$
f) $\sqrt{100} \div \sqrt{4}$

2) Simplify each expression to one natural number:

a) $1^3 + 0^2$
b) $2^3 - 2^2$
c) $3^2 - 2^2$
d) $5^2 + 5 - 2^3$
e) $2^2 + 3^3 - \sqrt{25}$
f) $6^2 - 4^2 - \sqrt{9}$

3) Simplify the following:

a) $\sqrt{36} + 2^3 - 3^2$
b) $3^2 - \sqrt[3]{27} + 12^2$
c) $\sqrt{4} + 3^2$
d) $5^2 + \sqrt{10^2} - 1^3$
e) $\sqrt{20+5} - \sqrt[3]{4^3}$
f) $\sqrt[3]{64} - \sqrt{16}$

Numbers in Exponential form.

a) $2^3 + 2^4 + 2^5$
$= 8 + 16 + 32$
$= 56$

b) $4^2 \times 10^2$
$= 16 \times 10\,000$
$= 160\,000$

c) $(8-5)^3$
$= 3^3$
$= 27$

Exercise 7

Calculate:

1) $3^4 + 10^2$

2) $5^3 \times 10^4$

3) 25×105

4) 112×105

5) $62 + 25$

6) $72 - 33$

7) $8^3 \div 2^4$

8) $9^2 \times 2^3$

9) $22 + 23 + 24$

10) $34 - 33 - 32$

11) $5^2 \times 2^3 \times 3^2$

12) $2^5 \div 2^3 \div 2^1$

RATIO AND RATE

DEFINITIONS

RATIO

- A ratio is used to compare the sizes of two or more quantities that use the same unit of measurement.
- A ratio of 5 : 6 means that for every 5 of the first quantity, there are 6 of the second quantity.
- Ratio can also be written as a fraction. In the ratio 5 : 6, the first quantity would be written as $\frac{5}{11}$. The second quantity would be written as $\frac{6}{11}$.
- Ratios can be simplified.
  e.g. 10 : 12 can be simplified to 5 : 6.

RATE

- A rate compares 2 or more quantities that use different units of measurement, e.g. km/l.; R/kg.

EXERCISE 1

<u>Calculating Ratio</u>

1) Simplify these ratios
   a) 3: 6: 12
   b) 24 : 60 : 84
   c) 10 cm : 2 m
   d) 40 m : 2km
   e) 12 : 40
   f) 38 : 90
   g) 25 : 1 000
   h) 50 c : R10,00
   i) 40g : 1kg

j) 8 l: 250 ml

k) 9 hours : 150 minutes

EXERCISE 2

<u>Calculating Rate</u>

a) A chef bakes a dozen cookies in 20 minutes. How many cookies does he bake in 3 hours?

b) A seamstress sews 16 tops in 1 hour. How many tops does she sew in:
- half an hour?
- 9 hours?

c) 500 pairs of shoes are sold for a total R2 500,00. How much does 1 pair of shoes cost?

d) A truck driver takes 4, 5 hour to travel 400 km. What is the average speed per hour?

e) 14 Sharpeners cost a total of R35,00. What will 6 sharpeners cost?

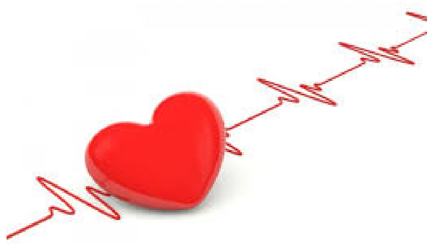
f) 4 passengers pay R380,00 as travel fare. What is the fee per passenger?

g) 6 kg of biltong costs R462,00. How much would you pay for 2,5 kg of biltong?

h) A car travels 360 km in 4 hours. How far does it travel in 6 hours?

i) Kyra must take 15 ml of medicine every 6 hours. How much medicine must Kyra take daily?

j) A butcher sells 15 kg of mince for R900,00. How much does the mince cost per kilogram?



Sharing a “whole” in a given ratio.

Share R 2 250,00 in the ratio 3:2:1

- This means 3:2:1 that 3+2+1= 6 parts of the whole 2 250.
- In fraction form, this means $\frac{3}{6}$ of 2 250

  = 3x2 250

  = 6 750 ÷ 6

  = R 1 125
- $\frac{2}{6}$ of 2 250

  = 2x2 250

  = 4 500 ÷ 6

  = R 750
- $\frac{1}{6}$ of 2 250

  = 2 250 ÷ 6

= R 375

EXERCISE 3

1) Divide R 200,00 between you and your best friend in the ratio 3:2
2) Divide R 240,00 in the ratio 3:4:5
3) Share 28 sweets between Joe and Amy in the ratio 3:1
4) Share an inheritance of R 50 000,00 between five children in the following ratio 7:9:3:2:4

CALCULATING PERCENTAGE INCREASE AND DECREASE

- When increasing or decreasing a number by a given percentage, write the percentage out of 100 and multiply it by the given number.

Example: Increase R 1 500 by 25%

$= \frac{25}{100} \times \frac{1500}{1}$ *Simplify / Cancel if possible

= R 375

Now add this amount to the original value:

i.e. R1500+R375

= R 1 875

- If decreasing, you would subtract this amount from the original value.

Example: Decrease R 3 000 by 45%

$= \frac{45}{100} \times \frac{3000}{1}$

= R 1 350

Decreased amount: R3000–R1350

= R1650

EXERCISE 4

Solve the problems below:

1) The building cost of houses has increased by 20%. If a house previously cost R 26 000,00 what will the new building cost be?

2) Civil servants are told they will get a salary increase of 4%. How much will they earn if their current salaries are R 4 500,00?
3) The value of a home entertainment system decreased by 15% after 1 year. If the price is R 17 800, what will its value be after a year?
4) The Grade 7 learners from last year made a profit of R 18 560,00 on Entrepreneurs' Day. This year's Grade 7 learners made 12% more profit than the previous group. How much profit was made?
5) Thandiwe bought a book that cost R 52,00. If she was given a cash discount of 8%, how much will she pay for the book?
6) Increase R 4 575 by 25%.
7) Decrease R 7 500 by 40%.
8) I bought a CD for R 120,00. I sell it for 15% less than what I paid for it. How much money will I receive?

CONCEPT : CALCULATING VAT, INTEREST AND DISCOUNT

- When calculating VAT, 14% of the article's value must be added to the cost of the article (i.e. increase by 14%).
- When calculating discount, we decrease the value of the article by the given percentage.
- Interest is added to the value of an article, normally when credit is given and the buyer cannot pay the full amount.

Example 1:

Calculate the VAT on the article costing R 500,00.

$$= \frac{14}{100} \times \frac{500}{1}$$

= 70

= R500+R70

= R 570

Example 2:

Calculate what you will pay for an item costing R 250,00 if a discount of 15% is given.

$$\frac{15}{100} \times \frac{250}{1}$$

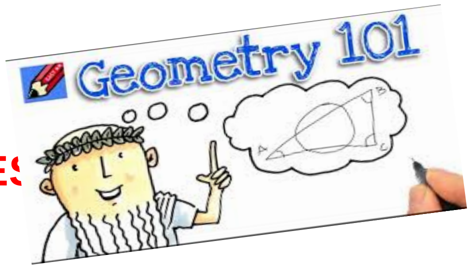
$$= \frac{75}{2}$$

= R 37,50

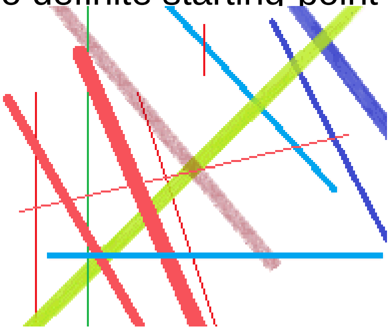
= R250–R37,50

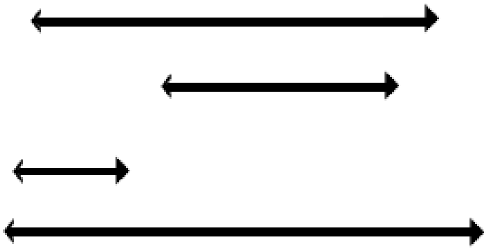
= R 212,50

GEOMETRY OF STRAIGHT LINES

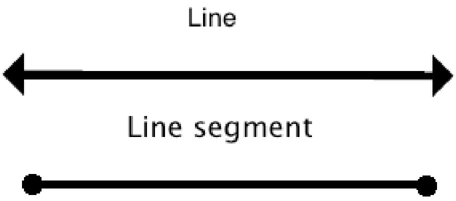


A straight line is the shortest distance between two points or a set of points with no definite starting-point or end point.

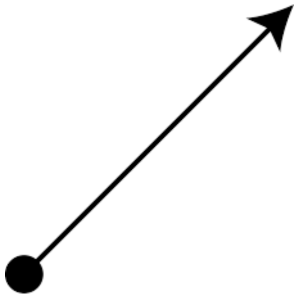




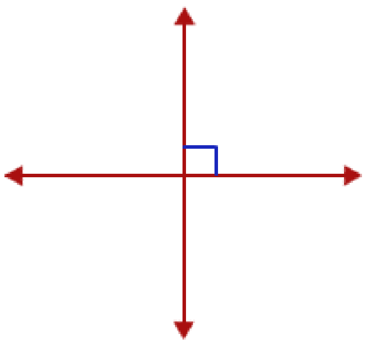
A line segment is a set of points with a definite starting point and end point.

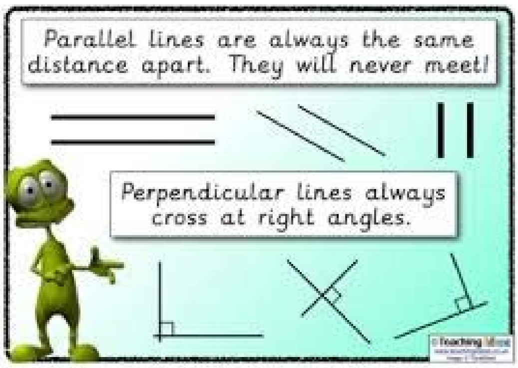


A ray is a set of points with a definite starting point and no definite end point.

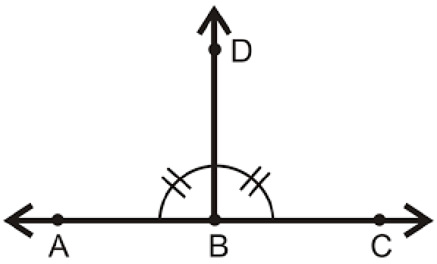


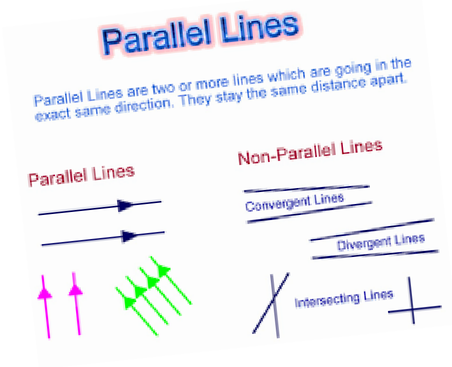
Two intersecting straight lines are said to be perpendicular when the angle they form is equal to 90º. We use a little square to indicate when an angle is a right angle (equal to 90º).





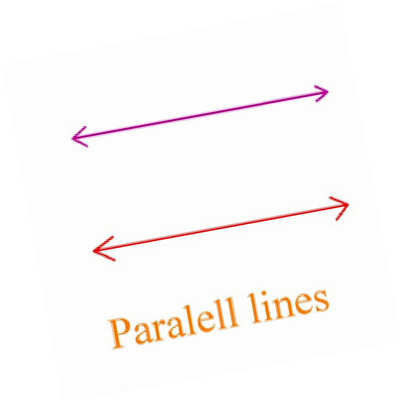
When two line segments are perpendicular, we indicate this with the ⊥ symbol.





We say two straight lines are parallel when the perpendicular distance between the two lines is constant. Notice that we indicate lines that are parallel with little arrows on the lines.

When two line segments AB and CD are parallel, we use the symbol ∥ to indicate this.



CONCEPT : GEOMETRY (LINES AND ANGLES)

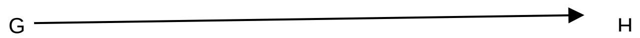
The world around us is made up of many two-dimensional (2D) shapes and three dimensional (3D) objects. Architecture, for example, shows how geometry concepts are applied to structures. It is therefore important to understand the various geometry concepts upon which our structures are based.

GEOMETRY CONCEPTS

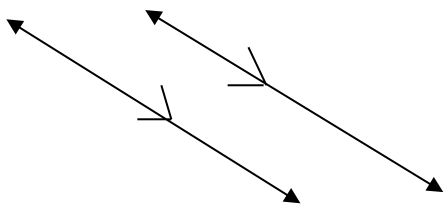
1. **Line**
   A line consists of an uncountable number of points in a straight line and it has length.

2. **Line Segment**
   A line segment is any part of a line. It has a beginning and an end, which is indicated by capital letters.

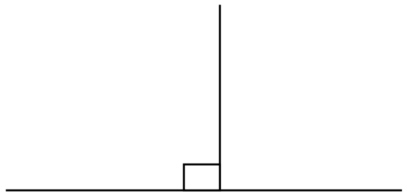
3. **Ray**
   A ray has a definite beginning but no end. A ray can be found in an angle.



4. **Parallel Lines**
   Parallel lines are lines that are the same distance apart and extend in the same direction. They never intersect and are indicated by the symbol ll or //. Arrows are drawn on the set of parallel lines to indicate that they are parallel.

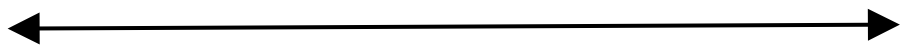


5. **Perpendicular Lines**
   Perpendicular lines are formed when 2 lines meet or intersect at 90^0. The symbol for perpendicular is $\perp$.

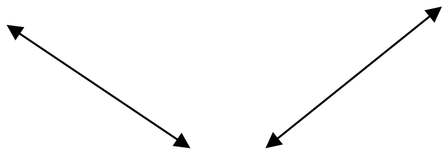


Vertical Lines

Horizontal Lines

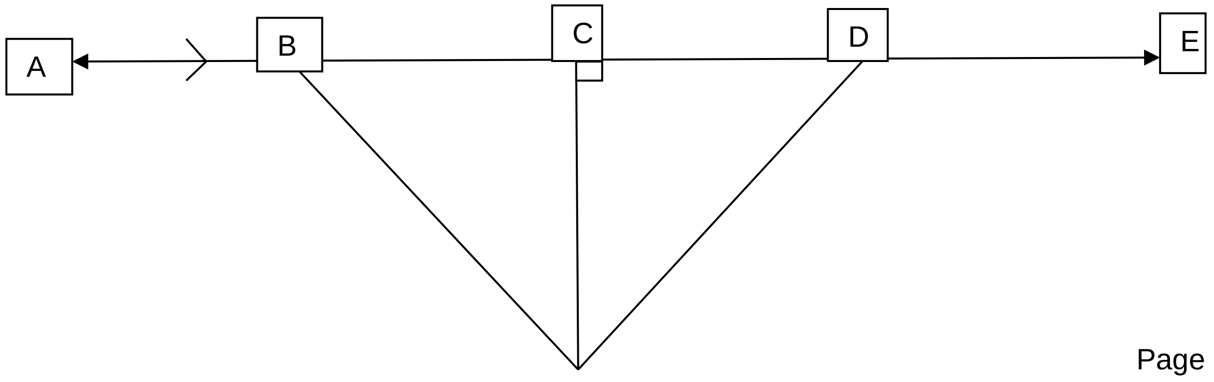


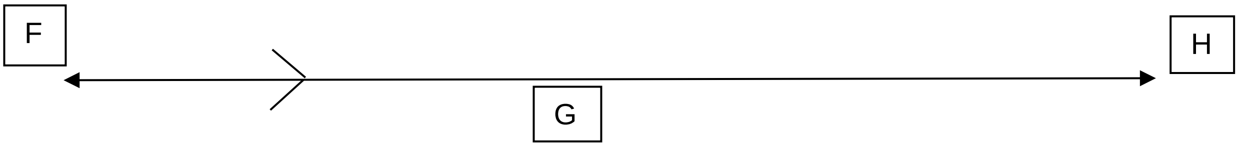
Diagonal Lines



Exercise 1

1. **Study the figure below and complete the table**

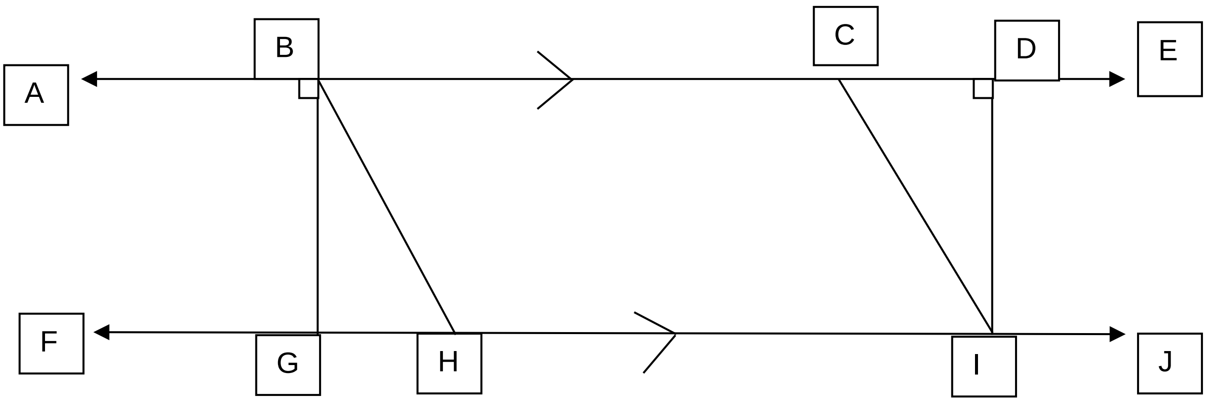




Two lines in the figure above	
One pair of perpendicular lines	
One pair of parallel lines	
Three line segments	
One vertical line / line segment	
One horizontal line / line segment	

2. <u>Look at the figure on page 3 and answer true or false for the mathematical statements. Give reasons for your answers.</u>

1. AE ll FH
2. CD ⊥ DI
3. BG intersects AB perpendicularly
4. GH is a vertical line
5. DI is a horizontal line



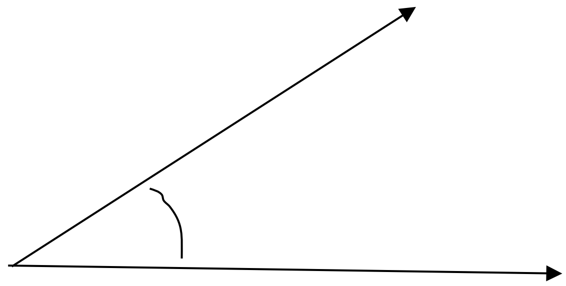
CLASSIFICATION OF ANGLES

WHAT IS AN ANGLE

An angle is formed when two rays start from the same point of origin, but rotate away from each other.

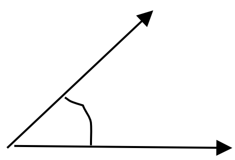
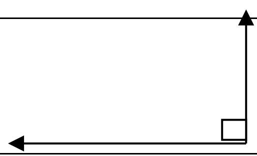
The magnitude or size of the angle is therefore the measurement that shows how far one of the rays has rotated.

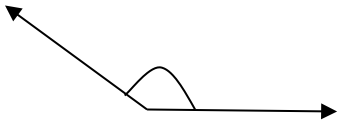
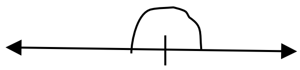
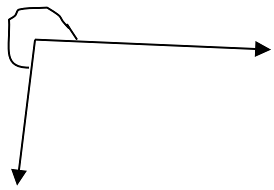
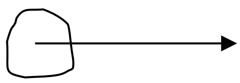
Angles are measured in degrees and we use a protractor to measure the size of the angle.



CLASSIFICATION OF ANGLES ACCORDING TO MAGNITUDE

There are 6 types of angles

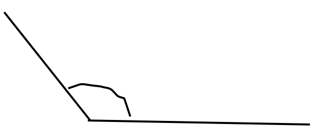
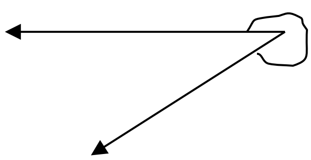
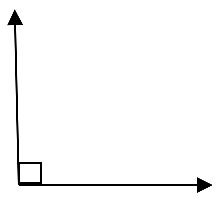
	Name of angle	Size of angle	Example
1)	ACUTE ANGLE	LESS THAN 90	
2)	RIGHT ANGLE	90	

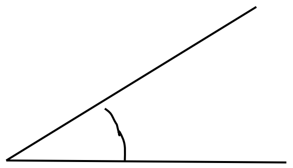
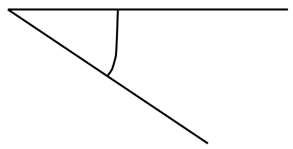
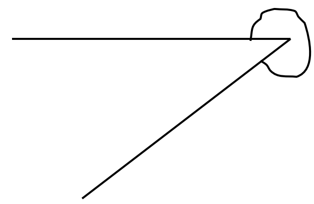
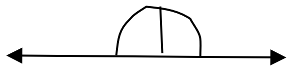
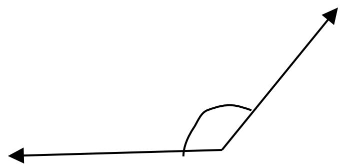
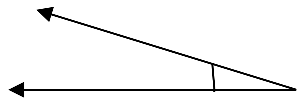
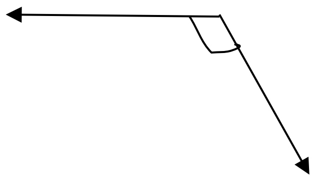
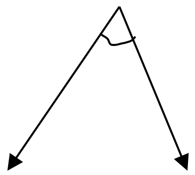
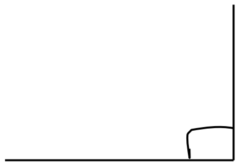
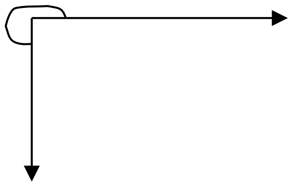
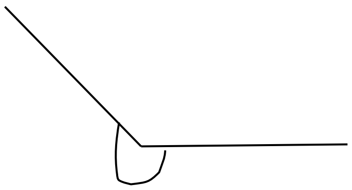
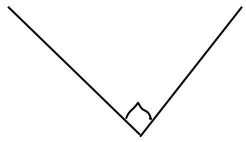
3)	**OBTUSE ANGLE**	BETWEEN 90 AND 179	
4)	**STRAIGHT ANGLE**	180	
5)	**REFLEX ANGLE**	BETWEEN 181 AND 359	
6)	**REVOLUTION**	360	

Exercise 2

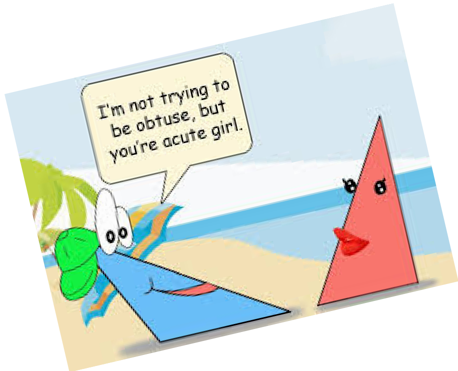
Measuring and naming angles

Name each of the following angles

1. 	2. 	3. 

4. 	5. 	6. 
7. 	8. 	9. 
10. 	11. 	12. 
13. 	14. 	15. 

CONSTRUCTIONS



MEASURE AND CLASSIFY ANGLES

Angles are formed wherever two straight lines or edges meet. The straight lines or edges that form the angle are called the arms, and the point where the arms meet are called the vertex. The plural of vertex is vertices.

You can indicate an angle by drawing an arc between its arms. An arc is a part of the circumference of a circle. The lines in the diagram below are identical, but the angles are different.

USING THE PROTRACTOR TO MEASURE ANGLES SMALLER THAN A REFLEX ANGLE.

- Place the centre midpoint of the protractor on the vertex or corner of the angle you are missing.
- Make sure the 180^0 line of the protractor is lying on the arm of the angle.
- If the angle moves in a clockwise direction, you will read he outer scale of numbers,
- If the angle moves in an anti-clockwise direction, you will read the inner scale of numbers.
- Look to see where the second rotating arm extends to. Whichever number it points to will tell you the size or magnitude of the angle.

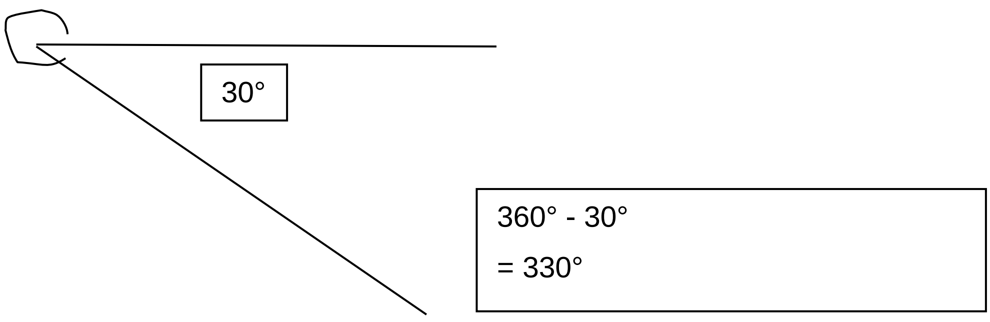
USING THE PROTRACTOR TO MEASURE REFLEX ANGLES

- The conventional protractor measures up to 180^0 only. Yet, a reflex angle is bigger than 180^0. There are 2 possible methods you can use.

Method 1

- Rotate your protractor so that the 180^0 line is lying on one of the arms of the angle.
- Look to see which number the second arm is pointing to. Subtract this number from 360^0. This will give you the measurement of the angle.

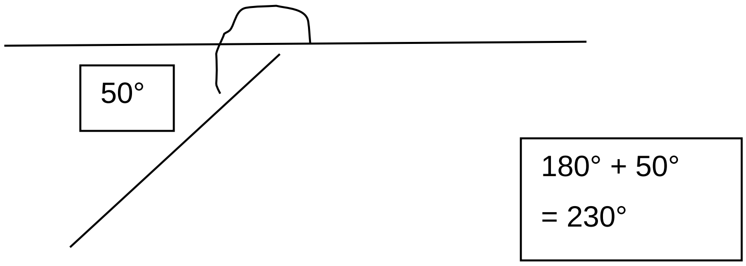
Example:



Method 2

- The arc of the angle indicates that one of the rays has rotated past the 180^0 mark.
- Rotate the protractor so that its 180^0 line is lying on one of the arms of the angle.
- Use the opposite scale of numbers to what you used in Method 1.
- Look to see which number the second arm is pointing to. Add this number to 180^0. This will give you the measurement of the angle.

Example:



Exercise 3: You will receive an activity sheet to paste into your book.

CONCEPT : POLYGONS – DEFINITION AND TYPES

WHAT ARE POLYGONS?

Polygons are shapes that have 3 or more sides. Polygons may be:

1. Regular: This means that the sides and angles of the polygon are equal in size.

2. Irregular: This means that the sides and angles of the polygon are not the same size.

CLASSIFICATION OF POLYGONS

Polygons are classified according to the number of sides and angles that they have.

Name of polygon	Number of sides/angles	Translation
Triangle	3	Tri = 3
Quadrilateral	4	Quad = 4
Pentagon	5	Penta = 5
Hexagon	6	Hexa = 6
Heptagon	7	Hepta = 7
Octagon	8	Oct = 8
Nonagon	9	Nona = 9
Decagon	10	Deca = 10

CLASSIFICATION OF TRIANGLES

WHAT IS A TRIANGLE?

A triangle is a shape that has 3 sides and 3 angles.

The interior angles of a triangle always adds up to 180^0.

CLASSIFICATION OF TRIANGLES

Triangles may be classified according to sides or angles.

1. Classification according to sides:

1.1. Equilateral triangles: All 3 sides are equal

1.2. Isosceles triangle: Two of the sides are equal
1.3. Scalene triangle: None of the sides are equal

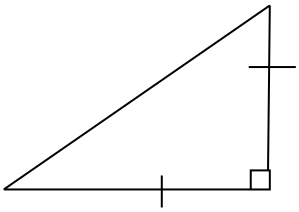
Classification according to angles

1.4. Acute-angled triangle: All 3 angles are acute (i.e. less than 90^0)
1.5. Obtuse-angled triangle: One of the angles is an obtuse angle (i.e. bigger than 90^0)
1.6. Right-angled triangle: One of the angles is a right angle (i.e. 90^0)

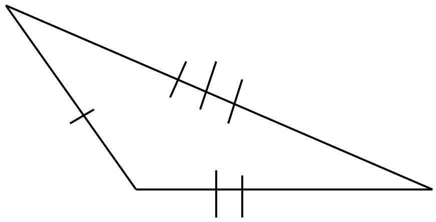
Exercise 4

Name each of the triangles below:

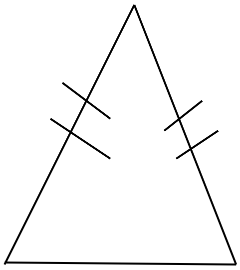
a.



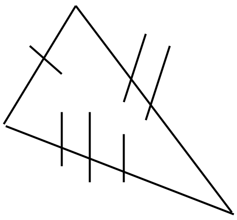
b.



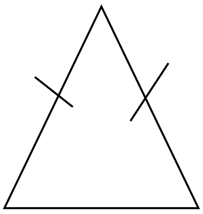
c.



d.



e.

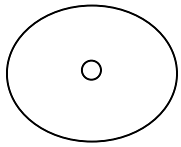
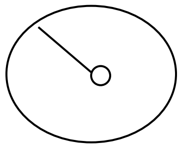
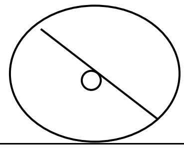
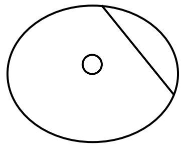
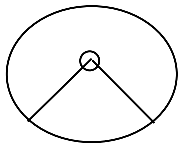
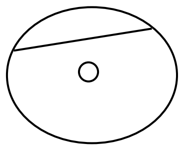
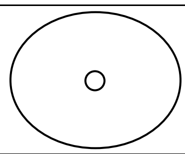
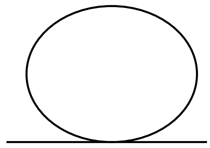
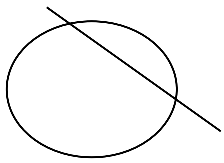


NB: You need to name triangles according to their sides and angles. (Right-angled isosceles triangle)

Acute-angled triangle - All 3 angles are acute (i.e. less than 90)
Obtuse-angled triangle - one of the angles is an obtuse angle (i.e. bigger than 90)
Right-angled triangle -One of the angles is a right angle (i.e. 90)

Equilateral triangles - all 3 sides are equal
Isosceles triangle - two of the sides are equal
Scalene triangle - none of the sides are equal

GEOMETRY – CIRCLES

PART OF CIRCLE	**DIAGRAM**
CENTRE	
RADIUS	
DIAMETER	
CHORD	
SECTOR	
SEGMENT	
ARC	
TANGENT	
SECANT	

Exercise 5

1) Circumference -

2) Diameter -

3) Radius -

4) Centre -

5) Arc -

a)	A line that forms the circle.
b)	A line that passes through the centre of the circle and joins two points on the circumference.
c)	Any straight line drawn from the centre of the circle to any point on the circumference.
d)	A point in the middle of the circle that is an equal distance from every point on the circumference
e)	Part of the circumference of the circle

Exercise 6

1. Draw circles with radii:

   a) 4cm b) 54mm

2. Draw circles with diameters:

   a) 88mm b) 10cm

3. Draw a circle; centre O, of radius 30mm. Draw radius OA. Mark a point B on OA so that OB = 10mm. Using B as a centre, draw a circle of radius 20mm.

   a) What do you notice about the two circles?

   b) Draw the common tangent PQ at A.

   c) Measure angle PAO.

4) Draw PQ = 7cm.

   a. Mark off angle QPS = 35° and draw PS = 8cm.

   b. Draw TS II PQ with TS = 6cm and angle TSP less than 90°. Draw TQ.

   c. Measure and write down the length of TQ.

5) Draw PQ = 7cm

   a. Mark off point S on PQ with PS = 3cm.

   b. Draw ST perpendicular to PQ with ST = 4cm.

   c. Draw AB through T parallel to PQ with AT = 5cm and AB = 8cm.

   d. Draw AQ. Measure and write down the length of AQ.