Worksheet 2.6 Factorizing Algebraic Expressions

Section 1 Finding Factors

Factorizing algebraic expressions is a way of turning a sum of terms into a product of smaller ones. The product is a multiplication of the factors. Sometimes it helps to look at a simpler case before venturing into the abstract. The number 48 may be written as a product in a number of different ways:

$$48 = 3 \times 16 = 4 \times 12 = 2 \times 24$$

So too can polynomials, unless of course the polynomial has no factors (in the way that the number 23 has no factors). For example:

$$x^{3} - 6x^{2} + 12x - 8 = (x - 2)^{3} = (x - 2)(x - 2)(x - 2) = (x - 2)(x^{2} - 4x + 4)$$

where $(x-2)^3$ is in fully factored form.

Occasionally we can start by taking common factors out of every term in the sum. For example,

$$3xy + 9xy^{2} + 6x^{2}y = 3xy(1) + 3xy(3y) + 3xy(2x)$$
$$= 3xy(1 + 3y + 2x)$$

Sometimes not all the terms in an expression have a common factor but you may still be able to do some factoring.

Example 1:

$$9a^2b + 3a^2 + 5b + 5b^2a = 3a^2(3b+1) + 5b(1+ba)$$

Example 2:

$$10x^{2} + 5x + 2xy + y = 5x(2x + 1) + y(2x + 1)$$
 Let $T = 2x + 1$
= $5xT + yT$
= $T(5x + y)$
= $(2x + 1)(5x + y)$

Example 3:

$$x^{2} + 2xy + 5x^{3} + 10x^{2}y = x(x + 2y) + 5x^{2}(x + 2y)$$
$$= (x + 5x^{2})(x + 2y)$$
$$= x(1 + 5x)(x + 2y)$$

- 1. Factorize the following algebraic expressions:
 - (a) 6x + 24
 - (b) $8x^2 4x$
 - (c) $6xy + 10x^2y$
 - (d) $m^4 3m^2$
 - (e) $6x^2 + 8x + 12yx$

For the following expressions, factorize the first pair, then the second pair:

- (f) $8m^2 12m + 10m 15$
- (g) $x^2 + 5x + 2x + 10$
- (h) $m^2 4m + 3m 12$
- (i) $2t^2 4t + t 2$
- (j) $6y^2 15y + 4y 10$

Section 2 Some standard factorizations

Recall the distributive laws of section 1.10.

Example 1:

$$(x+3)(x-3) = x(x-3) + 3(x-3)$$

$$= x^2 - 3x + 3x - 9$$

$$= x^2 - 9$$

$$= x^2 - 3^2$$

Example 2:

$$(x+9)(x-9) = x(x-9) + 9(x-9)$$

$$= x^{2} - 9x + 9x - 81$$

$$= x^{2} - 81$$

$$= x^{2} - 9^{2}$$

Notice that in each of these examples, we end up with a quantity in the form $A^2 - B^2$. In example 1, we have

$$A^{2} - B^{2} = x^{2} - 9$$

= $(x+3)(x-3)$

where we have identified A = x and B = 3. In example 2, we have

$$A^2 - B^2 = x^2 - 81$$

= $(x+9)(x-9)$

where we have identified A = x and B = 9. The result that we have developed and have used in two examples is called the difference of two squares, and is written:

$$A^2 - B^2 = (A + B)(A - B)$$

The next common factorization that is important is called a perfect square. Notice that

$$(x+5)^{2} = (x+5)(x+5)$$

$$= x(x+5) + 5(x+5)$$

$$= x^{2} + 5x + 5x + 25$$

$$= x^{2} + 10x + 25$$

$$= x^{2} + 2(5x) + 5^{2}$$

The perfect square is written as:

$$(x+a)^2 = x^2 + 2ax + a^2$$

Similarly,

$$(x-a)^{2} = (x-a)(x-a)$$

$$= x(x-a) - a(x-a)$$

$$= x^{2} - ax - ax + a^{2}$$

$$= x^{2} - 2ax + a^{2}$$

For example,

$$(x-7)^{2} = (x-7)(x-7)$$

$$= x(x-7) - 7(x-7)$$

$$= x^{2} - 7x - 7x + 7^{2}$$

$$= x^{2} - 14x + 49$$

- 1. Expand the following, and collect like terms:
 - (a) (x+2)(x-2)
 - (b) (y+5)(y-5)
 - (c) (y-6)(y+6)
 - (d) (x+7)(x-7)
 - (e) (2x+1)(2x-1)
 - (f) (3m+4)(3m-4)
 - (g) (3y+5)(3y-5)
 - (h) (2t+7)(2t-7)
- 2. Factorize the following:
 - (a) $x^2 16$
 - (b) $y^2 49$
 - (c) $x^2 25$
 - (d) $4x^2 25$

- (e) $16 y^2$
- (f) $m^2 36$
- (g) $4m^2 49$
- (h) $9m^2 16$
- 3. Expand the following and collect like terms:
 - (a) (x+5)(x+5)
 - (b) (x+9)(x+9)
 - (c) (y-2)(y-2)
 - (d) (m-3)(m-3)

- (e) (2m+5)(2m+5)
- (f) (t+10)(t+10)
- (g) $(y+8)^2$
- (h) $(t+6)^2$

- 4. Factorize the following:
 - (a) $y^2 6y + 9$
 - (b) $x^2 10x + 25$
 - (c) $x^2 + 8x + 16$
 - (d) $x^2 + 20x + 100$

- (e) $m^2 + 16m + 64$
- (f) $t^2 30t + 225$
- (g) $m^2 12m + 36$
- (h) $t^2 + 18t + 81$

Section 3 Introduction to Quadratics

In the expression $5t^2 + 2t + 1$, t is called the variable. Quadratics are algebraic expressions of one variable, and they have degree two. Having degree two means that the highest power of the variable that occurs is a squared term. The general form for a quadratic is

$$ax^2 + bx + c$$

Note that we assume that a is not zero because if it were zero, we would have bx + c which is not a quadratic: the highest power of x would not be two, but one. There are a few points to make about the quadratic $ax^2 + bx + c$:

- 1. a is the coefficient of the squared term and $a \neq 0$.
- 2. b is the coefficient of x and can be any number.
- 3. c is the called the constant term (even though a and b are also constant), and can be any number.

Quadratics may factor into two linear factors:

$$ax^2 + bx + c = a(x+k)(x+l)$$

where (x + k) and (x + l) are called the linear factors.

- 1. Which of the following algebraic expressions is a quadratic?
 - (a) $x^2 3x + 4$
- (c) $x^3 6x + 2$
- (e) $x^2 4$

- (b) $4x^2 + 6x 1$
- (d) $\frac{1}{x^2} + 2x + 1$
- (f) $6x^2$

Section 4 Factorizing Quadratics

Before we start factorizing quadratics, it would be a good idea to look for a pattern.

$$(x+2)(x+4) = x^2 + 4x + 2x + 8$$

= $x^2 + 6x + 8$

Notice that the numbers 2 and 4 add to give 6 and multiply to give 8.

$$(x+5)(x-3) = x^2 - 3x + 5x - 15$$

= $x^2 + 2x - 15$

Notice that the numbers 5 and -3 add to give 2 and multiply to give -15.

Let's try to factorize expressions similar to those above, where we will start with the expression in its expanded out form. To factorize the expression $x^2 + 7x + 12$, we will try to find numbers that multiply to give 12 and add to give 7. The numbers that we come up with are 3 and 4, so we write

$$x^{2} + 7x + 12 = (x+3)(x+4)$$

This equation should be verified by expanding the right hand side.

Example 1 : Factorize $x^2 + 9x + 14$.

We attempt to find two numbers that add to give 9 and multiply to give 14, and the numbers that do this are 2 and 7. Therefore

$$x^2 + 9x + 14 = (x+2)(x+7)$$

Again, this equation shouldn't be believed until the right hand side is expanded, and is shown to equal $x^2 + 9x + 14$.

Example 2: Factorize $x^2 + 7x - 18$.

We attempt to find two numbers that add to give 7 and multiply to give -18 (notice the minus!). The numbers that do this are -2 and 9. Therefore

$$x^2 + 7x - 18 = (x - 2)(x + 9)$$

This equation shouldn't be believed until the right hand side is expanded, and is shown to equal $x^2 + 7x - 18$.

1. Factorize the following quadratics:

(a)
$$x^2 + 4x + 3$$

(b)
$$x^2 + 15x + 44$$

(c)
$$x^2 + 11x - 26$$

(d)
$$x^2 + 7x - 30$$

(e)
$$x^2 + 10x + 24$$

(f)
$$x^2 - 14x + 24$$

(g)
$$x^2 - 7x + 10$$

(h)
$$x^2 - 5x - 24$$

(i)
$$x^2 + 2x - 15$$

(j)
$$x^2 - 2x - 15$$

The method that we have just described to factorize quadratics will work, if at all, only in the case that the coefficient of x^2 is 1. For other cases, we will need to factorize by

- 1. Using the 'ACE' method, or by
- 2. Using the quadratic formula

The 'ACE' method (pronounced a-c), unlike some other methods, is clear and easy to follow, as each step leads logically to the next. If you can expand an expression like (3x+4)(2x-3), then you will be able to follow this technique.

Example	Factorize $6x^2 - x - 12$
 Multiply the first term 6x² by the last term (-12) Find factors of -72x² which add to -x. 	$-72x^{2}$ $(-9x)(8x) = -72x^{2}$ $-9x + 8x = -x$
 3: Return to the original expression and replace -x with -9x + 8x. 4: Factorize (6x²-9x) and (8x-12). 5: One common factor is (2x - 3). The other factor, (3x+4), is found by dividing each term by (2x - 3). 	$6x^{2} - x - 12$ $= 6x^{2} - 9x + 8x - 12$ $= 3x(2x - 3) + 4(2x - 3)$ $= (2x - 3)(3x + 4)$
6: Verify the factorization by expansion	$(3x+4)(2x-3)$ $= 3x(2x-3) + 4(2x-3)$ $= 6x^2 - 9x + 8x - 12$ $= 6x^2 - x - 12$

Example 3 : Factorize $4x^2 + 21x + 5$.

- 1. Multiply first and last terms: $4x^2 \times 5 = 20x^2$
- 2. Find factors of $20x^2$ which add to 21x and multiply to give $20x^2$. These are 20x and x.
- 3. Replace 21x in the original expression with 20x + x:

$$4x^2 + 21x + 5 = 4x^2 + 20x + x + 5$$

4. Factorize the first two terms and the last two terms

$$4x^2 + 20x + x + 5 = 4x(x+5) + (x+5)$$

5. Factorize further:

$$4x(x+5) + (x+5) = (x+5)(4x+1)$$

Exercises:

1. Factorize the following quadratics using the 'ACE' method:

(a)
$$2x^2 + 11x + 12$$

(f)
$$2x^2 - 5x - 3$$

(b)
$$3x^2 + 16x + 5$$

(g)
$$3x^2 - 10x - 8$$

(c)
$$6x^2 + 17x + 12$$

(h)
$$3x^2 - 11x - 20$$

(d)
$$2x^2 + 9x + 10$$

(i)
$$5x^2 + 17x + 6$$

(e)
$$12x^2 + 11x + 2$$

(i)
$$10x^2 + 19x + 6$$

Section 5 The quadratic formula

When there is no obvious whole-number solution to the quadratic factorization, the quadratic formula must be used. It can be shown by the method of completing the square that the solutions to $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If we let the roots be k and l, say, then

$$k = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$
$$l = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Then

$$ax^2 + bx + c = a(x - k)(x - l)$$

When factorizing using this method be sure to multiply throughout by the coefficient of x^2 .

Example 1: Factorize $x^2 + 5x + 3$. We try the 'ACE' method. The obvious factors of $3x^2$ are 3x and 1x, which won't add to 5x. We abandon this method, and go to

the quadratic formula. Note that a = 1, b = 5, and c = 3.

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(3)}}{2(1)}$$
$$= -\frac{5}{2} \pm \frac{\sqrt{13}}{2}$$

so that the two roots are

$$k_1 = \frac{-5 + \sqrt{13}}{2}$$
 and $k_2 = \frac{-5 - \sqrt{13}}{2}$

Then

$$x^{2} + 5x + 3 = \left(x - \frac{-5 + \sqrt{13}}{2}\right)\left(x - \frac{-5 - \sqrt{13}}{2}\right)$$

Example 2 : Factorize $2x^2 - x - 5$.

Note that a=2, b=-1, and c=-5. Then the solutions to $2x^2-x-5=0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{(-1)^2 - 4 \times 2 \times (-5)}}{2 \times 2}$$

$$= \frac{1 \pm \sqrt{41}}{4}$$

So the two factors of $2x^2 - x - 5$ are

$$(x - \frac{1 + \sqrt{41}}{4})$$
 and $(x - \frac{1 - \sqrt{41}}{4})$

and so the factorization is

$$2x^{2} - x - 5 = 2\left(x - \frac{1 + \sqrt{41}}{4}\right)\left(x - \frac{1 - \sqrt{41}}{4}\right)$$

This right hand side of this equation should be expanded before it is believed!

1. Factorize the following quadratics using the quadratic formula:

(a)
$$3x^2 + 2x - 4$$

(f)
$$5x^2 + 7x - 2$$

(b)
$$x^2 + 3x + 1$$

(g)
$$3x^2 + 5x - 4$$

(c)
$$2x^2 + 8x + 3$$

(h)
$$2x^2 + 4x + 1$$

(d)
$$3x^2 + 5x + 1$$

(i)
$$5x^2 + 2x - 2$$

(e)
$$3x^2 + 6x + 2$$

(j)
$$2x^2 + x - 7$$

Section 6 Uses of factorization

We can use factorization of expressions in a variety of ways. One way is to simplify algebraic fractions.

Example 1:

$$\frac{x^2 - 9}{x - 3} = \frac{(x - 3)(x + 3)}{(x - 3)}$$
$$= \frac{x - 3}{x - 3} \times (x + 3)$$
$$= x + 3$$

Example 2:

$$\frac{x}{x^2 + 4x + 4} + \frac{x}{x + 2} = \frac{x}{(x + 2)^2} + \frac{x}{x + 2}$$

$$= \frac{x}{(x + 2)^2} + \frac{x}{x + 2} \times \frac{x + 2}{x + 2}$$

$$= \frac{x}{(x + 2)^2} + \frac{x^2 + 2x}{(x + 2)^2}$$

$$= \frac{x^2 + x + 2x}{(x + 2)^2}$$

$$= \frac{x(x + 3)}{(x + 2)^2}$$

Another way of using factorization is in solving quadratic equations.

Example 3 : Solve $(x + 3)^2 = x + 5$.

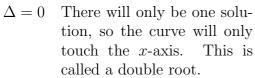
$$x^{2} + 6x + 9 = x + 5$$
$$x^{2} + 5x + 4 = 0$$

$$(x+4)(x+1) = 0$$

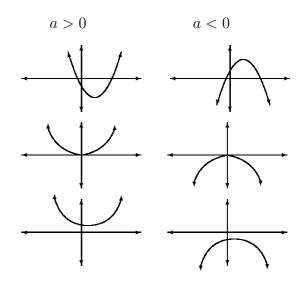
This is true when either x = -4 or x = -1. In other words, just one of the factors needs to be zero for the original equation that we started with to be true.

It is a good idea to know what to expect from the equation by first examining the discriminant $\Delta = b^2 - 4ac$. This is the expression under the square-root sign in the quadratic formula. Given the equation $y = ax^2 + bx + c$, and using our knowledge of square roots, we find the following:

 $\Delta > 0$ There will be 2 distinct solutions, so the curve crosses the x-axis twice.



 $\Delta < 0$ The curve does not touch the x-axis. We will deal with this case in detail later.



- 1. Factorize and then simplify the following algebraic expressions:
 - $\left(\mathbf{a}\right) \ \frac{x^2 + 3x}{x + 3}$
 - (b) $\frac{6x^2-8}{2x}$
 - (c) $\frac{x^2+3x+2}{3x+6}$
 - (d) $\frac{x^2-7x-18}{x^2-6x-27}$
 - (e) $\frac{x^2-16}{2x+8}$
 - (f) $\frac{3x^2-9x}{18x}$

- (g) $\frac{x^2-25}{x^2-3x-10}$
- (h) $\frac{2x^2-32}{x^2+6x+8}$
- (i) $\frac{x^3 9x^2}{3x 27}$
- (j) $\frac{2x^2-x-6}{x^2+x-6}$
- 2. Simplify the following by first finding a common denominator:
 - (a) $\frac{3}{x+2} + \frac{5x}{x+3}$
 - (b) $\frac{4x}{x-5} \frac{2}{x+2}$
 - (c) $\frac{x+1}{x+2} + \frac{x+3}{x+4}$
 - (d) $\frac{6}{x^2+5x+6} + \frac{2}{x^2+8x+15}$
 - (e) $\frac{4}{x^2-3x-10} \frac{1}{x^2+5x+6}$
 - (f) $\frac{x+3}{x^2+6x+9} \frac{2}{x+3}$
 - (g) $\frac{x^2+8x+15}{x^2+7x+10} \frac{x+3}{x+2}$
 - (h) $\frac{x^2-9}{2x+6} \frac{x^2}{x-3}$
 - $(i) \ \frac{x}{x+1} \frac{2x}{x+3}$
 - (j) $\frac{3x}{x^2+6x} \frac{2x+1}{x+6}$
- 3. Solve the following quadratic equations:
 - (a) $x^2 6x + 8 = 0$

(f) $2x^2 - x - 6 = 0$

(b) $x^2 + 8x + 15 = 0$

(g) $2x^2 - 13x - 7 = 0$

(c) $x^2 + 7x + 12 = 0$

(h) $3x^2 - 10x - 8 = 0$

(d) $x^2 + 9x - 22 = 0$

(i) $7x^2 + 13x - 2 = 0$

(e) $x^2 - 7x + 12 = 0$

- (j) $x^2 18x + 77 = 0$
- 4. Solve the following equations using the quadratic formula. Write the answers to two decimal places.
 - (a) $x^2 3x + 1 = 0$

(c) $3x^2 + 2x - 2 = 0$

(b) $2x^2 - 6x - 7 = 0$

(d) $2x^2 - 13x + 7 = 0$

Section 7 MULTIPLICATION AND DIVISION OF ALGEBRAIC FRACTIONS

We are often able to use factorization when we are multiplying or dividing algebraic expressions.

Example 1:

$$\frac{x^2 - 16}{x+3} \times \frac{x^2 + 5x + 6}{x+4} = \frac{(x+4)(x-4)}{x+3} \times \frac{(x+3)(x+2)}{x+4}$$
$$= (x-4)(x+2)$$

Example 2:

$$\frac{2x^2 + 12x + 16}{3x^2 + 6x} \times \frac{4x^2 - 100}{6x + 30} = \frac{2(x^2 + 6x + 8)}{3x(x + 2)} \times \frac{4(x^2 - 25)}{6(x + 5)}$$
$$= \frac{2(x + 4)(x + 2)}{3x(x + 2)} \times \frac{4(x + 5)(x - 5)}{6(x + 5)}$$
$$= \frac{4(x + 4)(x - 5)}{9x}$$

Example 3:

$$\frac{6x^2 + 9x}{x^2 + 8x + 15} \div \frac{4x + 6}{x^2 - 9} = \frac{6x^2 + 9x}{x^2 + 8x + 15} \times \frac{x^2 - 9}{4x + 6}$$
$$= \frac{3x(2x + 3)}{(x + 3)(x + 5)} \times \frac{(x + 3)(x - 3)}{2(2x + 3)}$$
$$= \frac{3x(x - 3)}{2(x + 5)}$$

- 1. Simplify the following expressions:

 - (a) $\frac{2x^2 5x 3}{x^2 + 2x} \times \frac{x^2 + 4x}{2x + 1}$ (b) $\frac{3x + 21}{x^2 7x + 12} \times \frac{4x 12}{9x + 63}$ (c) $\frac{x^2 + 2x}{x^2 + x 20} \times \frac{2x^2 5x 12}{5x + 10}$ (d) $\frac{3x^2 10x 8}{5x 15} \div \frac{2x^2 7x 4}{x^2 3x}$ (e) $\frac{x^2 16}{4x^2 1} \div \frac{x^2 + 11x + 28}{2x^2 + 5x + 2}$

Exercises 2.6 Factorizing Algebraic Expressions

1. Expand

(a)
$$(x+2)(x-2)$$

(b) (2x+4y)(2x-4y)

2. Factorize

(a)
$$144x^2 - y^2$$

(b)
$$16a^2 - 9b^2$$

(c)
$$x^2 + 7x + 10$$

(d)
$$b^2 + 9b + 14$$

(e)
$$x^2 + 14x + 49$$

(f)
$$x^2 + x - 6$$

(g)
$$2x^3 + 10x^2 - 48x$$

(h)
$$2x^2 + 7x + 3$$

(i)
$$5b^2 + 17b + 6$$

(j)
$$18y^2 + 12y + 2$$

(k)
$$12x^2 + 6x - 6$$

(1)
$$7a^2 - 9a - 10$$

(m)
$$-x^2 - x + 2$$

(n)
$$-2x^2 + 3x + 2$$

3. (a) Factorize $x^2 + 5x + 3$ using the quadratic formula.

- (b) Factorize $2x^2 x 1$.
- (c) Simplify $\frac{A^2-4}{3A-6}$.
- (d) Simplify $\frac{2x^2-8}{2x^2-x-6}$.
- (e) Simplify $\frac{x^2-x-6}{2xy} \times \frac{2x^2y}{x^2-9}$.
- (f) Simplify $\frac{2x^2+5x-3}{x^3+3x^2+2x} \div \frac{4x^2-1}{x^3+2x^2}$.
- (g) Simplify $\frac{3x}{x^2+6x+9} + \frac{x+3}{x^2-9}$.
- (h) Solve $x^2 + 2x 3 = 0$.

Answers 2.6

Section 1

1. (a)
$$6(x+4)$$

(b)
$$4x(2x-1)$$

(c)
$$2xy(3+5x)$$

(d)
$$m^2(m^2-3)$$

(e)
$$2x(3x+4+6y)$$

(f)
$$(4m+5)(2m-3)$$

(g)
$$(x+5)(x+2)$$

(h)
$$(m-4)(m+3)$$

Section 2

1. (a)
$$x^2 - 4$$

(b)
$$y^2 - 25$$

(c)
$$y^2 - 36$$

2. (a)
$$(x+4)(x-4)$$

(b)
$$(y+7)(y-7)$$

(c)
$$(x+5)(x-5)$$

3. (a)
$$x^2 + 10x + 25$$

(b)
$$x^2 + 18x + 81$$

(c)
$$y^2 - 4y + 4$$

4. (a)
$$(y-3)^2$$

(b)
$$(x-5)^2$$

(c)
$$(x+4)^2$$

(d)
$$x^2 - 49$$

(e)
$$4x^2 - 1$$

(f)
$$9m^2 - 16$$

(d)
$$(2x+5)(2x-5)$$

(e)
$$(4+y)(4-y)$$

(f)
$$(m+6)(m-6)$$

(d)
$$m^2 - 6m + 9$$

(e)
$$4m^2 + 20m + 25$$

(f)
$$t^2 + 20t + 100$$

(d)
$$(x+10)^2$$

(e)
$$(m+8)^2$$

(f)
$$(t-15)^2$$

(i) (t-2)(2t+1)

(j) (2y-5)(3y+2)

(g)
$$9y^2 - 25$$

(h)
$$4t^2 - 49$$

(g)
$$(2m+7)(2m-7)$$

(h)
$$(3m+4)(3m-4)$$

(g)
$$y^2 + 16y + 64$$

(h)
$$t^2 + 12t + 36$$

(g)
$$(m-6)^2$$

(h)
$$(t+9)^2$$

Section 3

Section 4 part 1

1. (a)
$$(x+3)(x+1)$$

(b)
$$(x+11)(x+4)$$

(c)
$$(x+13)(x-2)$$

(d)
$$(x+10)(x-3)$$

(e)
$$(x+6)(x+4)$$

(f)
$$(x-12)(x-2)$$

(g)
$$(x-5)(x-2)$$

(h)
$$(x-8)(x+3)$$

(j)
$$(x-5)(x+3)$$

(i)
$$(x+5)(x-3)$$

Section 4 part 2

1. (a)
$$(2x+3)(x+4)$$

(b)
$$(3x+1)(x+5)$$

(c)
$$(2x+3)(3x+4)$$

(d)
$$(2x+5)(x+2)$$

(e)
$$(3x+2)(4x+1)$$

(f)
$$(2x+1)(x-3)$$

(g)
$$(3x+2)(x-4)$$

(h)
$$(3x+4)(x-5)$$

(i)
$$(5x+2)(x+3)$$

(j)
$$(5x+2)(2x+3)$$

Section 5

1. (a)
$$3(x - \frac{-2 + \sqrt{52}}{6})(x - \frac{-2 - \sqrt{52}}{6})$$

(b)
$$(x - \frac{-3+\sqrt{5}}{2})(x - \frac{-3-\sqrt{5}}{2})$$

(c)
$$2(x - \frac{-8 + \sqrt{40}}{4})(x - \frac{-8 - \sqrt{40}}{4})$$

(d)
$$3(x - \frac{-5+\sqrt{13}}{6})(x - \frac{-5-\sqrt{13}}{6})$$

(e)
$$3(x - \frac{-6+\sqrt{12}}{6})(x - \frac{-6-\sqrt{12}}{6})$$

(f)
$$5(x-\frac{-7+\sqrt{89}}{10})(x-\frac{-7-\sqrt{89}}{10})$$

(g)
$$3(x - \frac{-5 + \sqrt{73}}{6})(x - \frac{-5 - \sqrt{73}}{6})$$

(h)
$$2(x - \frac{-4+\sqrt{8}}{4})(x - \frac{-4-\sqrt{8}}{4})$$

(i)
$$5(x - \frac{-2+\sqrt{44}}{10})(x - \frac{-2-\sqrt{44}}{10})$$

(j)
$$2(x - \frac{-1+\sqrt{57}}{4})(x - \frac{-1-\sqrt{57}}{4})$$

Section 6

1. (a)
$$x$$

(c)
$$\frac{x+1}{3}$$

(e)
$$\frac{x-4}{2}$$

$$(g) \frac{x+5}{x+2}$$

(i)
$$\frac{x^2}{2}$$

(a)
$$x$$
 (c) $\frac{x+1}{3}$ (e) $\frac{x-4}{2}$ (b) $\frac{3x^2-4}{x}$ (d) $\frac{x+2}{x+3}$ (f) $\frac{x-3}{6}$

(d)
$$\frac{x+2}{x+3}$$

(f)
$$\frac{x-3}{6}$$

(g)
$$\frac{x+5}{x+2}$$
 (i) $\frac{x^2}{3}$ (h) $\frac{2(x-4)}{x+2}$ (j) $\frac{2x+3}{x+3}$

$$(j) \quad \frac{2x+3}{x+3}$$

2. (a)
$$\frac{5x^2+13x+9}{(x+2)(x+3)}$$

(d)
$$\frac{2(4x+17)}{(x+2)(x+3)(x+5)}$$
 (h) $\frac{-x^2-6x+9}{2(x-3)}$

(e)
$$\frac{3x+17}{(x+2)(x+3)(x-5)}$$

(e)
$$\frac{3x+1t}{(x+2)(x+3)(x-5)}$$

(f) $\frac{-1}{x+3}$

(i)
$$\frac{-x(x-1)}{(x+1)(x+3)}$$

(c)
$$\frac{2(x^2+5x+5)}{(x+2)(x+4)}$$

(b) $\frac{2(2x^2+3x+5)}{(x+2)(x-5)}$

$$(j) \quad \frac{-2(x-1)}{(x+6)}$$

3. (a)
$$4, 2$$
 (c) $-4, -3$ (e) $4, 3$

(c)
$$-4, -3$$

(e)
$$4, 3$$

(g)
$$7, -\frac{1}{2}$$

(g)
$$7, -\frac{1}{2}$$
 (i) $-2, \frac{1}{7}$

(b)
$$-5, -3$$
 (d) $-11, 2$ (f) $-2, \frac{3}{2}$ (h) $4, -\frac{2}{3}$ (j) $11, 7$

$$(d) -11, 2$$

(f)
$$-2, \frac{3}{2}$$

(h)
$$4, -\frac{2}{3}$$

4. (a)
$$\frac{3\pm\sqrt{5}}{2}$$

(b)
$$\frac{3}{2} \pm \frac{\sqrt{92}}{4}$$

(b)
$$\frac{3}{2} \pm \frac{\sqrt{92}}{4}$$
 (c) $-\frac{1}{3} \pm \frac{\sqrt{28}}{6}$ (d) $\frac{13}{4} \pm \frac{\sqrt{113}}{4}$

(d)
$$\frac{13}{4} \pm \frac{\sqrt{113}}{4}$$

Section 7

1. (a)
$$\frac{(x+4)(x-3)}{x+2}$$
 (b) $\frac{4}{3(x-4)}$

(c)
$$\frac{x(2x+3)}{5(x+5)}$$

(e)
$$\frac{(x-4)(x+2)}{(2x-1)(x+7)}$$

(b)
$$\frac{4}{3(x-4)}$$

(d)
$$\frac{x(3x+2)}{5(2x+1)}$$

Exercises 2.6

1. (a)
$$x^2 - 4$$

2. (a)
$$(12x - y)(12x + y)$$

(b)
$$(4a - 3b)(4a + 3b)$$

(c)
$$(x+5)(x+2)$$

(d)
$$(b+7)(b+2)$$

(e)
$$(x+7)^2$$

(f)
$$(x+3)(x-2)$$

(g)
$$2x(x-3)(x+8)$$

3. (a)
$$\left[x - \left(\frac{-5 - \sqrt{13}}{2}\right)\right] \left[x - \left(\frac{-5 + \sqrt{13}}{2}\right)\right]$$
 or $\left(\frac{2x + 5 + \sqrt{13}}{2}\right) \left(\frac{2x + 5 - \sqrt{13}}{2}\right)$ or $\frac{1}{4}(2x + 5 + \sqrt{13})(2x + 5 - \sqrt{13})$

(b)
$$(2x+1)(x-1)$$

(c)
$$\frac{A+2}{3}$$

(c)
$$\frac{A+2}{3}$$
 (d) $\frac{2(x+2)}{2x+3}$

(b)
$$4x^2 - 16y^2$$

(h)
$$(x+3)(2x+1)$$

(i)
$$(b+3)(5b+2)$$

(j)
$$2(3y+1)^2$$

(k)
$$6(x+1)(2x-1)$$

(1)
$$(7a+5)(a-2)$$

(m)
$$(x+2)(1-x)$$

(n)
$$(2x+1)(2-x)$$

(e)
$$\frac{x(x+2)}{x+3}$$

(f)
$$\frac{x(x+3)}{(2x+1)(x+1)}$$

(g)
$$\frac{4x^2-3x+9}{(x+3)^2(x-3)}$$

(h)
$$x = -3 \text{ or } x = 1$$