Math 100 – worksheet 8 –

Stocks, Bonds, and Cash

1. The book (p.226) provides historical return data for stocks, bonds and cash investments. In particular, over the period 1900 – 2012, the "average annual return" for stocks is given as 6.3%. Let us see where may this data come from.

   - The S&P on Jan 1 1900 was 6.10, and on Jan 1 2013 1480.40. Calculate the annual return.

   - The Dow on Jan 2 1900 was 68.13, and on Dec 31 2012 13,104.14. Calculate the annual return.

   - The discrepancy with the data from the book comes from our inability to take into the account *dividends*. They varied over years, and it may be quite a complicated procedure to incorporate them into this calculation.

     In order to appreciate the growth, however, one should take inflation into the account: the buying power of a 1900 dollar was quite different from that of 2012.

     After all that, these long range numbers become a bit fuzzy, and difficult to make sense of, but we are not into stock trading, we are here into calculations instead.

   - What if we end up today, another 4 years later? S&P on Jan 1 2017 was 2275.12, and Dow on Dec 30 2016 was 19762.60. Calculate the annual returns.

   - It seems that, in the last 4 years, Dow did better than previously. Calculate its annual return during the last 4 years.

   - What about the performance of S&P during these recent years? Calculate its annual return during the last 4 years.

   - The calculation above demonstrate, and common sense suggest that better performance over a period of time improves overall performance. We will look at how exactly these numbers add up. We will consider stocks because they are volatile, and examples become more impressive.

2. Assume that XYZ stock lost 80% of its value over a first year, and gained 80% over a second year. Calculate its annual return during these 2 years, and find out that it is not at all zero. Disregard dividends.

   - Over the period of two years the price has been multiplied by a factor of 1.8 (first year), and then again multiplied by a factor of 0.2 (second year). What is the ratio A/P after all?

   - Use our formula
   $$\text{annual return} = (A/P)^{(1/Y)} - 1$$
   to calculate the annual return.

   - The performance is the same as if the tock was loosing 40% of its value every year. Check that.

   - That suggest that simply averaging the performance over years is not a good idea. Why people still so frequently do that? That is because the change in value over a year is seldom that big, and naive averaging almost works if the percentage changes are small.

     Repeat the previous calculations with 0.8% loss and gain over the two consecutive years and convince yourself that the price remained almost flat (with zero annual return) over the two years period.

   - Assume again that the stock has lost 80% of its value over the first year. How much should it gain over the second year in order make it up and to end up with zero annual return over the period of two years?

3. Assume again that XYZ stock has lost 10% of its value over the first year. How much should it gain over the second year in order make it up and to end up with at least 10% annual return over the period of two years? How far your answer is from naive and incorrect 20%?

4. BACK TO SAVING PLANS. The book provides (p.219) a derivation of the saving plan formula. The whole thing boils down to a nice formula "*sum of geometric series*" which we now try to work out. We for a quantity x, and a positive integer number n, we want to calculate the sum which we call S

$$S = 1 + x + x^2 + x^3 + x^4 + \ldots + x^{n-1} + x^n$$

Of course, one can simply calculate the sum if one needs it, but we want a shortcut, or a formula for that.

- Let us start with an experiment. Take $x = 2$, and try to calculate our sum for various n, namely $n = 1, 2, 3, 4 \ldots$. Write your answers down and try to guess a pattern. The existence of such pattern suggests that some shortcut really exists.

- Now let us look at our sum closer. If we multiply it by x, it changes, but we can keep the changes under our control:

$$S \times x = x + x^2 + x^3 + \ldots + x^n + x^{n+1}$$

  Specifically, 1 disappeared, but an extra term x^{n+1} showed up. The rest, whatever lengthy it was, remains the same. We can thus produce Sx from S simply subtracting 1 and adding x^{n+1}:

$$Sx = S - 1 + x^{n+1}$$

  This is an equation which relates the three quantities under discussion, namely S, x, and n. We are interested to find S in terms of x and n which means we want to solve this equation for S. That is easy:

$$S = \frac{x^{n+1} - 1}{x - 1}$$

- Again set $x = 2$, and try now to calculate this same sum S with the formula for $n = 1, 2, 3, 4 \ldots$. Convince yourself that the values coincides with those you calculated above, and the pattern which you have probably guessed before is now explained.