THE SCHOOL DISTRICT OF PHILADELPHIA

Grade: 8 **Subject:** Math (worksheets taken from enVision Mathematics, Grade 8 and Pearson A1)

Topic: Analyze and Solve Linear Equations

What Your Student is Learning:

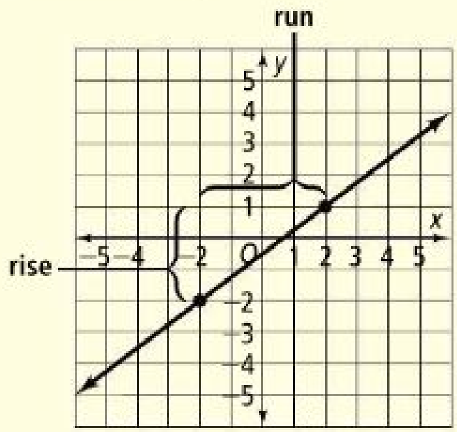
- Finding slope using a formula and a graph.
- Analyzing various slopes and describe their meaning.
- The equation of a line gives you the slope and the y-intercept.
- Make predictions based on scatter plots using the line of best fit and trends in the scatter plot.

Background and Context for Parents:

Slope

The slope of a line represents the constant rate of change of a linear function.

$$\text{slope} = \frac{\text{vertival change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}$$



The slope of the line above can be determined from the two points shown, $(-2, -2)$ and $(2, 1)$.

The vertical change between $(-2, -2)$ and $(2, 1)$ is the difference in the y-values, $1 - (-2) = 3$. The horizontal change is the difference in the x-values, $2 - (-2) = 4$. So, slope $= \frac{3}{4}$. It does not matter which point is identified as (x_1, y_1) and which one is defined as (x_2, y_2).

For example, set $(x_1, y_1) = (-2, -2)$ and $(x_2, y_2) = (2, 1)$.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{2 - (-2)} = \frac{1 + 2}{2 + 2} = \frac{3}{4}$$

Common Errors With Slope

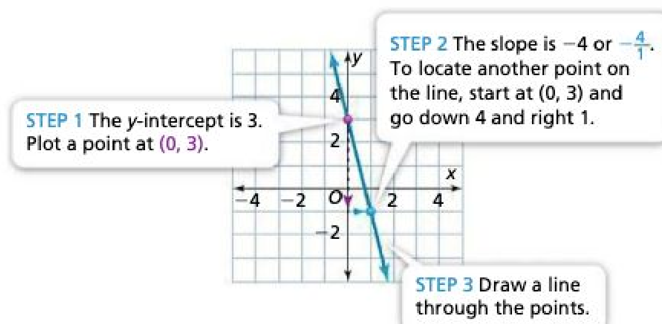
Although it does not matter which point is identified as (x_1, y_1) and which one is defined as (x_2, y_2), it is important that the same order is used for the numerator as for the denominator.

If students mix up the elements in the ordered pairs, the terms in the numerator and denominator are subtracted in reverse order from each other, and the slope is incorrect.

Represent Linear Equations

- **Equations** In Lesson 2-7, students relate slope to proportional relationships as they review the equation for a line passing through the origin. They write equations in the form $y = mx$ to represent given scenarios. In Lesson 2-9, students write the equation of a line in the form $y = mx + b$ to represent nonproportional relationships, substituting a value for one of the unknowns.
- **Graphs** In Lesson 2-7, students analyze and solve linear equations by plotting and graphing their solutions. In Lesson 2-8, they analyze structures needed to create models and make sense of the y-intercept. In Lesson 2-9, students graph nonproportional linear equations and write equations for a given graph.

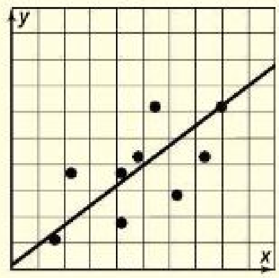
Graph the equation $y = -4x + 3$.



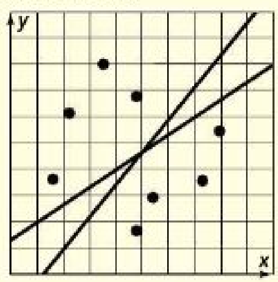
Scatter Plots

The line of best fit for a scatter plot can be drawn with varying degrees of precision. If you plot the data points using pencil and paper, you may simply draw a line of best fit by eye.

The stronger the correlation, the easier it is to draw the line of best fit by eye.



The weaker the correlation, the harder it is to draw the line of best fit by eye. It is not clear which of the lines best fit the scatter plot below.



Common Errors With Scatter Plots

Negative correlation does not mean lack of correlation but rather correlation in a negative direction: it refers to a scatter plot whose line of best fit has a negative slope.

Ways to support your student:

- Provide blank/ lined paper so they have enough space to try different strategies.
- Read the problem outloud to them.
- Watch the Khan Academy video together (linked below).
- Before giving your student the answer to their question or specific help, ask them "What have you tried so far?, What do you know?, What might be a next step?
- After your student has solved it, and before you tell them it's correct or not, have them explain to you how they got their solution and if they think their answer makes sense.

Online Resources for Students:

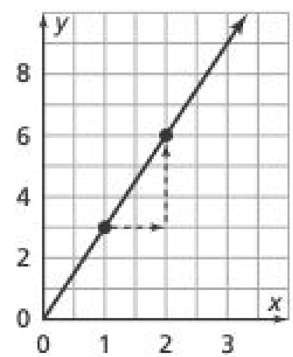
- **https://www.desmos.com/calculator** -Free online graphing calculator that allows students to check their work by inputting the equations of the lines. Remind students that unlike a graphing calculator, inputs into desmos don't have to be in y=mx+b (point slope form).
- https://www.khanacademy.org/math/8th-engage-ny/engage-8th-module-4 - Topics A-C (D is the next section)

Name ____________________________________

Additional Vocabulary Support
2-6

Use each of these terms or phrases once to complete the sentences.

proportional relationship	rise	run
slope	$x_2 - x_1$	$y_2 - y_1$



1. The __________ of a line is the measure of the steepness of the line.

2. The slope is the same as the unit rate and constant of proportionality in a ______________________________.

3. In the slope ratio, the change in the *x*-coordinates from one point to another on a line is the __________.

4. In the slope ratio, the change in the *y*-coordinates from one point to another on a line is the __________.

5. The slope between two points on a line can be found using the ratio $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$ or $\frac{\boxed{}}{\boxed{}}$.

Name ______________________________

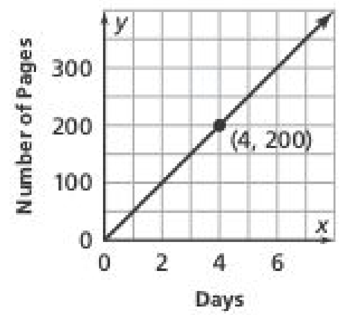
Reteach to Build Understanding
2-6

In order to finish reading a book by the assigned date, Caleb plans to read the same number of pages each day, as shown in the graph. What is the slope of the line?

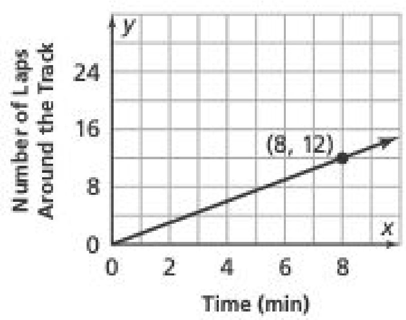
The rise is 200 pages. The run is 4 days.

$\frac{\text{rise}}{\text{run}} = \frac{200 \text{ pages}}{4 \text{ days}} = 50$ pages per day

The slope of the line is 50.



The graph relates the time in minutes and the number of laps Jerry must run around the track in order to meet his goal for the track meet. What is the slope of the line?



1. What is the rise between the points (0, 0) and (8, 12)?

2. What is the run between the points (0, 0) and (8, 12)?

3. What is the slope of the line?

On the Back!

4. Christina made a graph to describe the distance she walked on her backpacking trip. She plotted miles on the *y*-axis and the number of days on the *x*-axis. She graphed a point at (0, 0) and another point to show she walked 14 miles in 8 days and drew a line that passes through the points. What is the slope of the line?

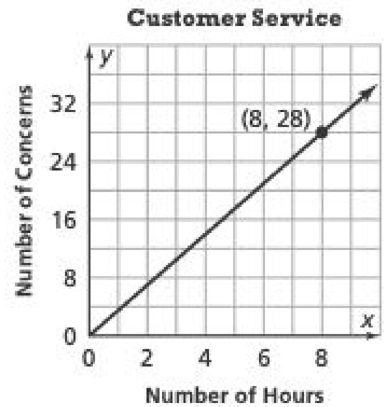
Name ______________________________

Reteach to Build Understanding
2-7

The manager of a customer service center made this graph to show the average number of customer concerns, y, he would like his department to resolve each hour, x. Write the equation of the line that represents this relationship.

The slope is $\frac{\text{rise}}{\text{run}} = \frac{28}{8} = 3.5$.

The equation of the line is $y = 3.5x$.



Each week, the same amount of money is automatically taken out of Geraldo's paycheck and deposited into his savings account. This graph shows the relationship between the total amount Geraldo has saved in dollars, y, to time in weeks, x. Write the equation of the line that represents the relationship.

1. The graph starts at (0, 0). What other point is shown on the graph?

2. What are the rise and run between (0, 0) and the point you identified in Exercise 1?

3. What is the slope of the line?

4. Write the equation of the line.

On the Back!

5. Penny wrote the same number of holiday cards each day for 6 days, and she wrote a total of 42 cards. Graph the line relating the number of cards, y, to the number of days, x. Write the equation of the line that represents the relationship?

Name ______________________________

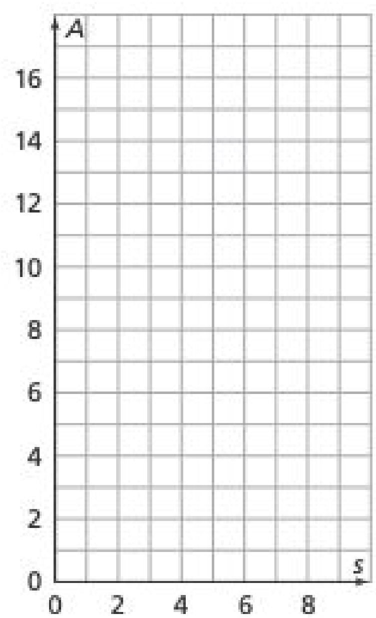
Enrichment
2-7

1. For each square below, s represents the side length. Find the area A and the perimeter P of each square.

$s = 1$ unit	$s = 2$ units	$s = 3$ units	$s = 4$ units
$A =$ ☐ sq. unit	$A =$ ☐ sq. units	$A =$ ☐ sq. units	$A =$ ☐ sq. units
$P =$ ☐ units	$P =$ ☐ units	$P =$ ☐ units	$P =$ ☐ units

2. Use the perimeters you found in Exercise 1 to graph each point (s, P) on a line. Write an equation to describe the relationship between a square's side length and its perimeter. Use the slope to justify your answer.

3. Use the areas you found in Exercise 1 to graph each point (s, A). Do the points lie on a line? Use the slope to justify your answer.



4. What formula do you use to find the area of a square, A, when given its side length, s? Rewrite this formula by replacing s with x and A with y. Is this equation in the form $y = mx$? Explain.

5. Which relationships are proportional? Select all that apply.

☐ The relationship between a square's side length and its perimeter

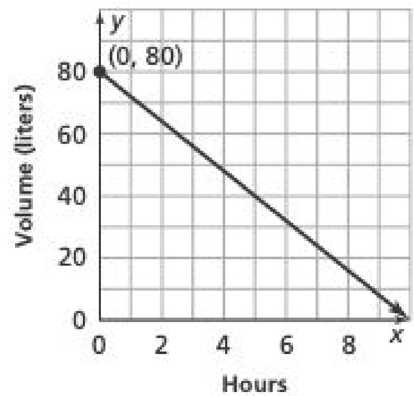
☐ The relationship between a square's side length and its area

Name ______________________________

Build Mathematical Literacy
2-8

Read the problem. Then answer the questions to help you understand the problem.

A pump is removing water from a tank at a constant rate. The graph shows the relationship between the volume of water in the tank and the number of hours. What is the *y*-intercept of the graph, and what does it represent?



1. Underline each question you need to answer.

2. On the graph, circle the point that corresponds to the *y*-intercept. Explain how to use this point to find the y-intercept.

3. What does the graph show about how the volume of water in the tank changes over time?

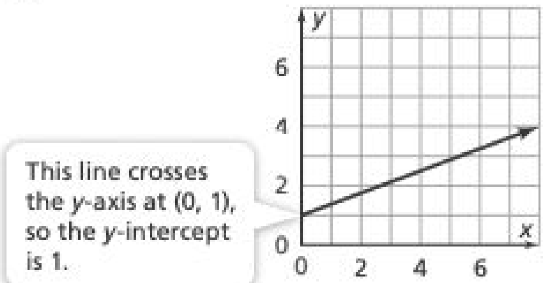
4. Does the *y*-intercept represent a starting value or an ending value? Explain.

5. On the graph, highlight the information that describes the quantity represented on the *y*-axis. How does this help you determine the meaning of the *y*-intercept?

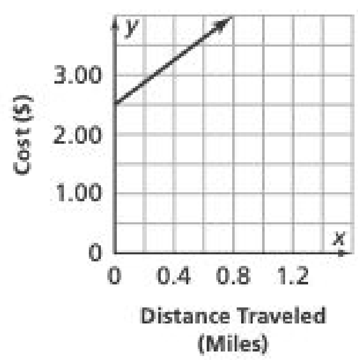
Name ______________________________

Reteach to Build Understanding
2-8

The y-intercept of a line is the y-coordinate of the point where the line crosses the y-axis.



In New York City, there is an initial fee for a daytime taxi plus a charge per mile traveled, as shown in the graph below. What is the y-intercept of the graph, and what does it represent?



1. At what point does the line cross the y-axis?

2. What is the y-intercept of the line?

3. The y-intercept gives the cost of a taxi for what distance traveled?

4. What does the y-intercept represent?

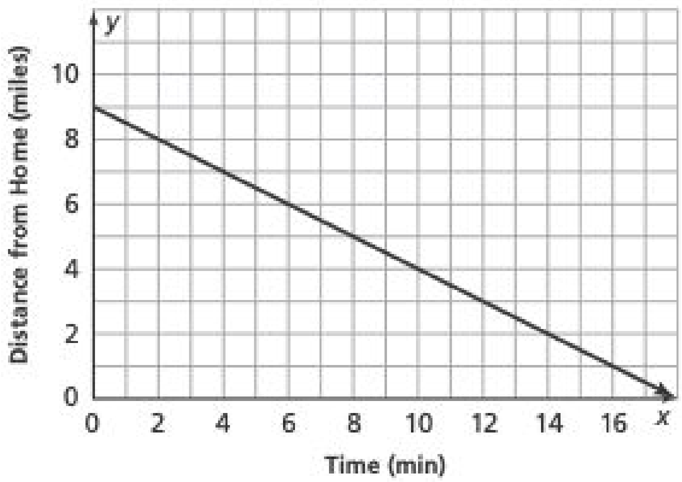
On the Back!

5. For a science experiment, Claudia observes the effects of temperature on a substance. The original temperature is 9°C, and Claudia increases the temperature by 2°C each hour. Then Claudia graphs the relationship between time and temperature. What is the y-intercept of Claudia's graph and what does it represent?

Name ______________________________

Enrichment
2-8

Karina leaves the soccer practice field to drive to her house, 9 miles away, at a speed of 30 miles per hour. Her distance from home is represented by the graph below. At the same time, her brother Jonah leaves their house and drives to the same practice field along the same route as Karina. His speed is 24 miles per hour.



1. What are Karina's and Jonah's speeds in miles per minute?

2. Graph Jonah's distance from home on the axes above. What is the *y*-intercept of your graph and what does it represent?

3. Write an equation that represents Jonah's distance from home.

4. What is the *y*-intercept of the graph that represents Karina's distance from home and what does it represent?

5. At what point do the two graphs intersect, and what does this mean in terms of the situation?

6. Which graph represents a proportional relationship? Explain.

Name ____________________

Build Mathematical Literacy
2-9

Answer the questions to help you understand characteristics of equations in the form of $y = mx$ and $y = mx + b$.

Characteristics of the Graph of the Equation	$y = mx$	$y = mx + b, b \neq 0$
Slope	m	m
y-intercept	0	b
$\frac{\text{change in } y}{\text{change in } x}$ is constant.	yes	yes
Shows a proportional relationship between x and y	yes	no

1. Circle the general forms of the equations of the lines shown in the table.

2. What information about slope is summarized in the table?

3. What information about the y-intercept is summarized in the table?

4. Why does the table have separate columns for the equations $y = mx$ and $y = mx + b$, where $b \neq 0$?

5. Write an equation that represents a proportional relationship when $m = 3$.

6. Write an equation that represents a relationship that is not proportional when $m = 3$ and $b = -6$.

Name ______________________________

Enrichment
2-9

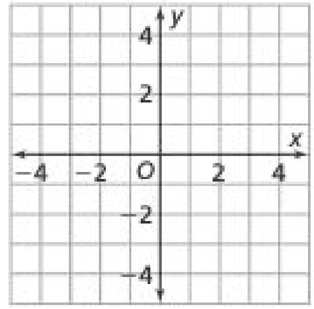
1. Graph the following equations on the coordinate plane at right.

$y = -\frac{1}{5}x - 2$

$y = x - 2$

$y = 2x - 2$

$y = -3x - 2$



2. What do all of the equations in Exercise 1 have in common? How is this shown in the graphs?

3. Write an equation for another line that shares the property you described in Exercise 2.

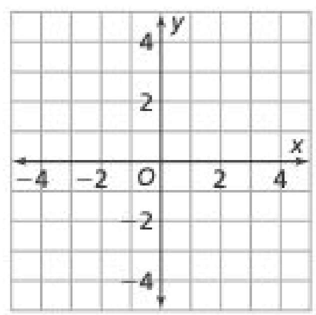
4. Graph the following equations on the coordinate plane at right.

$y = -\frac{1}{2}x + 3$

$y = -\frac{1}{2}x$

$y = -\frac{1}{2}x - 1$

$y = -\frac{1}{2}x - 2$



5. What do all of the equations in Exercise 4 have in common? How is this shown in the graphs?

6. Write an equation for a line that has each of the properties from Exercises 2 and 5.

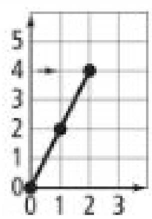
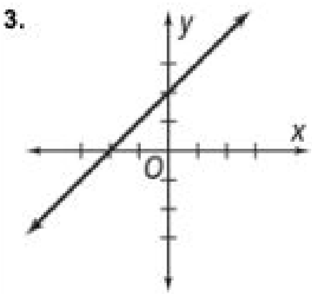
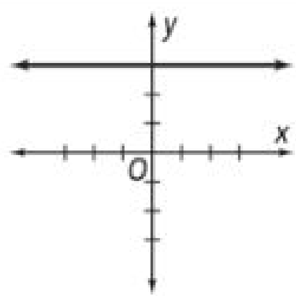
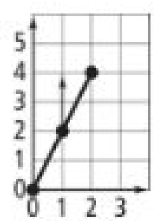
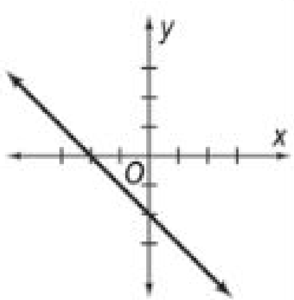
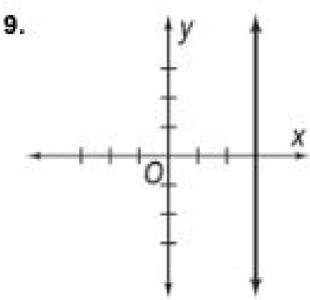
Name ______________________ Class ____________ Date ____________

5-1 Additional Vocabulary Support

Concept List

negative slope	**positive slope**	**rate of change**
rise	**run**	**slope**
slope formula	**slope of horizontal line**	**slope of vertical line**

Choose the concept from the list above that best represents the item in each box.

1. $\frac{y_2 - y_1}{x_2 - x_1}$	**2.** 	**3.** 
4. $\frac{\text{vertical change}}{\text{horizontal change}}$	**5.** 	**6.** 
7. 	**8.** $\frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$	**9.** 

5-1 Think About a Plan

Profit John's business made $4500 in January and $8600 in March. What is the rate of change in his profit for this time period?

Understanding the Problem

1. What is the formula for finding rate of change?

___

2. What are the two changing quantities that affect rate of change in this problem? What are the units of each quantity?

___

___

3. Will the rate of change be positive or negative? Explain.

___

___

Planning the Solution

4. Which quantity is the dependent variable? Which quantity is the independent variable? Explain.

___

___

5. What is the general equation that represents the rate of change?

___

Getting an Answer

6. Substitute values into your general equation and simplify. Show your work.

___

___

7. If you were to graph this relationship, what would the rate of change be in relation to your graph?

___

___

5-1 Reteaching

The rate of the vertical change to the horizontal change between two points on a line is called the slope of the line.

$$\textbf{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}$$

There are two special cases for slopes.

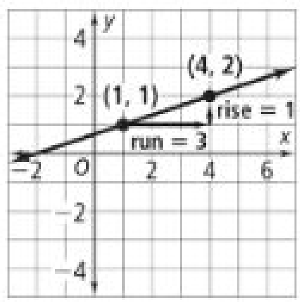
- A horizontal line has a slope of 0.
- A vertical line has an undefined slope.

Problem

What is the slope of the line?

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{1}{3}$$



The slope of the line is $\frac{1}{3}$.

In general, a line that slants upward from left to right has a positive slope.

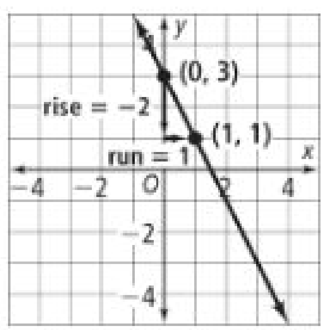
Problem

What is the slope of the line?

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{-2}{1}$$

$$= -2$$



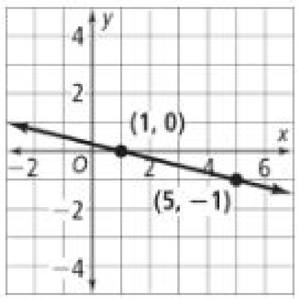
The slope of the line is –2.

In general, a line that slants downward from left to right has a negative slope.

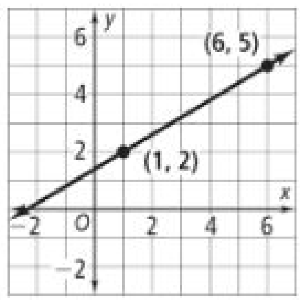
Exercises

Find the slope of each line.

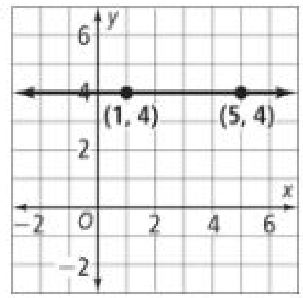
1.



2.



3.



Suppose one point on a line has the coordinates (x_1, y_1) and another point on the same line has the coordinates (x_2, y_2). You can use the following formula to find the slope of the line.

$$\textbf{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_2 - x_1 \neq 0$$

Problem

What is the slope of the line through $R(2, 5)$ and $S(-1, 7)$?

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7-5}{-1-2} \qquad \text{Let } y_2 = 7 \text{ and } y_1 = 5. \quad \text{Let } x_2 = -1 \text{ and } x_1 = 2.$$

$$= \frac{2}{-3} = -\frac{2}{3}$$

Exercises

Find the slope of the line that passes through each pair of points.

4. (0, 0), (4, 5)

5. (2, 4), (7, 8)

6. (–2, 0), (–3, 2)

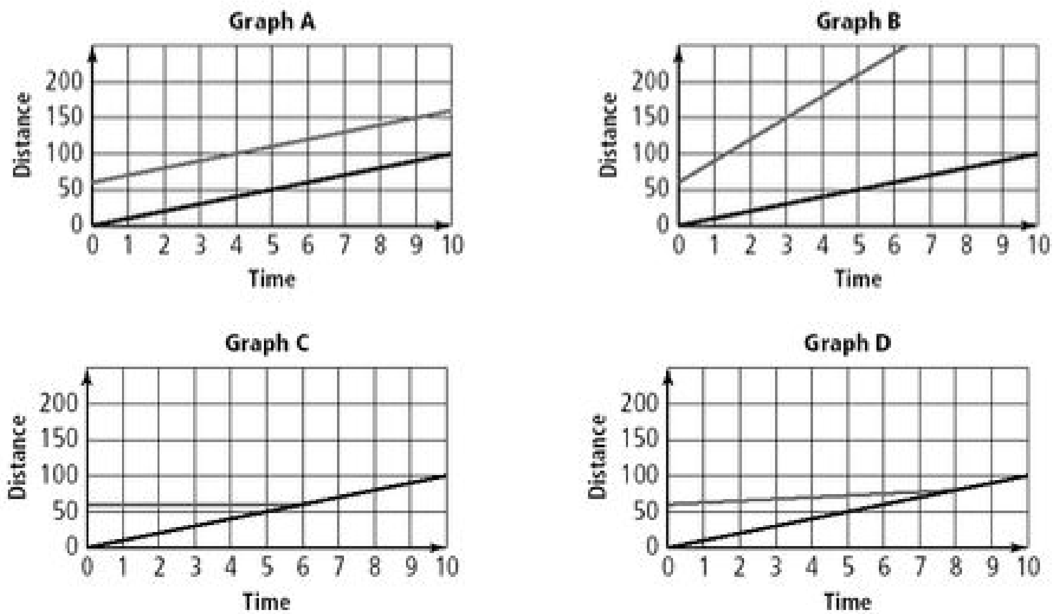
7. (–2, –3), (1, 1)

8. (1, 4), (2, –3)

9. (3, 2), (–5, 3)

5-1 Enrichment

A wildlife biologist is observing interactions among animals in a forest. She notices a fox sniffing around, in search of lunch. She also notices a rabbit happily chewing on some grass about 60 meters away. The fox looks up, notices the rabbit and starts heading towards it at a constant rate. There are a number of different possible outcomes regarding this situation and some are modeled in the graphs shown below. The fox's path is shown in black and the rabbit's is shown in gray.



1. Explain what is happening regarding rate of change and slope of each line.

2. In each graph, what will happen regarding the fox and rabbit? How does the slope assure this outcome?

3. Assume the rabbit was a bit confused and wound up running towards the fox at a rate of 10 meters per second. Sketch the graph. Note the slope. Determine the approximate time when the fox would catch the rabbit.

Name ______________________ Class ____________ Date ____________

Chapter 5 Find The Errors!

For use with Lessons 5-1 through 5-2

For each exercise, identify the error(s) in planning the solution or solving the problem. Then write the correct solution.

1. The table shows the price for different amounts of cashew nuts. What is the rate of change in price with respect to weight? What does that rate of change represent?

Weight (1bs.)	Price ($)
1	3.5
2	7
3	10.5
4	14

$$\text{rate of change} = \frac{\text{change in price}}{\text{change in weight}}$$

$$\frac{2-1}{7-3.5} \approx 0.286 \qquad \frac{3-2}{10.5-7} \approx 0.286 \qquad \frac{4-3}{14-10.5} \approx 0.286$$

The rate of change is approximately $0.286 per pound.

It represents the unit price of the cashews.

2. What is the slope of the line through (9, −3) and (7, −4)?

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-(-3)}{9-7} = -\frac{1}{2}$$

The slope of the line is $-\frac{1}{2}$.

3. Does the equation $-x + y = 3$ represent a direct variation? If so, find the constant of variation.

$$-x + y = 3$$

$$y = x + 3$$

The equation is a direct variation because y is always 3 more than x.

4. A rectangular photo is 3 inches high and 5 inches long. As the photo is enlarged, the height varies directly with the width. What is an equation that relates the length (in inches) x and the height (in inches) y?

$$y = kx$$

$$5 = 3k$$

$$\frac{5}{3} = k$$

The equation $y = \frac{5}{3}x$ gives the height y (in inches) of an enlargement that is x inches wide.

Name ______________________ Class __________ Date __________

5-3 Think About a Plan

Hobbies Suppose you are doing a 5000-piece puzzle. You have already placed 175 pieces. Every minute you place 10 more pieces.

a. Write an equation in slope-intercept form to model the number of pieces placed. Graph the equation.

b. After 50 more minutes, how many pieces will you have placed?

Understanding the Problem

1. Is this relationship linear? How do you know?

____

____

Planning the Solution

2. How many pieces have you already placed? What does this represent in the slope-intercept form?

____

____

3. What two quantities are used to find the rate of change or slope? What is the slope of this relationship?

____

____

Getting an Answer

4. Use your answers in Steps 2 and 3 to write an equation in slope-intercept form to model the number of pieces placed.

____

5. Graph the equation on a coordinate grid.

6. How many pieces will you have placed after 50 more minutes?

____

5-3 Reteaching

The **slope-intercept form** of a linear equation is $y = mx + b$. In this equation, m is the slope and b is the y-intercept.

Problem

What are the slope and y-intercept of the graph of $y = -2x - 3$?

The equation is solved for y, but it is easier to determine the y-intercept if the right side is written as a sum instead of a difference.

$y = -2x - 3$	
$y = -2x + (-3)$	Write the subtraction as addition.

The slope is -2 and the y-intercept is -3.

Problem

What is an equation for the line with slope $\frac{2}{3}$ and y-intercept 9?

When the slope and y-intercept are given, substitute the values into the slope-intercept form of a linear equation.

$y = mx + b$	
$y = \frac{2}{3}x + 9$	Substitute $\frac{2}{3}$ for m and 9 for b.

Problem

What is an equation in slope-intercept form for the line that passes through the points (1, −3) and (3, 1)?

Substitute the two given points into the slope formula to find the slope of the line.

$$m = \frac{1-(-3)}{3-1} = \frac{4}{2} = 2$$

Then substitute the slope and the coordinates of one of the points into the slope-intercept form to find b.

$y = mx + b$	Use slope-intercept form.
$-3 = 2(1) + b$	Substitute 2 for m, 1 for x, and −3 for y.
$-5 = b$	Solve for b.

Substitute the slope and y-intercept into the slope-intercept form.

$y = mx + b$	Use slope-intercept form.
$y = 2x + (-5)$	Substitute 2 for m and −5 for b.

Reteaching (continued)

Exercises

Find the slope and y-intercept of the graph of each equation.

1. $y = \frac{1}{2}x + 7$

2. $y = -5x + 1$

3. $y = -\frac{2}{5}x - 3$

4. $y = x + 5$

5. $y = \frac{1}{6}x - 2$

6. $y = 4x$

Write an equation for the line with the given slope m and y-intercept b.

7. $m = -3, b = 7$

8. $m = \frac{2}{3}, b = 8$

9. $m = 4, b = -3$

10. $m = -\frac{1}{5}, b = -1$

11. $m = -\frac{5}{6}, b = 0$

12. $m = 7, b = -2$

Write an equation in slope-intercept form for the line that passes through the given points.

13. (1, 3) and (2, 5)

14. (2,−1) and (4, 0)

15. (1, 2) and (2,−1)

16. (1,−5) and (3,−3)

17. (3, 3) and (6, 5)

18. (4,−3) and (8,−4)

Name ______________________ Class __________ Date __________

Chapter 5 Find The Errors!

For use with Lessons 5-3 through 5-5

For each exercise, identify the error(s) in planning the solution or solving the problem. Then write the correct solution.

1. What is an equation in slope-intercept form of a line that passes through the points (0, 5) and (4, 0)?

Step 1 Find the slope.

$\frac{0-5}{4-0} = -\frac{5}{4}$

Step 2 Find b.

$y = mx + b$

$5 = -\frac{5}{4}(4) + b$

$5 = -5 + b$

$10 = b$

Step 3 Substitute into the slope-intercept form.

$y = mx + b$

$y = -\frac{5}{4}x + 10$

An equation of the line is $y = -\frac{5}{4}x + 10$.

2. A student starts with \$20 and saves \$10 each week. What graph models the amount of money she has after x weeks?

Step 1 Identify the slope and the y-intercept.

The slope is 20.

The y-intercept is 10.

Step 2 Substitute into the slope-intercept form.

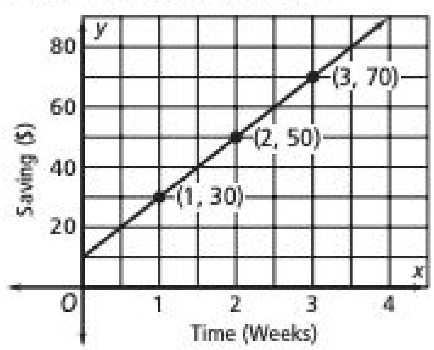
$y = mx + b$

$y = 20x + 10$

Step 3 Graph the equation.

The y-intercept is 10. Graph the point (0, 10).

The slope is 20. Graph a point 20 units above and 1 unit to the right of (0, 10).



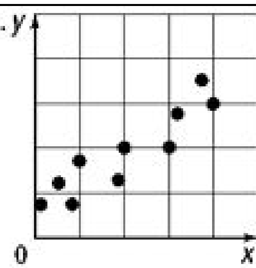
5-7 Additional Vocabulary Support

Concept List

causal relationship	correlation coefficient	extrapolation
interpolation	line of best fit	negative correlation
no correlation	positive correlation	trend line

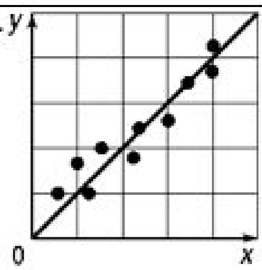
Choose the concept from the list above that best represents the item in each box.

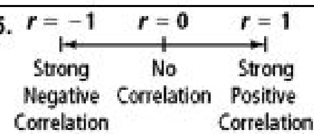
1. LinReg $(ax + b)$

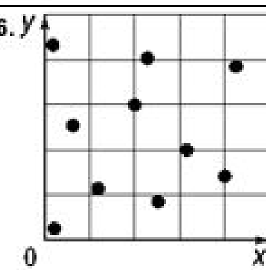
2. 

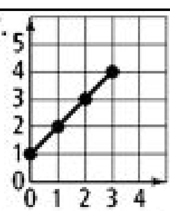
3.

Practice Time (hours)	Grade in Band Class
10	C
20	B
40	A

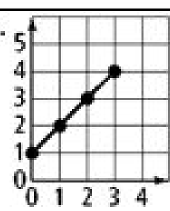
4. 

5. 

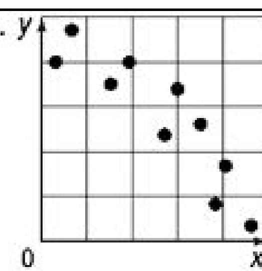
6. 

7. 

Estimate the value for y when $x = 3.5$.

8. 

Estimate the value for y when $x = 1.5$.

9. 

5-7 Think About a Plan

U.S. Population Use the data below.

Estimated Population of the United States (thousands)

Year	2000	2001	2002	2003	2004	2005	2006
Male	138,482	140,079	141,592	142,937	144,467	145,973	147,512
Female	143,734	145,147	146,533	147,858	149,170	150,533	151,886

SOURCE: U.S. Census Bureau

a. Make a scatter plot of the data pairs (male population, female population).
b. Draw a trend line and write its equation.
c. Use your equation to predict the U.S. female population if the U.S. male population increases to 150,000,000.
d. Reasoning Consider a scatter plot of the data pairs (year, male population). Would it be reasonable to use this scatter plot to predict the U.S. male population in 2035? Explain your reasoning.

1. Make a scatter plot of the data pairs using the male population for the x-coordinates and the female population for the y-coordinates for each year.

2. Draw the trend line onto the scatter plot.

3. How do you determine the equation of a trend line? What is the equation of this trend line? Show your work.

____

4. Substitute 150 for x to predict the female population.______________________

5. Make a scatter plot of the data pairs (year, male population).

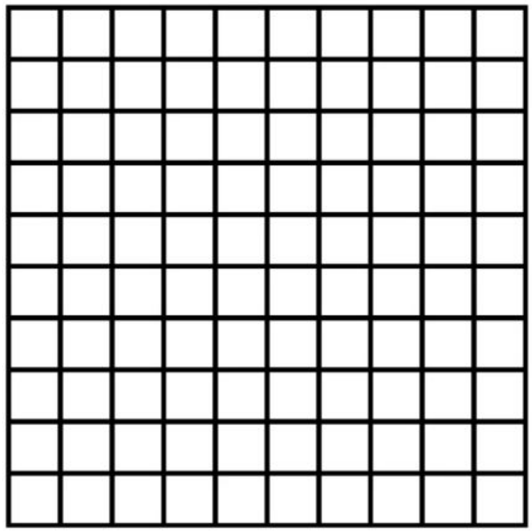
6. Would it be reasonable to use this scatter plot to predict the U.S. male population in 2035? Explain your reasoning.

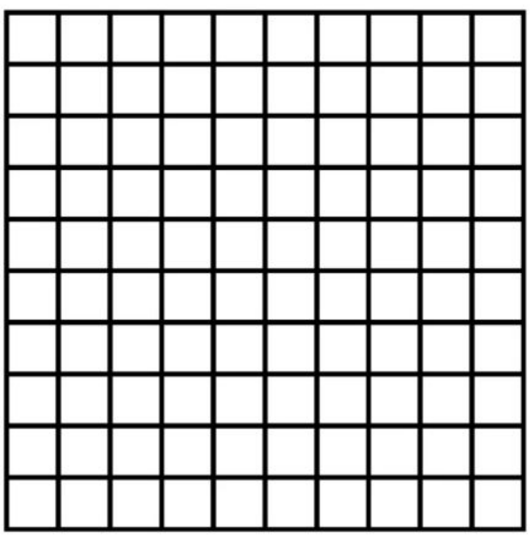
____

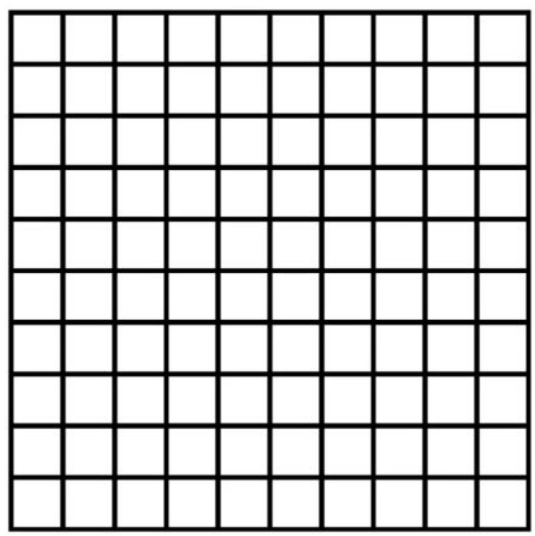
____

THE SCHOOL DISTRICT OF PHILADELPHIA

Graph paper for the “Think About a Plan” and the “Reteaching”







5-7 Reteaching

A **scatter plot** is a graph that relates two different sets of data by displaying them as ordered pairs. A scatter plot can show a trend or correlation, which may be either positive or negative. Or the scatter plot may show no trend or correlation. It is often easier to determine whether there is a correlation by looking at a scatter plot than it is to determine by looking at the numerical data.

If the points on a scatter plot generally slope up to the right, the two sets of data have a positive correlation. If the points on a scatter plot generally slope down to the right, the two sets of data have a negative correlation. If the points on a scatter plot do not seem to generally rise or fall in the same direction, the two sets of data have no correlation.

Problem

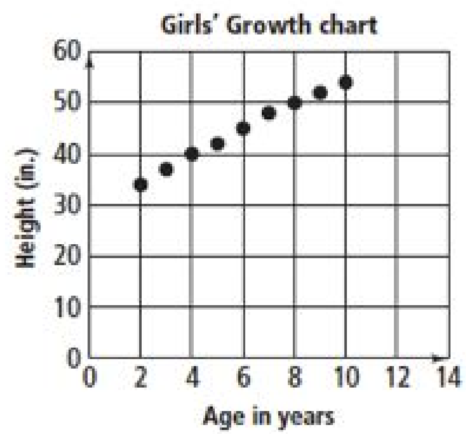
The table below compares the average height of girls at different ages. Make a scatter plot of the data. What type of correlation does the scatter plot indicate?

Age in years	2	3	4	5	6	7	8	9	10
Height in Inches	34	37	40	42	45	48	50	52	54

Treat the data as ordered pairs. The average height of a 2-year old girl is 34 inches, so one ordered pair is (2, 34). Plot this point. Then plot (3, 37), (4, 40), (5, 42), (6, 45), (7, 48), (8, 50), (9, 52), and (10, 54).

Notice that the height increases as the age increases. There is a positive correlation for this data.

A **trend line** is a line on a scatter plot that is drawn near the points. You can use a trend line to estimate other values.



5-7 Reteaching (continued)

Problem

Draw a trend line for the scatter plot in the previous problem. What is the equation for your trend line? What would you estimate to be the average height of a girl who is 12 years old?

Draw a line that seems to fit the data. The line drawn for this data goes through (4, 40) and (8, 50). Use these points to write an equation.

$$m = \frac{50 - 40}{8 - 4} = 2.5$$

Use the point-slope form of the line.

$$y - y_1 = m(x - x_1)$$

$$y - 40 = 2.5(x - 4)$$

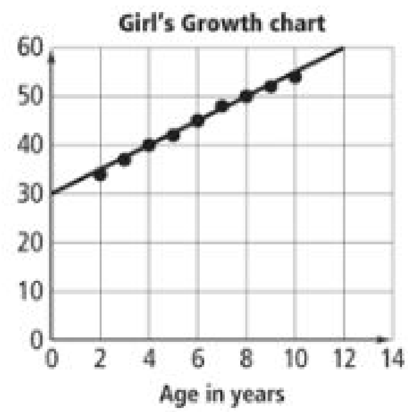
$$y - 40 = 2.5x - 10$$

$$y = 2.5x + 30$$

Use this equation to estimate the average height of 12-year old girls.

$$y = 2.5(12) + 30$$

$$y = 60$$



Exercises

Ryan practices throwing darts. From each distance listed below, he throws 10 darts and records how many times he hits the center.

Distance (in feet)	2	5	7	8	10	12	15
Number of Center Hits	10	9	8	6	5	1	2

1. Use the space at the right to make a scatter plot of the data.

2. Describe the type of correlation that is shown in the scatter plot.

3. Draw a trend line.

4. What equation represents your trend line?

5. How many hits do you estimate Ryan would make from 6 feet?

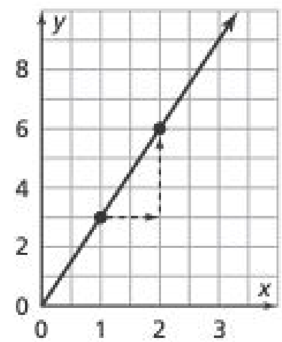
Answer Keys

Name ______________________________

Additional Vocabulary Support
2-6

Use each of these terms or phrases once to complete the sentences.

proportional relationship	rise	run
slope	$x_2 - x_1$	$y_2 - y_1$



1. The **slope** of a line is the measure of the steepness of the line.

2. The slope is the same as the unit rate and constant of proportionality in a **proportional relationship**.

3. In the slope ratio, the change in the x-coordinates from one point to another on a line is the **run**.

4. In the slope ratio, the change in the y-coordinates from one point to another on a line is the **rise**.

5. The slope between two points on a line can be found using the ratio $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$ or $\frac{y_2 - y_1}{x_2 - x_1}$.

Name ______________________________

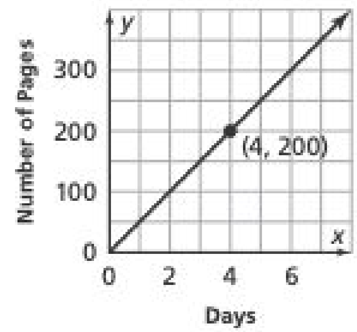
Reteach to Build Understanding
2-6

In order to finish reading a book by the assigned date, Caleb plans to read the same number of pages each day, as shown in the graph. What is the slope of the line?

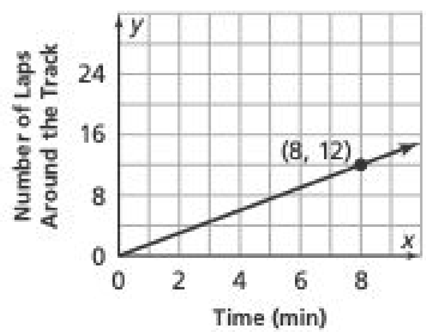
The rise is 200 pages. The run is 4 days.

$\frac{\text{rise}}{\text{run}} = \frac{200 \text{ pages}}{4 \text{ days}} = 50$ pages per day

The slope of the line is 50.



The graph relates the time in minutes and the number of laps Jerry must run around the track in order to meet his goal for the track meet. What is the slope of the line?



1. What is the rise between the points (0, 0) and (8, 12)?
   12 times around the track

2. What is the run between the points (0, 0) and (8, 12)?
   8 minutes

3. What is the slope of the line?
   $\frac{\textbf{12 times around the track}}{\textbf{8 minutes}} = \frac{3}{2}$

On the Back!

4. Christina made a graph to describe the distance she walked on her backpacking trip. She plotted miles on the *y*-axis and the number of days on the *x*-axis. She graphed a point at (0, 0) and another point to show she walked 14 miles in 8 days and drew a line that passes through the points. What is the slope of the line?
   $\frac{7}{4}$, or $1\frac{3}{4}$

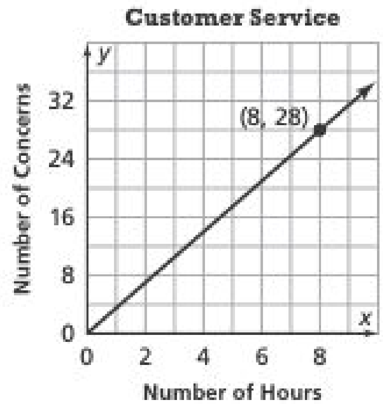
Name ____________________

Reteach to Build Understanding
2-7

The manager of a customer service center made this graph to show the average number of customer concerns, y, he would like his department to resolve each hour, x. Write the equation of the line that represents this relationship.

The slope is $\frac{\text{rise}}{\text{run}} = \frac{28}{8} = 3.5$.

The equation of the line is $y = 3.5x$.



Each week, the same amount of money is automatically taken out of Geraldo's paycheck and deposited into his savings account. This graph shows the relationship between the total amount Geraldo has saved in dollars, y, to time in weeks, x. Write the equation of the line that represents the relationship.

1. The graph starts at (0, 0). What other point is shown on the graph?
   (4, 600)

2. What are the rise and run between (0, 0) and the point you identified in Exercise 1?
   rise = 600; run = 4

3. What is the slope of the line?
   $\frac{600}{4} = 150$

4. Write the equation of the line.
   $y = 150x$

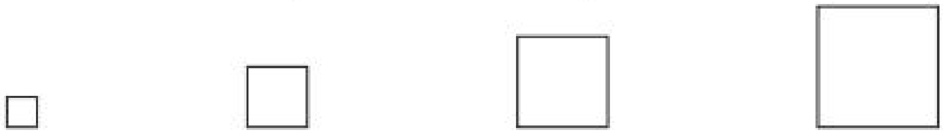
On the Back!

5. Penny wrote the same number of holiday cards each day for 6 days, and she wrote a total of 42 cards. Graph the line relating the number of cards, y, to the number of days, x. Write the equation of the line that represents the relationship?
   Check students' graphs; $y = 7x$.

Name ______________________________

Enrichment
2-7

1. For each square below, s represents the side length. Find the area A and the perimeter P of each square.



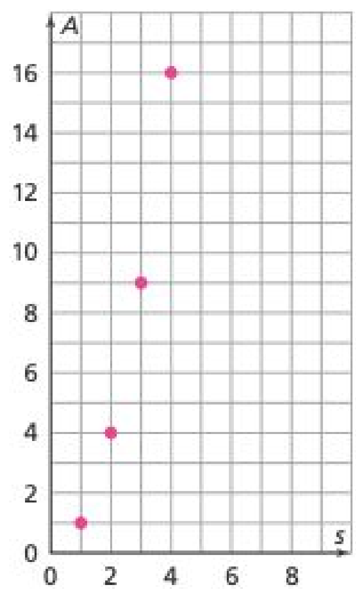
$s = 1$ unit	$s = 2$ units	$s = 3$ units	$s = 4$ units
$A =$ **1** sq. unit	$A =$ **4** sq. units	$A =$ **9** sq. units	$A =$ **16** sq. units
$P =$ **4** units	$P =$ **8** units	$P =$ **12** units	$P =$ **16** units

2. Use the perimeters you found in Exercise 1 to graph each point (s, P) on a line. Write an equation to describe the relationship between a square's side length and its perimeter. Use the slope to justify your answer.

Check students' graphs; The slope between each pair of points (s, P) is equal to 4; $P = 4s$.

3. Use the areas you found in Exercise 1 to graph each point (s, A). Do the points lie on a line? Use the slope to justify your answer.

No; Sample answer: The slope between (1, 1) and (2, 4) is different than the slope between (2, 4) and (3, 9).



4. What formula do you use to find the area of a square, A, when given its side length, s? Rewrite this formula by replacing s with x and A with y. Is this equation in the form $y = mx$? Explain.

$A = s^2$; $y = x^2$; no; The equation $y = x^2$ has an exponent other than 1, and the equation $y = mx$ does not.

5. Which relationships are proportional? Select all that apply.

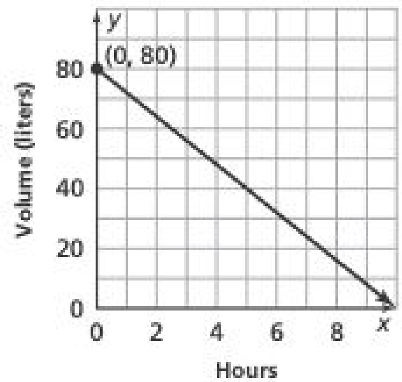
- [x] The relationship between a square's side length and its perimeter
- [ ] The relationship between a square's side length and its area

Name ______________________________

Build Mathematical Literacy
2-8

Read the problem. Then answer the questions to help you understand the problem.

A pump is removing water from a tank at a constant rate. The graph shows the relationship between the volume of water in the tank and the number of hours. What is the *y*-intercept of the graph, and what does it represent?



1. Underline each question you need to answer.
   Check students' work.

2. On the graph, circle the point that corresponds to the *y*-intercept. Explain how to use this point to find the y-intercept.
   Check students' work; The *y*-coordinate of this point is the *y*-intercept.

3. What does the graph show about how the volume of water in the tank changes over time?
   It shows that the volume of water decreases over time.

4. Does the *y*-intercept represent a starting value or an ending value? Explain.
   A starting value; The time is 0 hours at the *y*-intercept.

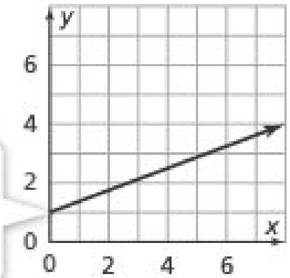
5. On the graph, highlight the information that describes the quantity represented on the *y*-axis. How does this help you determine the meaning of the *y*-intercept?
   Check students' work; Knowing that the *y*-axis represents volume tells you that the *y*-intercept is the starting volume of water in the tank.

Name ______________________________

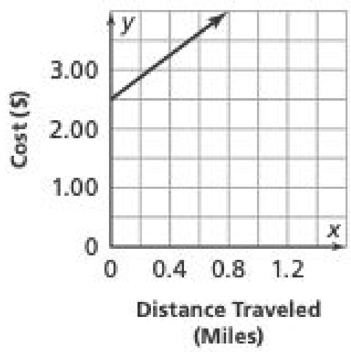
Reteach to Build Understanding
2-8

The *y*-intercept of a line is the *y*-coordinate of the point where the line crosses the *y*-axis.

This line crosses the *y*-axis at (0, 1), so the *y*-intercept is 1.



In New York City, there is an initial fee for a daytime taxi plus a charge per mile traveled, as shown in the graph below. What is the *y*-intercept of the graph, and what does it represent?



1. At what point does the line cross the *y*-axis?
   (0, 2.5)
2. What is the *y*-intercept of the line?
   2.5
3. The *y*-intercept gives the cost of a taxi for what distance traveled?
   0 miles
4. What does the *y*-intercept represent?
   The initial fee for a daytime taxi, $2.50

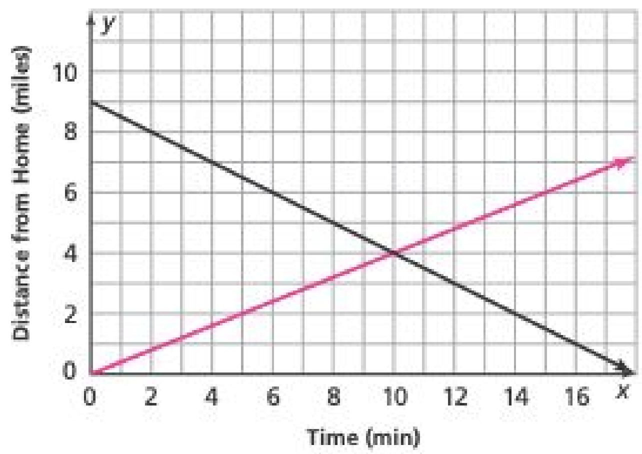
On the Back!

5. For a science experiment, Claudia observes the effects of temperature on a substance. The original temperature is 9°C, and Claudia increases the temperature by 2°C each hour. Then Claudia graphs the relationship between time and temperature. What is the *y*-intercept of Claudia's graph and what does it represent?
   9; the original temperature of the substance, 9°C

Name ______________________________

Enrichment
2-8

Karina leaves the soccer practice field to drive to her house, 9 miles away, at a speed of 30 miles per hour. Her distance from home is represented by the graph below. At the same time, her brother Jonah leaves their house and drives to the same practice field along the same route as Karina. His speed is 24 miles per hour.



1. What are Karina's and Jonah's speeds in miles per minute?
   Karina's speed is 0.5 mile per minute; Jonah's speed is 0.4 mile per minute.
2. Graph Jonah's distance from home on the axes above. What is the *y*-intercept of your graph and what does it represent?
   See graph above. 0; Jonah's initial distance from home is 0 (because he starts at home).
3. Write an equation that represents Jonah's distance from home.
   $y = 0.4x$
4. What is the *y*-intercept of the graph that represents Karina's distance from home and what does it represent?
   9; Karina's initial distance from home is 9 miles.
5. At what point do the two graphs intersect, and what does this mean in terms of the situation?
   (10, 4); After 10 minutes, they are both 4 miles from home.
6. Which graph represents a proportional relationship? Explain.
   Jonah's graph; Sample answer: The *y*-intercept is always 0 for proportional relationships.

Build Mathematical Literacy

2-9

Name ______________________________

Answer the questions to help you understand characteristics of equations in the form of $y = mx$ and $y = mx + b$.

Characteristics of the Graph of the Equation	$y = mx$	$y = mx + b, b \neq 0$
Slope	m	m
y-intercept	0	b
$\frac{\text{change in } y}{\text{change in } x}$ is constant.	yes	yes
Shows a proportional relationship between x and y	yes	no

1. Circle the general forms of the equations of the lines shown in the table.
   Check students' work.
2. What information about slope is summarized in the table?
   Slope is represented by m in each equation.
3. What information about the y-intercept is summarized in the table?
   For an equation in the form $y = mx$, the y-intercept is 0. For an equation in the form $y = mx + b$, the y-intercept is b.
4. Why does the table have separate columns for the equations $y = mx$ and $y = mx + b$, where $b \neq 0$?
   Sample answer: The equation $y = mx$ represents a proportional relationship, while the equation $y = mx + b$, where $b \neq 0$, does not.
5. Write an equation that represents a proportional relationship when $m = 3$.
   $y = 3x$
6. Write an equation that represents a relationship that is not proportional when $m = 3$ and $b = -6$.
   $y = 3x - 6$

Name ______________________

Enrichment **2-9**

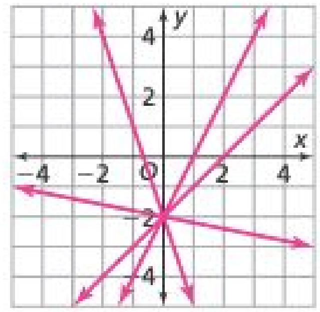
1. Graph the following equations on the coordinate plane at right.

$y = -\frac{1}{5}x - 2$

$y = x - 2$

$y = 2x - 2$

$y = -3x - 2$



2. What do all of the equations in Exercise 1 have in common? How is this shown in the graphs?

All of the equations are written in the form $y = mx + b$ with $b = -2$. The point $(0, -2)$ is on all of the lines, which represents the same y-intercept for each graph.

3. Write an equation for another line that shares the property you described in Exercise 2.

Sample answer: $y = -x - 2$

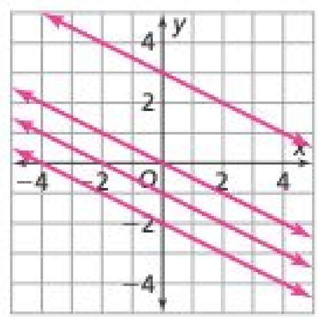
4. Graph the following equations on the coordinate plane at right.

$y = -\frac{1}{2}x + 3$

$y = -\frac{1}{2}x$

$y = -\frac{1}{2}x - 1$

$y = -\frac{1}{2}x - 2$



5. What do all of the equations in Exercise 4 have in common? How is this shown in the graphs?

All of the equations are written in the form $y = mx + b$ with $m = -\frac{1}{2}$. All of the lines have a slope of $-\frac{1}{2}$.

6. Write an equation for a line that has each of the properties from Exercises 2 and 5.

$y = -\frac{1}{2}x - 2$

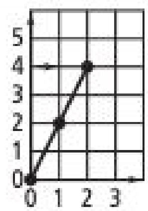
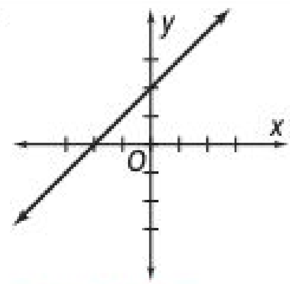
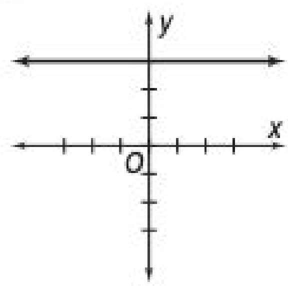
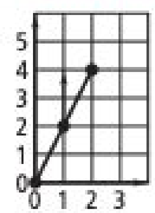
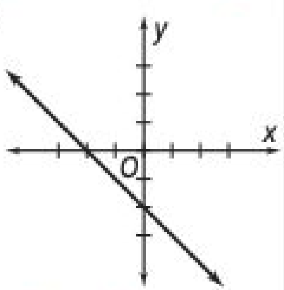
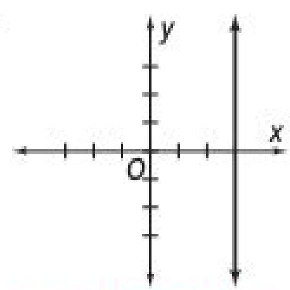
5-1 Additional Vocabulary Support

Rate of Change and Slope

Concept List

negative slope	positive slope	rate of change
rise	run	slope
slope formula	slope of horizontal line	slope of vertical line

Choose the concept from the list above that best represents the item in each box.

1. $\frac{y_2 - y_1}{x_2 - x_1}$ slope formula	2.  run	3.  positive slope
4. $\frac{\text{vertical change}}{\text{horizontal change}}$ slope or rate of change	5.  slope of horizontal line	6.  rise
7.  negative slope	8. $\frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$ rate of change or slope	9.  slope of vertical line

5-1 Think About a Plan

Rate of Change and Slope

Profit John's business made \$4500 in January and \$8600 in March. What is the rate of change in his profit for this time period?

Understanding the Problem

1. What is the formula for finding rate of change?

   $\frac{\text{change in dependent variable}}{\text{change in independent variable}}$

2. What are the two changing quantities that affect rate of change in this problem? What are the units of each quantity?

   profit, time; dollars, months

3. Will the rate of change be positive or negative? Explain.

   positive; profit increase over time

Planning the Solution

4. Which quantity is the dependent variable? Which quantity is the independent variable? Explain.

   Profit depends on time, so profit is dependent and time is independent

5. What is the general equation that represents the rate of change?

   $\text{rate} = \frac{\text{change in } y}{\text{change in } x}$

Getting an Answer

6. Substitute values into your general equation and simplify. Show your work.

   $r = \$2050$ per month

7. If you were to graph this relationship, what would the rate of change be in relation to your graph?

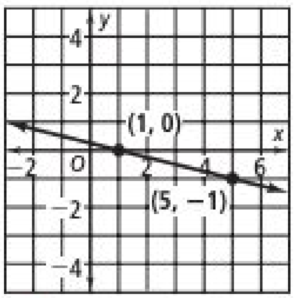
   the slope

5-1 Reteaching (continued)

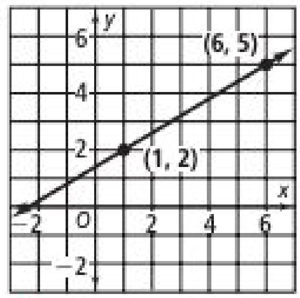
Rate of Change and Slope

Exercises

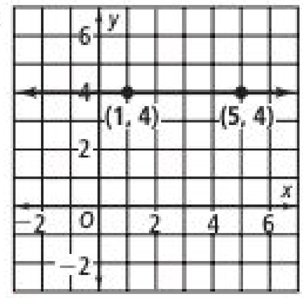
Find the slope of each line.

1. 

$-\frac{1}{4}$

2. 

$\frac{3}{5}$

3. 

0

Suppose one point on a line has the coordinates (x_1, y_1) and another point on the same line has the coordinates (x_2, y_2). You can use the following formula to find the slope of the line.

$$\textbf{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_2 - x_1 \neq 0$$

Problem

What is the slope of the line through $R(2, 5)$ and $S(-1, 7)$?

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7 - 5}{-1 - 2}$$ Let $y_2 = 7$ and $y_1 = 5$. Let $x_2 = -1$ and $x_1 = 2$.

$$= \frac{2}{-3} = -\frac{2}{3}$$

Exercises

Find the slope of the line that passes through each pair of points.

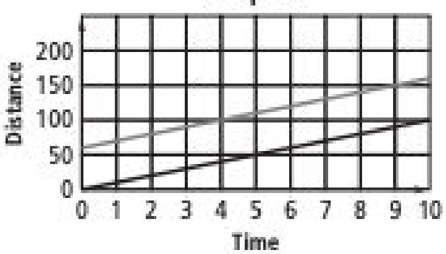
4. $(0, 0), (4, 5)$ $\frac{5}{4}$
5. $(2, 4), (7, 8)$ $\frac{4}{5}$
6. $(-2, 0), (-3, 2)$ -2
7. $(-2, -3), (1, 1)$ $\frac{4}{3}$
8. $(1, 4), (2, -3)$ -7
9. $(3, 2), (-5, 3)$ $-\frac{1}{8}$

5-1 Enrichment

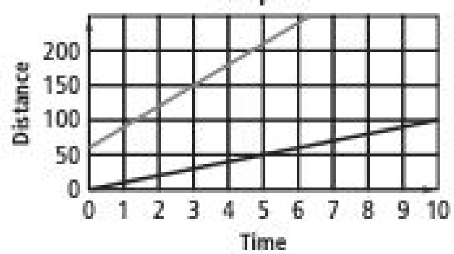
Rate of Change and Slope

A wildlife biologist is observing interactions among animals in a forest. She notices a fox sniffing around, in search of lunch. She also notices a rabbit happily chewing on some grass about 60 meters away. The fox looks up, notices the rabbit and starts heading towards it at a constant rate. There are a number of different possible outcomes regarding this situation and some are modeled in the graphs shown below. The fox's path is shown in black and the rabbit's is shown in gray.

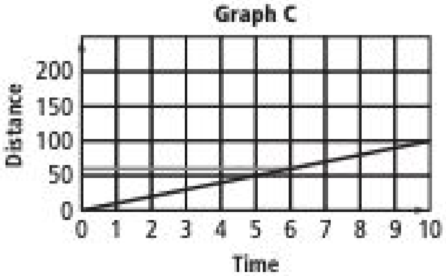
Graph A



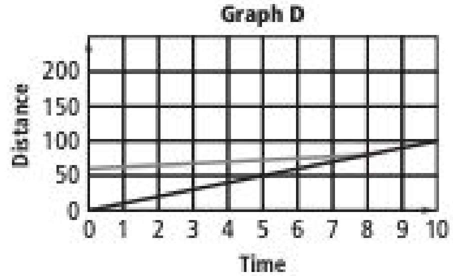
Graph B



Graph C



Graph D



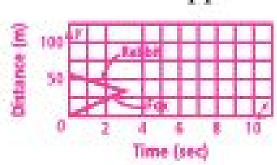
1. Explain what is happening regarding rate of change and slope of each line.

A: Fox and rabbit run at the same rate, 10 m/s, so the slopes are the same.
B: Fox runs at 10 m/s and rabbit runs at 30 m/s, so the rabbit's graph is steeper.
C: Rabbit remains still, and fox runs at 10 m/s, so the rabbit's graph is horizontal.
D: Fox runs at 10 m/s and rabbit runs at about 2 m/s, so the fox's graph is steeper.

2. In each graph, what will happen regarding the fox and rabbit? How does the slope assure this outcome?

In A and B the rabbit gets away; in C and D the fox catches the rabbit. When the slope of the fox's line is greater than the slope of the rabbit's line, then the lines intersect at a time $t > 0$ and the fox catches the rabbit. When the slope of the fox's line is less than the slope of the rabbit's line, the lines do not intersect for $t > 0$ and the rabbit escapes.

3. Assume the rabbit was a bit confused and wound up running towards the fox at a rate of 10 meters per second. Sketch the graph. Note the slope. Determine the approximate time when the fox would catch the rabbit.



slope: −10, 3 s

ANSWERS

Chapter 5 Find The Errors!

For use with Lessons 5-1 through 5-2

1. The changes in price and weight were reversed in the ratio.

$$\text{rate of change} = \frac{\text{change in price}}{\text{change in weight}}$$

$\frac{7 - 3.5}{2 - 1} = 3.5$ $\frac{10.5 - 7}{3 - 2} = 3.5$ $\frac{14 - 10.5}{4 - 3} = 3.5$

The rate of change is $3.50 per pound. It represents the unit price of the cashews.

2. In line 2, the points representing (x_1, y_1) and (x_2, y_2) are not consistent. y_2 is from $(7, -4)$ and y_1 is from $(9, -3)$, but x_2 is from $(9, -3)$ and x_1 is from $(7, -4)$.

$$\begin{aligned} \text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4 - (-3)}{7 - 9} \\ &= \frac{-1}{-2} \end{aligned}$$

The slope of the line is $\frac{1}{2}$.

3. The conclusion is incorrect. To be a direct variation, the equation must be able, to be written in the form $y = kx$.

$$\begin{aligned} -x + y &= 3 \\ y &= x + 3 \end{aligned}$$

The equation is not a direct variation.

4. The x- and y-values were reversed when using the direct variation equation to find k.

$$y = kx$$

$$3 = 5k$$

$$\frac{3}{5} = k$$

The equation $y = \frac{3}{5}x$ gives the height y (in inches) of an enlargement that is x inches wide.

5-3 Think About a Plan

Slope-Intercept Form

Hobbies Suppose you are doing a 5000-piece puzzle. You have already placed 175 pieces. Every minute you place 10 more pieces.

a. Write an equation in slope-intercept form to model the number of pieces placed. Graph the equation.

b. After 50 more minutes, how many pieces will you have placed?

Understanding the Problem

1. Is this relationship linear? How do you know?

yes; the rate of change (10 pieces/min) is constant

Planning the Solution

2. How many pieces have you already placed? What does this represent in the slope-intercept form?

175; the y-intercept

3. What two quantities are used to find the rate of change or slope? What is the slope of this relationship?

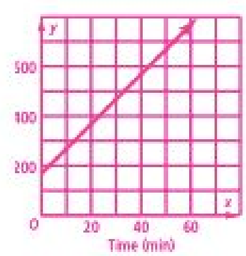
number of pieces placed and change in time; 10

Getting an Answer

4. Use your answers in Steps 2 and 3 to write an equation in slope-intercept form to model the number of pieces placed.

$y = 175 + 10x$

5. Graph the equation on a coordinate grid.



6. How many pieces will you have placed after 50 more minutes?

675 pieces

5-3 Reteaching (continued)

Slope-Intercept Form

Exercises

Find the slope and y-intercept of the graph of each equation.

1. $y = \frac{1}{2}x + 7$

   $m = \frac{1}{2}$; $b = 7$

2. $y = -5x + 1$

   $m = -5$; $b = 1$

3. $y = -\frac{2}{5}x - 3$

   $m = -\frac{2}{5}$; $b = -3$

4. $y = x + 5$

   $m = 1$; $b = 5$

5. $y = \frac{1}{6}x - 2$

   $m = \frac{1}{6}$; $b = -2$

6. $y = 4x$

   $m = 4$; $b = 0$

Write an equation for the line with the given slope m and y-intercept b.

7. $m = -3, b = 7$

   $y = -3x + 7$

8. $m = \frac{2}{3}, b = 8$

   $y = \frac{2}{3}x + 8$

9. $m = 4, b = -3$

   $y = 4x - 3$

10. $m = -\frac{1}{5}, b = -1$

    $y = -\frac{1}{5}x - 1$

11. $m = -\frac{5}{6}, b = 0$

    $y = -\frac{5}{6}x$

12. $m = 7, b = -2$

    $y = 7x - 2$

Write an equation in slope-intercept form for the line that passes through the given points.

13. $(1, 3)$ and $(2, 5)$

    $y = 2x + 1$

14. $(2, -1)$ and $(4, 0)$

    $y = \frac{1}{2}x - 2$

15. $(1, 2)$ and $(2, -1)$

    $y = -3x + 5$

16. $(1, -5)$ and $(3, -3)$

    $y = x - 6$

17. $(3, 3)$ and $(6, 5)$

    $y = \frac{2}{3}x + 1$

18. $(4, -3)$ and $(8, -4)$

    $y = -\frac{1}{4}x - 2$

ANSWERS

Chapter 5 Find The Errors!

For use with Lessons 5-3 through 5-5

1. In line 2 of Step 2, the coordinates used are not from the same point. 5 was used for y and 4 was used for x.

Step 1 Find the slope.

$$\frac{0-5}{4-0} = -\frac{5}{4}$$

Step 2 Find b.

$$y = mx + b$$

$$0 = -\frac{5}{4}(4) + b$$

$$0 = -5 + b$$

$$5 = b$$

Step 3 Substitute into the slope-intercept form.

$$y = mx + b$$

$$y = -\frac{5}{4}x + 5$$

An equation of the line is $y = -\frac{5}{4}x + 5$.

2. In Step 1, the slope and y-intercept were reversed. The slope should be the rate of change, \$10 per week, and the y-intercept should be the starting value, \$20.

Step 1 Identify the slope and the y-intercept.

The slope is the rate of change. She saves at a rate of \$10 per week.

The y-intercept is the starting value, \$20.

Step 2 Substitute into the slope-intercept form.

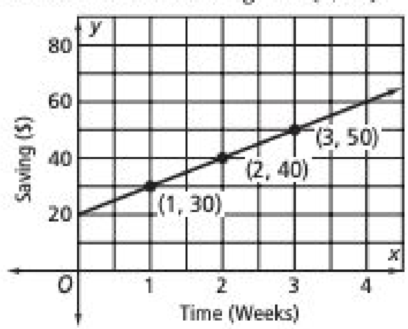
$$y = mx + b$$

$$y = 10x + 20$$

Step 3 Graph the equation.

The y-intercept is 20. Graph the point (0, 20).

The slope is 10. Graph a point 10 units above and 1 unit to the right of (0, 20).



THE SCHOOL DISTRICT OF PHILADELPHIA

Name ______________________ Class ____________ Date ____________

5-7 Additional Vocabulary Support

Scatter Plots and Trend Lines

Concept List

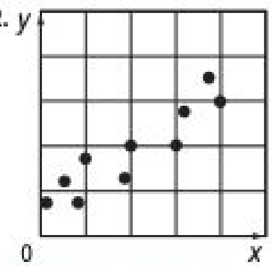
causal relationship	correlation coefficient	extrapolation
interpolation	line of best fit	negative correlation
no correlation	positive correlation	trend line

Choose the concept from the list above that best represents the item in each box.

1. LinReg $(ax + b)$

line of best fit

2.



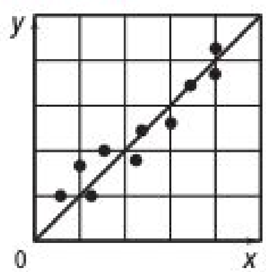
positive correlation

3.

Practice Time (hours)	Grade in Band Class
10	C
20	B
40	A

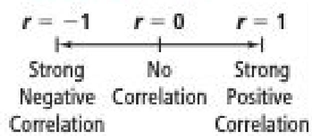
causal relationship

4.



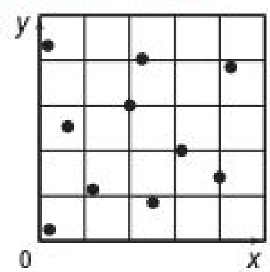
trend line

5.



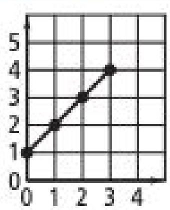
correlation coefficient

6.



no correlation

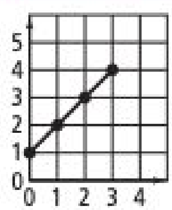
7.



Estimate the value for y when $x = 3.5$.

extrapolation

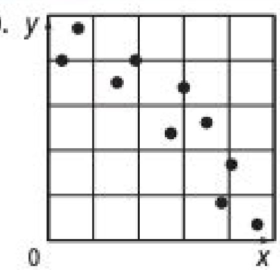
8.



Estimate the value for y when $x = 1.5$.

interpolation

9.



negative correlation

Name ______________________ Class __________ Date __________

5-7 Think About a Plan

Scatter Plots and Trend Lines

U.S. Population Use the data below.

Estimated Population of the United States (thousands)

Year	2000	2001	2002	2003	2004	2005	2006
Male	138,482	140,079	141,592	142,937	144,467	145,973	147,512
Female	143,734	145,147	146,533	147,858	149,170	150,533	151,886

SOURCE: U.S. Census Bureau

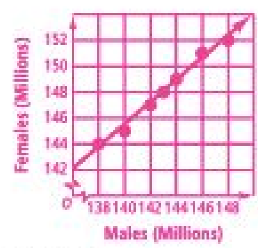
a. Make a scatter plot of the data pairs (male population, female population).

b. Draw a trend line and write its equation.

c. Use your equation to predict the U.S. female population if the U.S. male population increases to 150,000,000.

d. Reasoning Consider a scatter plot of the data pairs (year, male population). Would it be reasonable to use this scatter plot to predict the U.S. male population in 2035? Explain your reasoning.

1. Make a scatter plot of the data pairs using the male population for the x-coordinates and the female population for the y-coordinates for each year. **See points at right for scatter plot.**

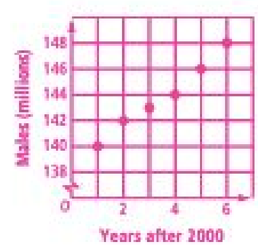


2. Draw the trend line onto the scatter plot.
See graph in Exercise 1.

3. How do you determine the equation of a trend line? What is the equation of this trend line? Show your work.
Choose 2 points on the line. Find the slope and use point-slope form; Answers may vary. Sample: $y = 0.9x + 23$

4. Substitute 150 for x to predict the female population. ______________
Answers vary depending on equation used: 158,000,000 females

5. Make a scatter plot of the data pairs (year, male population).



6. Would it be reasonable to use this scatter plot to predict the U.S. male population in 2035? Explain your reasoning.
The data go from $x = 0$ to $x = 6$; projecting to $x = 35$ is a very large extrapolation.

5-7 Reteaching (continued)

Scatter Plots and Trend Lines

Problem

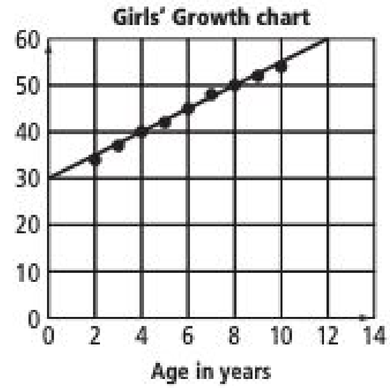
Draw a trend line for the scatter plot in the previous problem. What is the equation for your trend line? What would you estimate to be the average height of a girl who is 12 years old?

Draw a line that seems to fit the data. The line drawn for this data goes through (4, 40) and (8, 50). Use these points to write an equation.

$$m = \frac{50 - 40}{8 - 4} = 2.5$$

Use the point-slope form of the line.

$$y - y_1 = m(x - x_1)$$
$$y - 40 = 2.5(x - 4)$$
$$y - 40 = 2.5x - 10$$
$$y = 2.5x + 30$$



Use this equation to estimate the average height of 12-year-old girls.

$$y = 2.5(12) + 30$$
$$y = 60$$

Exercises

Ryan practices throwing darts. From each distance listed below, he throws 10 darts and records how many times he hits the center.

Distance (in feet)	2	5	7	8	10	12	15
Number of Center Hits	10	9	8	6	5	1	2

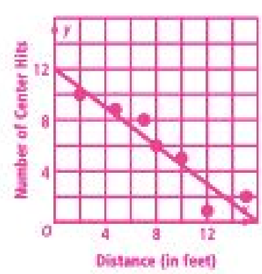
1. Use the space at the right to make a scatter plot of the data.
   See points on graph in Ex. 3.

2. Describe the type of correlation that is shown in the scatter plot.
   negative correlation; as the distance from the target increases, the number of center hits out of 10 decreases

3. Draw a trend line.

4. What equation represents your trend line? $y = -\frac{3}{4}x + 12$

5. How many hits do you estimate Ryan would make from 6 feet?
   about 7 hits



THE SCHOOL DISTRICT OF PHILADELPHIA

Grade: 8 **Subject:** Math (worksheets taken from enVision Mathematics, Grade 8 and from Pearson A1)

Topic: Systems of Linear Functions/Equations

What Your Student is Learning:
- Examine graphs of linear systems to determine the number of solutions and the solution.
- Solve systems of equations by graphing, substitution, and elimination.
- Write equations to represent situations.

Background and Context for Parents:

Solve Systems of Linear Equations Graphically

- **Analyze Graphs of Linear Systems** In Lesson 5-1, students relate the number of solutions to a linear system to the slopes and *y*-intercepts of the graphed lines. They see that a solution is any ordered pair that makes all equations in the system true. Students then apply these skills to systems of linear equations that represent real-world situations.

The system of equations $\begin{matrix} 6x + 2y = 12 \\ 12x + 4y = 24 \end{matrix}$ represents the situation.

Write each equation in slope-intercept form.

$6x + 2y = 12$	$12x + 4y = 24$
$2y = -6x + 12$	$4y = -12x + 24$
$y = -3x + 6$	$y = -3x + 6$

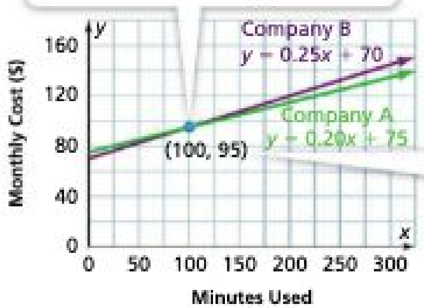
The equations represent the same line. Every (x, y) pair on the line is a solution.

Corey and Winnie bought the same amount of cheese and tomatoes.

- **Graph Linear Systems** In Lesson 5-2, students graph systems of linear equations and see that the relationship of the two lines represents the solution. Students interpret the meaning of the relationship to a given scenario.

STEP 2 Graph the system.

The point of intersection, (100, 95), is the solution of the system.



The lines intersect, so there is **one solution** for this system of equations.

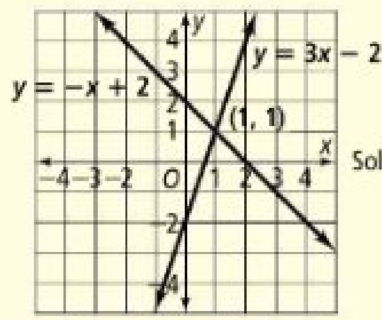
If Li uses 100 minutes, both plans cost $95. She could choose either plan.
If Li uses fewer than 100 minutes, she should choose Company B.
If Li uses more than 100 minutes, she should choose Company A.

Solving Systems of Equations Graphically

To solve a linear system by graphing, graph each equation and look for the point at which the lines intersect.

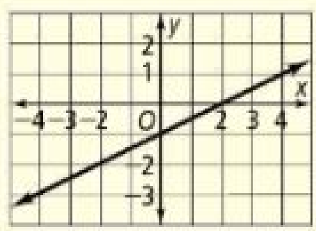
There are three possibilities when solving a linear system.

1 One Solution This occurs when the graphs of the equations intersect in one point. In this case, the coordinates of the intersection point are the solution to the system.



Solution: (1, 1)

2 Infinitely Many Solutions This occurs when two equations have the same graph. All points that lie on the line are solutions to the system.

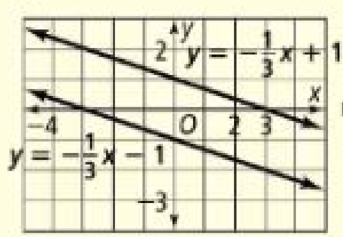


$x - 2y = 2$

$y = \frac{1}{2}x - 1$

All points on the line are solutions.

3 No Solution If the graphs of the equations are parallel, then the system has no solution.



no solution

Solve Systems of Linear Equations Algebraically

- **Solve by Substitution** In Lesson 5-3, students write systems of linear equations that describe the relationship between two variables. They learn to solve the system by solving one equation for one variable, and then substituting the solution into the other equation. Students then solve the second equation, which now contains only one variable. They use the solution to the system of equations to answer a given question.

STEP 1 Write a system of linear equations to represent the situation. Let x equal the number of hours and y equal Seiko's cost.

$$y = 14x$$
$$y = \frac{1}{2}(28x + 15)$$

STEP 2 Use substitution to solve one of the equations for one variable.

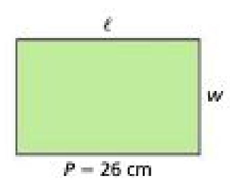
$$14x = \frac{1}{2}(28x + 15)$$
$$14x = 14x + 7.50$$
$$0 \neq 7.50$$

Substitute $14x$ for y in the second equation.

The result is not a true statement, so the system has no solution. There is no number of hours for which Seiko's cost is the same at both studios.

- **Solve by Elimination** In Lesson 5-4, students extend the skill of writing a system of linear equations to systems that can be solved using elimination. The goal is to eliminate one of the variables, creating a new equation in one variable. Students first learn to use addition and subtraction to eliminate one variable. They then multiply one or both equations to create opposites, and add to eliminate a variable.

The difference of the length and width of the rectangle is 3 centimeters. What are the length and width of the rectangle?



STEP 1 Write a system of equations to relate the length and width.

$$2\ell + 2w = 26$$
$$\ell - w = 3$$

STEP 2 Eliminate one variable. The coefficients of ℓ and w are not the same or opposites. Multiply one or both of the equations so that the variables are the same or opposites.

$$\begin{array}{r} 2\ell + 2w = 26 \\ +2\ell - 2w = 6 \\ \hline 4\ell + 0 = 32 \\ \ell = 8 \end{array}$$

Multiply each term of the second equation by 2 so that the w terms are opposites.

STEP 3 Solve for the other variable.

$$\ell - w = 3$$
$$8 - w = 3$$
$$w = 5$$

The length of the rectangle is 8 centimeters and the width is 5 centimeters.

Ways to support your student:

- Provide blank/ lined paper so they have enough space to try different strategies.
- Watch the Khan Academy video together (linked below).
- Read the problem outloud to them.
- Before giving your student the answer to their question or specific help, ask them "What have you tried so far?, What do you know?, What might be a next step?
- After your student has solved it, and before you tell them it's correct or not, have them explain to you how they got their solution and if they think their answer makes sense.

Online Resources for Students:

- **https://www.desmos.com/calculator** -Free online graphing calculator that allows students to check their work by inputting the equations of the lines. Remind students that unlike a graphing calculator, inputs into desmos don't have to be in y=mx+b (point slope form).
- https://www.khanacademy.org/math/8th-engage-ny/engage-8th-module-4/8th-module-4-topic-d/v/trolls-tolls-and-systems-of-equations - Topic D as A-C align to the prior content.

6-1 Think About a Plan

Cell Phone Plans A cell phone provider offers plan 1 that costs $40 per month plus $.20 per text message sent or received. A comparable plan 2 costs $60 per month but offers unlimited text messaging.

a. How many text messages would you have to send or receive in order for the plans to cost the same each month?

b. If you send and receive an average of 50 text messages each month, which plan would you choose? Why?

Know

1. What equations can you write to model the situation?

	times		plus		=	y (total)

Cell phone plan #2 cost per month ________________

Cell phone plan #1 cost per month ________________

2. How will graphing the equations help you find the answers?

____

Need

3. How will you find the best plan?

____

Plan

4. What are the equations that represent the two plans? __________ and______________

5. Graph your equations.

6. Where will the solution be on the graph?

____

____

7. What is the solution?

____

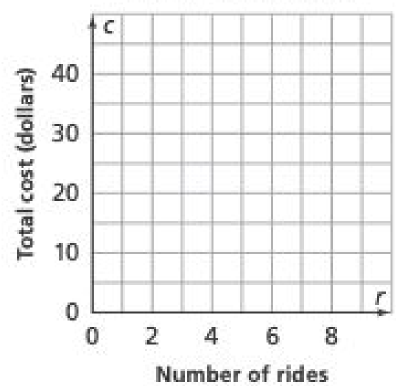
Name ______________________________

Build Mathematical Literacy
5-2

Read the problem below. Then answer the questions to help you understand the problem.

At Family Fun Park, admission is free, but ride tickets cost \$6 each and parking costs \$10. At Paradise Rides, the admission and parking is free, but the ride tickets cost \$8 each. The total cost, c, to visit a theme park and go on r rides can be represented by a system of equations. Write and graph the system of equations. For how many rides will the cost at both parks be the same?

Theme Park Costs



1. Circle the important information about the cost of Family Fun Park.

2. Underline the important information about the cost of Paradise Rides.

3. What needs to be included in the answer to this problem?

4. What will each equation in the system of equations represent?

5. How is the cost of attending Family Fun Park similar to the cost of attending Paradise Rides?

6. How is the cost of attending the theme parks different?

6-1 Reteaching

Graphing is useful for solving a system of equations. Graph both equations and look for a point of intersection, which is the solution of that system. If there is no point of intersection, there is no solution.

Problem

What is the solution to the system? Solve by graphing. Check.

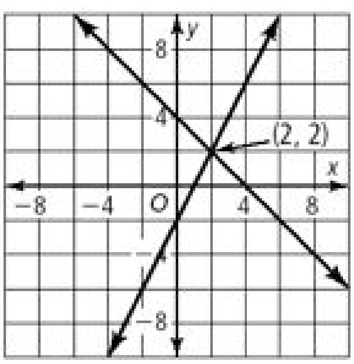
$x + y = 4$

$2x - y = 2$

Solution

$y = -x + 4$ $y = 2x - 2$	Put both equations into *y*-intercept form, $y = mx + b$.
$y = -x + 4$	The first equation has a *y*-intercept of (0, 4).
$0 = -x + 4$	Find a second point by substituting in 0 for *y* and solve for *x*.
$x = 4$	You have a second point (4, 0), which is the *x*-intercept.
$y = 2x - 2$	The second equation has a *y*-intercept of (0, –2).
$0 = 2(x) - 2$	Find a second point by substituting in 0 for *y* and solve for *x*.
$2 = 2x, x = 1$	You have a second point for the second line, (1, 0).

$2 = 2x, x = 1$ You have a second point for the second line, (1, 0).



Plot both sets of points and draw both lines. The lines appear to intersect (2, 2), so (2, 2) is the solution.

Check

If you substitute in the point (2, 2), for *x* and *y* in your original equations, you can double-check your answer.

$x + y = 4$	$2 + 2 \stackrel{?}{=} 4,$	$4 = 4$ ✓
$2x - y = 2$	$2(2) - 2 \stackrel{?}{=} 2,$	$2 = 2$ ✓

6-1 Reteaching (continued)

If the equations represent the same line, there is an infinite number of solutions, the coordinates of any of the points on the line.

Problem

What is the solution to the system? Solve by graphing. Check.

$2x - 3y = 6$

$4x - 6y = 18$

Solution

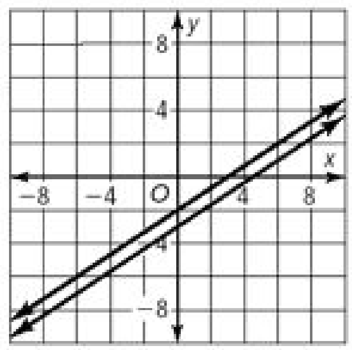
What do you notice about these equations? Using the y-intercepts and solving for the x-intercepts, graph both lines using both sets of points.

$y = \frac{2}{3}x - 2$

$y = \frac{2}{3}x - 3$

Graph equation 1 by finding two points: (0, –2) and (3, 0). Graph equation 2 by finding two points (0, –3) and (4.5, 0).

Is there a solution? Do the lines ever intersect? Lines with the same slope are parallel. Therefore, there is no solution to this system of equations



Exercises

Solve each system of equations by graphing. Check.

1. $2x = 2 - 9y$
$21y = 4 - 6x$

2. $2x = 3 - y$
$y = 4x - 12$

3. $y = 1.5x + 4$
$0.5x + y = -2$

4. $6y = 2x - 14$
$x - 7 = 3y$

5. $3y = -6x - 3$
$y = 2x - 1$

6. $2x = 3y - 12$
$\frac{1}{3}x = 4y + 5$

7. $2x + 3y = 11$
$x - y = -7$

8. $3y = 3x - 6$
$y = x - 2$

9. $y = \frac{1}{2}x + 9$
$2y - x = 1$

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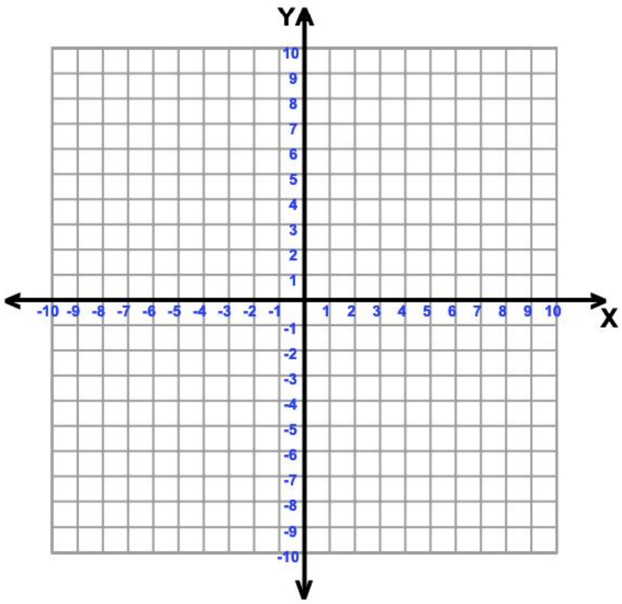
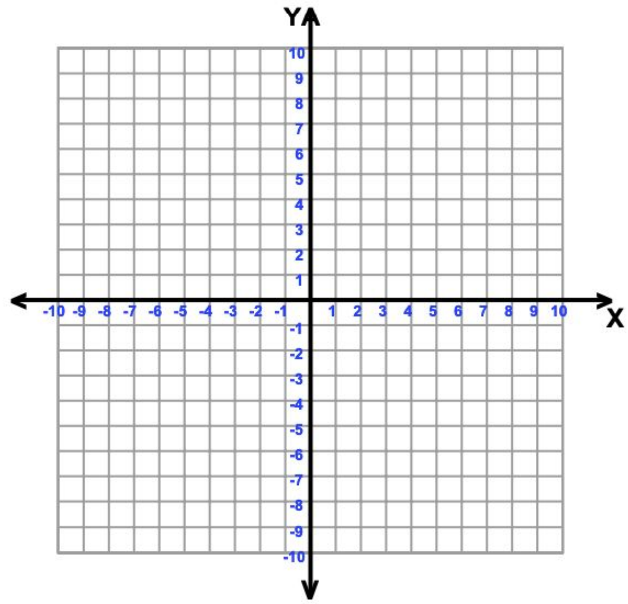
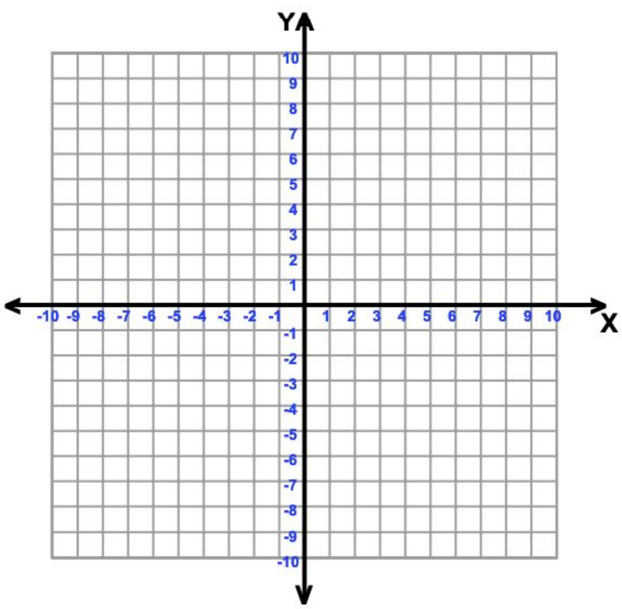
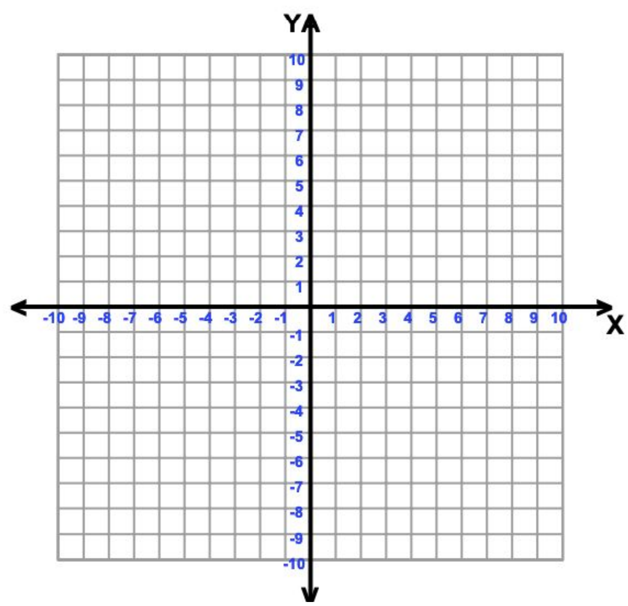
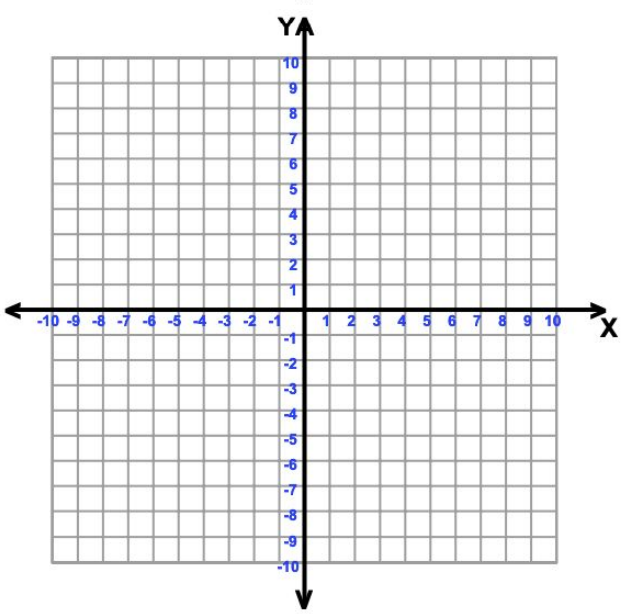
Graphs for Reteaching Problems (there are more than needed)

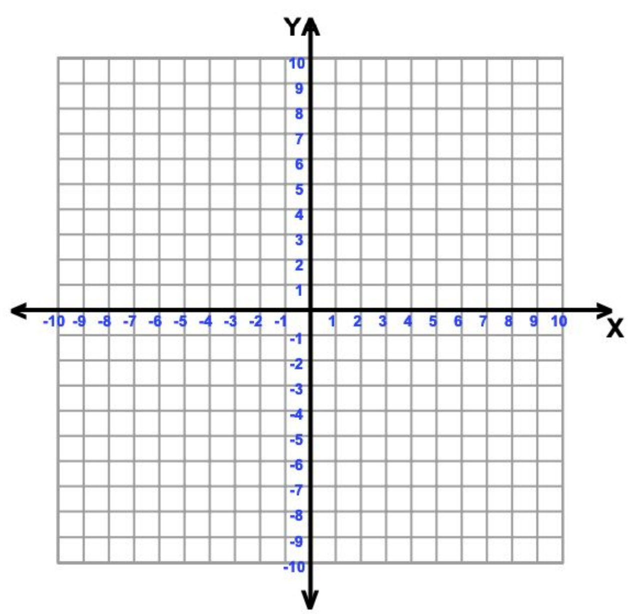












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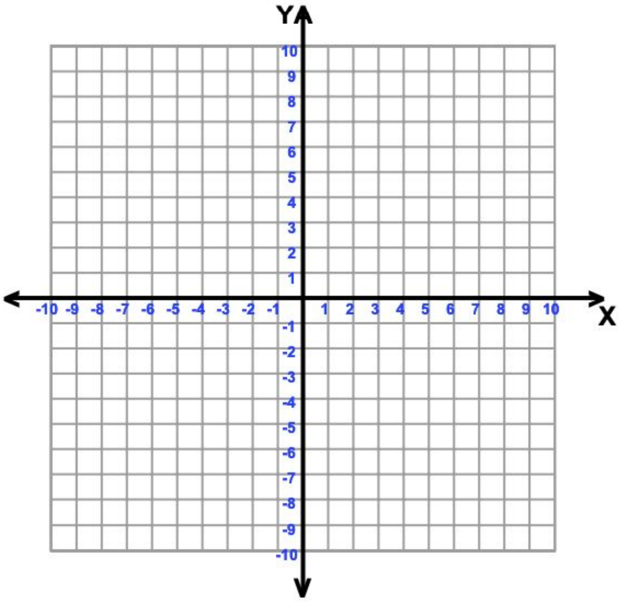

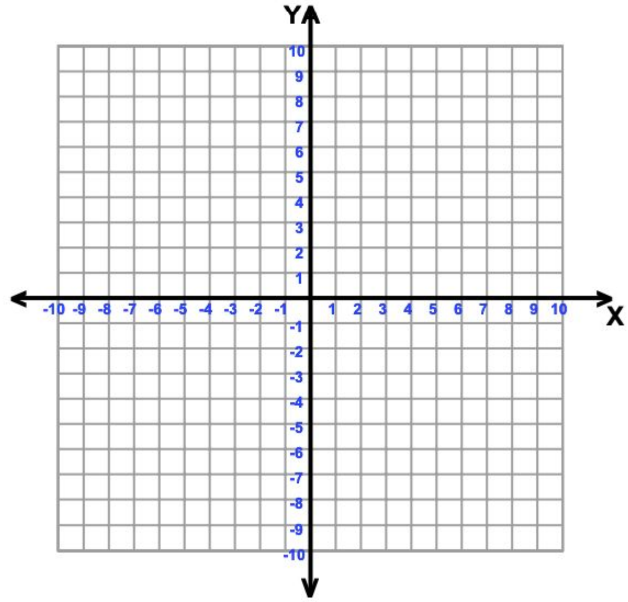

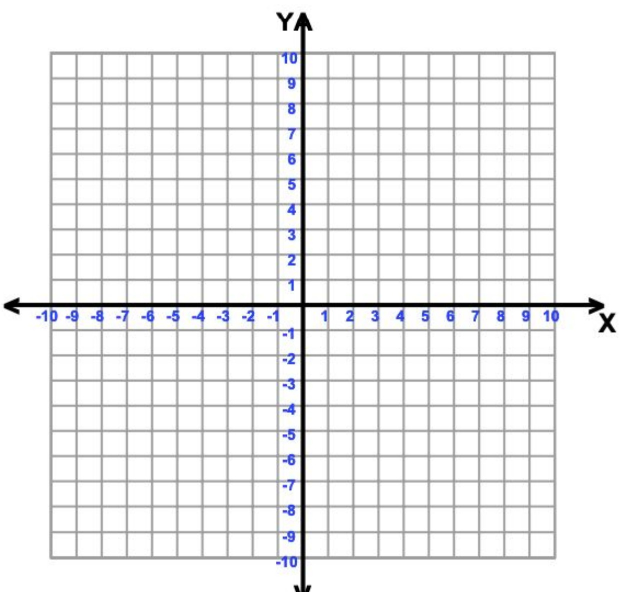

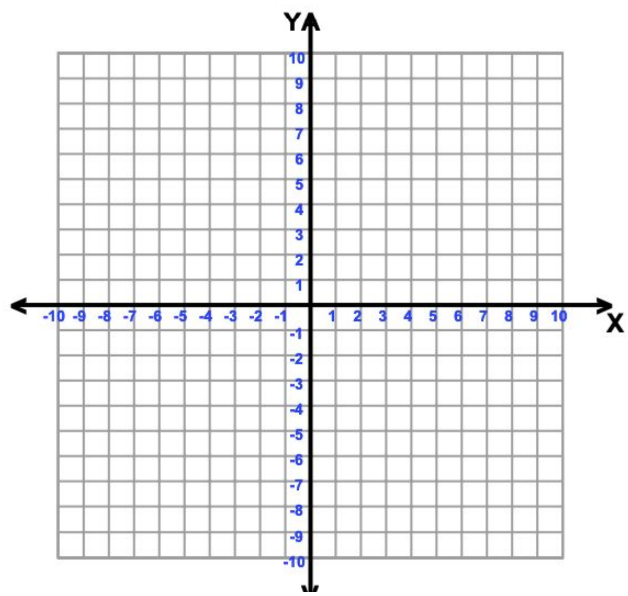

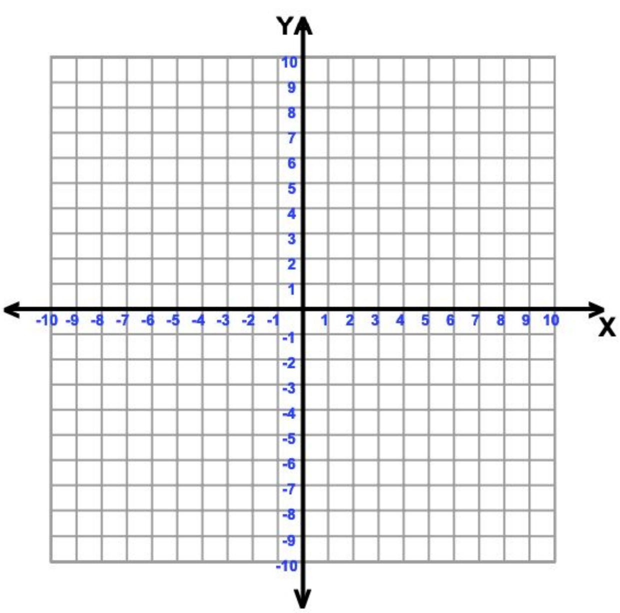

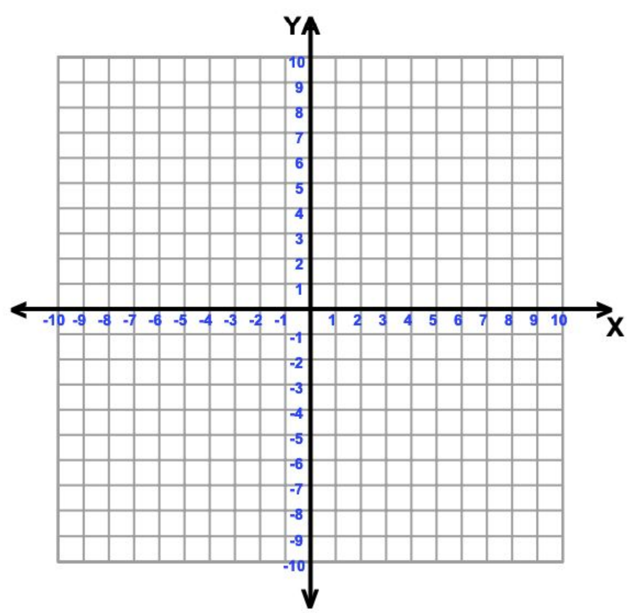

Name ________________________________

Build Mathematical Literacy
5-3

Read the problem below. Then answer the questions to help you understand the problem.

Members of the school band are selling pencils and erasers to raise money for a trip. On the first day, Misha sold a total of 30 items and collected \$6.40. How many of each type of item did he sell?

Help us march at the Playoff Parade!

Pencils: 25¢ each

Erasers: 80¢ each

1. Underline the question you need to answer.

2. How many quantities is the problem asking you to find? Explain.

3. Highlight the information you can use to write a system of equations. Is all of the necessary information given in the text of the problem? Explain.

4. To solve this problem, you can write a system of equations that contains the equation $x + y = 30$, where x represents the number of pencils Misha sold and y represents the number of erasers. Circle the information in the problem that is represented by this equation.

5. Explain how the equation $0.25x + 0.80y = 6.40$ represents the situation.

6. Of the equations in Exercises 3 and 4, which equation would you solve for one of the variables in terms of the other variable? Explain.

6-2 Think About a Plan

Art An artist is going to sell two sizes of prints at an art fair. The artist will charge $20 for a small print and $45 for a large print. The artist would like to sell twice as many small prints as large prints. The booth the artist is renting for the day costs $510. How many of each size print must the artist sell in order to break even at the fair?

Understanding the Problem

1. How much will the artist spend to rent a booth? ________________________________

2. What do you know about selling prices of the prints? ______________________________

3. What do you know about the number of prints the artist would like to sell?

   ____

4. What is the problem asking you to determine?

   ____

Planning the Solution

5. What variables are needed? ____

6. What equation can be used to determine the number of prints that the artist would like to sell based on size? ____________________

7. What equation can be used to determine how many prints the artist has to sell to break even?

   ____

Getting an Answer

8. What is the solution to the system of equations?

Name ______________________________

Reteach to Build Understanding
5-3

Together, Amy and Mai have saved \$490 for a trip this summer. Amy has saved \$54 more than Mai. How much has each person saved?

Let a = the amount Amy has saved. Let m = the amount Mai has saved.

Write a system of linear equations to represent the situation.

$$a + m = 490$$
$$a = m + 54$$

Use substitution to find the value of m.

$$(m + 54) + m = 490$$
$$2m + 54 = 490$$
$$m = 218$$

Substitute to find the value of a.

$$a = 218 + 54$$
$$a = 272$$

Amy has saved \$272. Mai has saved \$218.

A parking lot charges \$7 per day to park on weekdays and \$12 per day on weekends. Jamal parked his car in this lot on 6 days last week and spent a total of \$52. How many weekdays and weekend days did Jamal park?

1. Let x = the number of weekdays and let y = the number of weekend days. Fill in the boxes to write a system of equations.

   $x + y = \square$

   $\square x + \square y = 52$

2. Solve the equation for y in terms of x. $y = \square - \square$

3. Rewrite the second equation by substituting your value for y in terms of x found in Exercise 2. $7x + 12(\square) = 52$

4. Solve your equation in Exercise 3. What is the value of x?

5. How can you find the value of y? What is the value of y?

6. How many weekdays and weekend days did Jamal park?

On the Back!

7. Krysta bought 24 notebooks and spent \$104. The large notebooks cost \$6 each and the small notebooks cost \$2 each. How many of each type of notebook did she buy?

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6-2 Reteaching

You can solve a system of equations by substituting an equivalent expression for one variable.

Problem

Solve and check the following system:

$x + 2y = 4$
$2x - y = 3$

Solution	$x + 2y = 4$	The first equation is easiest to solve in terms of one variable.
	$x = 4 - 2y$	Get x to one side by subtracting $2y$.
	$2(4 - 2y) - y = 3$	Substitute $4 - 2y$ for x in the second equation.
	$8 - 4y - y = 3$	Distribute.
	$8 - 5y = 3$	Simplify.
	$8 - 8 - 5y = 3 - 8$	Subtract 8 from both sides.
	$-5y = -5$	Divide both sides by 25.
	$y = 1$	You have the solution for y. Solve for x.
	$x + 2(1) = 4$	Substitute in 1 for y in the first equation.
	$x + 2 - 2 = 4 - 2$	Subtract 2 from both sides.
	$x = 2$	The solution is (2, 1).

Check Substitute your solution into either of the given linear equations.

$x + 2y = 4$	
$2 + 2(1) \stackrel{?}{=} 4$	Substitute (2, 1) into the first equation.
$4 = 4$ ✓	You check the second equation.

Exercises

Solve each system using substitution. Check your answer.

1. $x + y = 3$
$2x - y = 0$

2. $x - 3y = -14$
$x - y = -2$

3. $2x - 2y = 10$
$x - y = 5$

4. $4x + y = 8$
$x + 2y = 5$

6-2 Reteaching (continued)

Problem

Solve and check the following system:

$\frac{x}{2} - 3y = 10$

$3x + 4y = -6$

Solve	$\frac{x}{2} - 3y = 10$	First, isolate x in the first equation.
	$\frac{x}{2} = 10 + 3y$	Add $3y$ to both sides and simplify.
	$x = 20 + 6y$	Multiply by 2 on both sides.
	$3x + 4y = -6$	Substitute $20 + 6y$ for x in second equation.
	$3(20 + 6y) + 4y = -6$	Simplify.
	$60 + 22y = -6$	Subtract 60 from both sides.
	$22y = -66,\ y = -3$	Divide by 22 to solve for y.
	$\frac{x}{2} - 3(-3) = 10$	Substitute 23 in the first equation.
	$\frac{x}{2} + 9 = 10$	Simplify.
	$x = 2$	Solve for x.

The solution is (2, –3).

Check $3(2) + 4(-3) \stackrel{?}{=} 26$

$-6 = -6$ ✓

Now you check the first equation.

Exercises

Solve each system using substitution. Check your answer.

5. $-2x + y = 8$
$3x + y = -2$

6. $3x - 4y = 8$
$2x + y = 9$

7. $3x + 2y = 25$
$2x + 3y = -6$

8. $6x - 5y = 3$
$x - 9y = 25$

6-3 Additional Vocabulary Support

Solving System Using Elimination

Tony is trying to find the solution of the system using elimination.

$2x - 4y = 12$ **$3x + 4y = 48$**

He wrote these steps to solve the problem on note cards, but they got mixed up.

Solve for x.

Eliminate one variable. Since the sum of the coefficients of y is 0, add the equations to eliminate y.

Simplify.

Substitute 12 for x to solve for the eliminated variable.

Solve for y.

Since $x = 12$ and $y = 3$, the solution is (12, 3).

Use the note cards to complete the steps below.

1. First,

____

____

2. Second,

____

3. Third,

____

____

4. Then,

____

5. Next,

____

6. Finally,

____

____

Name ______________________________

Build Mathematical Literacy
5-4

Read the problem below. Then answer the questions to help you understand the problem.

Lily exercises for a total of 150 minutes each week. She does weight training and cardio workouts. She spends more time doing cardio workouts. The difference between the time she spends on cardio workouts and the time she spends weight training is 72 minutes. How much time does Lily spend on each type of activity?

1. Highlight the question that you need to answer.

2. You can write a system of equations to represent this problem. What quantities should you represent with variables?

3. Circle the information in the problem that is represented by the equation $x + y = 150$ where x represents cardio workouts and y represents weight training.

4. Underline the information in the problem that you will use to write another equation.

5. Circle the equation that can represent the information you underlined in Exercise 4.

   $x + y = 72$ $y = 72x$ $x - y = 72$ $y = 72 - x$

6. Describe the correct answer to this problem, including the units.

6-3 Think About a Plan

Nutrition Half a pepperoni pizza plus three fourths of a ham-and-pineapple pizza contains 765 Calories. One fourth of a pepperoni pizza plus a whole ham-and-pineapple pizza contains 745 Calories. How many Calories are in a whole pepperoni pizza? How many Calories are in a whole ham-and-pineapple pizza?

Know

1. What equation will represent the 765 Calories combination of pizza? ____________________

2. What equation will represent the 745 Calories combination of pizza? ____________________

Need

3. What possible methods can you use to solve the system of equations?

Plan

4. How can you solve the system of equations by elimination?

____

____

5. How can you eliminate one of the variables to solve the system of equations?

____

6. Solve the system of equations.

7. What is the solution of the system?

____

8. How many Calories are in each kind of pizza?

____

____

6-3 Reteaching

Elimination is one way to solve a system of equations. Think about what the word "eliminate" means. You can eliminate either variable, whichever is easiest.

Problem

Solve and check the following system of linear equations.

$$4x - 3y = -4$$
$$2x + 3y = 34$$

Solution The equations are already arranged so that like terms are in columns.

Notice how the coefficients of the y-variables have the opposite sign and the same value.

$$\begin{array}{r} 4x - 3y = -4 \\ 2x + 3y = 34 \\ \hline 6x = 30 \end{array}$$

Add the equations to eliminate y.

Divide both sides by 6 to solve for x.

$$x = 5$$

$$4(5) - 3y = -4$$
$$20 - 3y = -4$$
$$-3y = -24$$
$$y = 8$$

Substitute 5 for x in one of the original equations and solve for y.

The solution is (5, 8).

Check

Substitute your solution into both of the original equations to check.

$$4x - 3y = -4$$
$$4(5) - 3(8) \stackrel{?}{=} -4$$
$$20 - 24 \stackrel{?}{=} -4$$
$$-4 = -4 \checkmark$$

You can check the other equation

Exercises

Solve and check each system.

1. $3x + y = 3$
   $-3x + y = 3$

2. $6x - 3y = -14$
   $6x - y = -2$

3. $3x - 2y = 10$
   $x - 2y = 6$

4. $4x + y = 8$
   $x + y = 5$

6-3 Reteaching (continued)

If none of the variables has the same coefficient, you have to multiply before you eliminate.

Problem

Solve the following system of linear equations. $-2x + 3y = -1$
$5x + 4y = 6$

Solution

$5(-2x - 3y) = (-1)5$ $2(5x + 4y) = (6)2$	Multiply the first equation by 5 (all terms, both sides) and the second equation by 2. You can eliminate the *x* variable when you add the equations together.
$-10x - 15y = -5$ $\underline{10x + 8y = 12}$ $-7y = 7$	Distribute, simplify and add.
$y = -1$	Divide both sides by 7.
$5x + 4(-1) = 6$	Substitute –1 in for y in the second equation to find the value of *x*.
$5x - 4 = 6$	Simplify.
$5x = 10$	Add 4 to both sides.
$x = 2$	Divide by 5 to solve for *x*.

The solution is (2, –1).

Check $-2x + 3y = -1$	Substitute your solution into both original equations.
$-2(2) - 3(-1) \stackrel{?}{=} -1$ $-1 = -1$ ✓	You can check the other equation.

Exercises

Solve and check each system.

5. $x - 3y = -3$
$-2x + 7y = 10$

6. $-2x - 6y = 0$
$3x + 11y = 4$

7. $3x + 10y = 5$
$7x + 20y = 11$

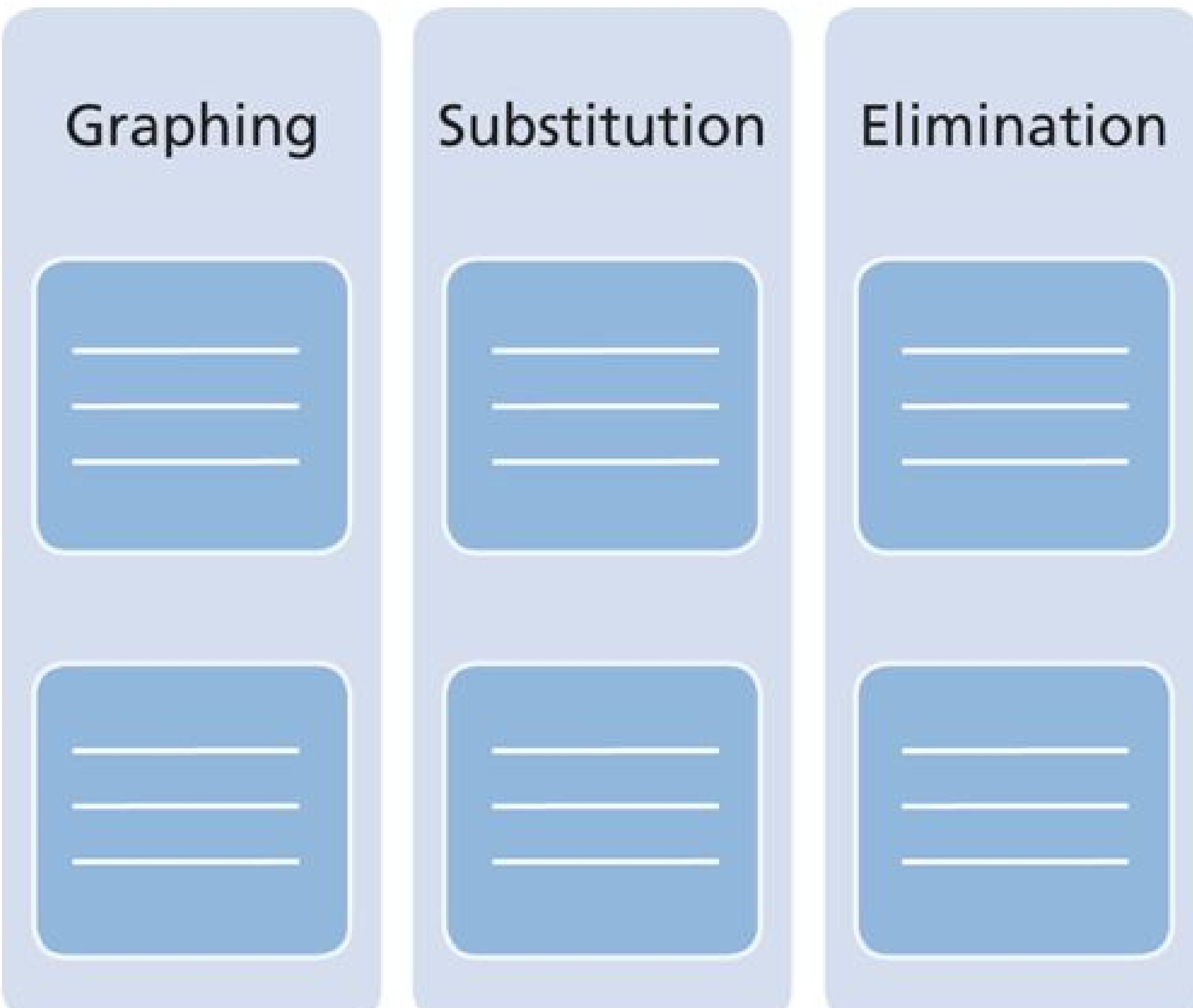
8. $4x + y = 8$
$x + y = 5$

6-4 Additional Vocabulary Support

Use the list below to complete the diagram.

use when you want a visual display of the equations	use when it is easy to solve for one of the variables	use when you want an estimation of the solution
use when one equation is already solved for one of the variables	use when the coefficients of one variable are the same or opposites	use when it is not convenient to use graphing or substitution

Choosing a Method for Solving Linear Systems



6-4 Reteaching

You can solve systems of linear equations by graphing, substitution, or elimination. Deciding which method to use depends on the exactness needed and the form of the equations.

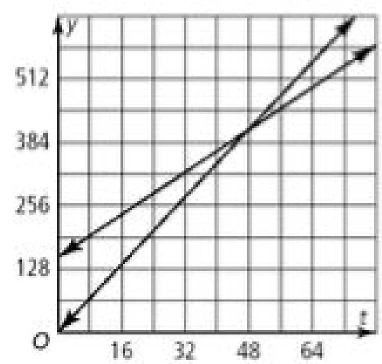
You just bought a coffee shop for \$153,600. The prior owner had an average monthly revenue of \$8600 and an average monthly cost of \$5400. If your monthly costs and revenues remain the same, how long will it take you to break even?

Write equations for revenue and costs, including the price you paid for the shop, after t months. Then solve the system by graphing.

$y = 8600\,t$ Equation for revenue

$y = 5400t + 153{,}600$ Equation for cost

It appears that the point of intersection is where t is equal to 48 months. Substitute $t = 48$ into either equation to find the other coordinate (y), which is 412.8.
Therefore, your breakeven point is after you have run the
shop for 48 months, at which point your revenue and cost
are the same: \$412,800.



Problem

A perfume is made from t ounces of 15% scented Thalia and b ounces of 40% Thalia. You want to make 60 oz of a perfume that has a 25% blend of the Thalia. How many ounces of each concentration of Thalia are needed to get 60 oz of perfume that is 25% strength of Thalia?

Write your systems of equations:

$$60(0.25) = 0.15t + 0.4b$$
$$60 = t + b$$

Solve the system by using substitution:

$60(0.25) = 0.15t + 0.4b$	Solve the second equation for t and substitute in the first equation.
$15 = 0.15(60 - b) + 0.4b$	Substitute $60 - b$ for t in the first equation.
$15 = 9 - 0.15b + 0.4b$	Distributive property
$24 = b$	Solve for b.

Substitute 24 for b in second equation to find that $t = 36$. The answer is (36, 24). The blend requires 36 oz of the 15% perfume and 24 oz of the 25% perfume.

6-4 Reteaching (continued) Exercises

1. You have a coin bank that has 275 dimes and quarters that total \$51.50. How many of each type of coin do you have in the bank?

2. Open-Ended Write a break-even problem and use a system of linear equations to solve it.

3. You earn a fixed salary working as a sales clerk making \$11 per hour. You get a weekly bonus of \$100. Your expenses are \$60 per week for groceries and \$200 per week for rent and utilities. How many hours do you have to work in order to break even?

4. Reasoning Find A and B so that the system below has the solution $(1, -1)$.

$Ax + 2By = 0$

$2Ax - 4By = 16$

5. You own an ice cream shop. Your total cost for 12 double cones is $24 and you sell them for $2.50 each. How many cones do you have to sell to break even?

6. Multi-Step A skin care cream is made with vitamin C. How many ounces of a 30% vitamin C solution should be mixed with a 10% vitamin C solution to make 50 ounces of a 25% vitamin C solution?

- Define the variables.
- Make a table or drawing to help organize the information.

7. Your hot-air balloon is rising at the rate of 4 feet per second. Another aircraft nearby is at 7452 feet and is losing altitude at the rate of 30 feet per second. In how many seconds will your hot-air balloon be at the same altitude as the other aircraft?

Answer Keys

6-1 Think About a Plan

Solving Systems by Graphing

Cell Phone Plans A cell phone provider offers plan 1 that costs \$40 per month plus \$.20 per text message sent or received. A comparable plan 2 costs \$60 per month but offers unlimited text messaging.

a. How many text messages would you have to send or receive in order for the plans to cost the same each month?

b. If you send and receive an average of 50 text messages each month, which plan would you choose? Why?

Know

1. What equations can you write to model the situation?

Cost per text message	times	Number of text messages	plus	Monthly fee	=	Total cost y (total)

Cell phone plan #2 cost per month $y = 60$

Cell phone plan #1 cost per month $y = 0.20x + 40$

2. How will graphing the equations help you find the answers?

The intersection of the graphs is the point at which the costs of the two plans are equal, based on the number of text messages.

Need

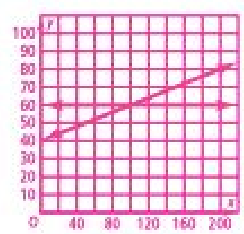
3. How will you find the best plan?

Graph the two equations. Use the graph to find which plan is cheaper if the number of text messages is 50.

Plan

4. What are the equations that represent the two plans? $y = 60$ and $y = 0.20x + 40$

5. Graph your equations.



6. Where will the solution be on the graph?

The graphs intersect at (100, 60). When the number of text messages is 100, the costs of the two plans are equal.

7. What is the solution?

If the number of text messages is 50, choose plan 1, because the cost is lower.

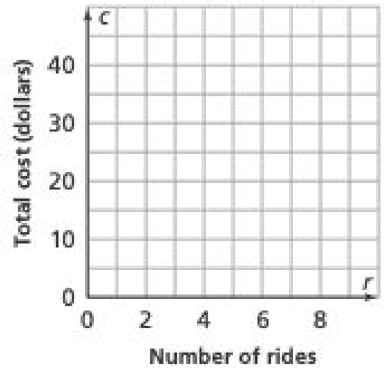
Name ___________________________________

Build Mathematical Literacy
5-2

Read the problem below. Then answer the questions to help you understand the problem.

At Family Fun Park, admission is free, but ride tickets cost \$6 each and parking costs \$10. At Paradise Rides, the admission and parking is free, but the ride tickets cost \$8 each. The total cost, *c*, to visit a theme park and go on *r* rides can be represented by a system of equations. Write and graph the system of equations. For how many rides will the cost at both parks be the same?

Theme Park Costs



1. Circle the important information about the cost of Family Fun Park.
   Check students' work.
2. Underline the important information about the cost of Paradise Rides.
   Check students' work.
3. What needs to be included in the answer to this problem?
   The system of equations, a graph of the system of equations, and if applicable, the number of rides at each park that will cost the participant the same amount of money
4. What will each equation in the system of equations represent?
   One equation will represent the cost of attending Family Fun Park and the other will represent the cost of attending Paradise Rides.
5. How is the cost of attending Family Fun Park similar to the cost of attending Paradise Rides?
   The total cost of attending both theme parks depends on how many rides you go on.
6. How is the cost of attending the theme parks different?
   The total cost of only Family Fun Park includes parking.

6-1 Reteaching (continued)

Solving Systems by Graphing

If the equations represent the same line, there is an infinite number of solutions, the coordinates of any of the points on the line.

Problem

What is the solution to the system? Solve by graphing. Check.

$2x - 3y = 6$

$4x - 6y = 18$

Solution

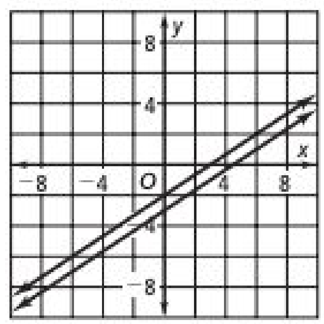
What do you notice about these equations? Using the y-intercepts and solving for the x-intercepts, graph both lines using both sets of points.

$y = \frac{2}{3}x - 2$

$y = \frac{2}{3}x - 3$

Graph equation 1 by finding two points: (0, −2) and (3, 0). Graph equation 2 by finding two points (0, −3) and (4.5, 0).

Is there a solution? Do the lines ever intersect? Lines with the same slope are parallel. Therefore, there is no solution to this system of equations.



Exercises

Solve each system of equations by graphing. Check.

1. $2x = 2 - 9y$
$21y = 4 - 6x$
$\left(-\frac{1}{2}, \frac{1}{3}\right)$

2. $2x = 3 - y$
$y = 4x - 12$
$\left(\frac{5}{2}, -2\right)$

3. $y = 1.5x + 4$
$0.5x + y = -2$
$\left(-3, -\frac{1}{2}\right)$

4. $6y = 2x - 14$
$x - 7 = 3y$
infinitely many solutions

5. $3y = -6x - 3$
$y = 2x - 1$
$(0, -1)$

6. $2x = 3y - 12$
$\frac{1}{3}x = 4y + 5$
$(-9, -2)$

7. $2x + 3y = 11$
$x - y = -7$
$(-2, 5)$

8. $3y = 3x - 6$
$y = x - 2$
infinitely many solutions

9. $y = \frac{1}{2}x + 9$
$2y - x = 1$
no solution

Name ______________________________

Build Mathematical Literacy
5-3

Read the problem below. Then answer the questions to help you understand the problem.

Members of the school band are selling pencils and erasers to raise money for a trip. On the first day, Misha sold a total of 30 items and collected \$6.40. How many of each type of item did he sell?

Help us march at the Playoff Parade!

Pencils: 25¢ each
Erasers: 80¢ each

1. Underline the question you need to answer.
Check students' work.

2. How many quantities is the problem asking you to find? Explain.
Two; The problem asks for the number of pencils and the number of erasers Misha sold.

3. Highlight the information you can use to write a system of equations. Is all of the necessary information given in the text of the problem? Explain.
Check students' work; No; The prices of each item are given in the sign.

4. To solve this problem, you can write a system of equations that contains the equation $x + y = 30$, where x represents the number of pencils Misha sold and y represents the number of erasers. Circle the information in the problem that is represented by this equation.
Check students' work.

5. Explain how the equation $0.25x + 0.80y = 6.40$ represents the situation.
$0.25x$ is the amount earned from selling x pencils and $0.80y$ is the amount earned by selling y erasers. The sum is the total amount Misha collected, \$6.40.

6. Of the equations in Exercises 3 and 4, which equation would you solve for one of the variables in terms of the other variable? Explain.
$x + y = 30$; Sample answer: Neither x nor y has a coefficient, so you can easily solve for either variable.

6-2 Think About a Plan

Solving Systems Using Substitution

Art An artist is going to sell two sizes of prints at an art fair. The artist will charge $20 for a small print and $45 for a large print. The artist would like to sell twice as many small prints as large prints. The booth the artist is renting for the day costs $510. How many of each size print must the artist sell in order to break even at the fair?

Understanding the Problem

1. How much will the artist spend to rent a booth? $510
2. What do you know about selling prices of the prints? small print: $20; large print: $45
3. What do you know about the number of prints the artist would like to sell? The artist wants to sell twice as many small prints as large prints.
4. What is the problem asking you to determine? how many of each size print the artist must sell to break even

Planning the Solution

5. What variables are needed? s = number of small prints sold; d = number of large prints sold.
6. What equation can be used to determine the number of prints that the artist would like to sell based on size? $s = 2d$
7. What equation can be used to determine how many prints the artist has to sell to break even? $20s + 45d = 510$

Getting an Answer

8. What is the solution to the system of equations?

   The artist must sell 6 large prints and 12 small prints to break even.

Name ______________________________

Reteach to Build Understanding
5-3

Together, Amy and Mai have saved \$490 for a trip this summer. Amy has saved \$54 more than Mai. How much has each person saved?

Let a = the amount Amy has saved. Let m = the amount Mai has saved.

Write a system of linear equations to represent the situation.

$$a + m = 490$$
$$a = m + 54$$

Use substitution to find the value of m.

$$(m + 54) + m = 490$$
$$2m + 54 = 490$$
$$m = 218$$

Substitute to find the value of a.

$$a = 218 + 54$$
$$a = 272$$

Amy has saved \$272. Mai has saved \$218.

A parking lot charges \$7 per day to park on weekdays and \$12 per day on weekends. Jamal parked his car in this lot on 6 days last week and spent a total of \$52. How many weekdays and weekend days did Jamal park?

1. Let x = the number of weekdays and let y = the number of weekend days. Fill in the boxes to write a system of equations.
   $x + y = \boxed{6}$
   $\boxed{7}x + \boxed{12}y = 52$
2. Solve the equation for y in terms of x.
   $y = \boxed{6} - \boxed{x}$
3. Rewrite the second equation by substituting your value for y in terms of x found in Exercise 2.
   $7x + 12(\boxed{6 - x}) = 52$
4. Solve your equation in Exercise 3. What is the value of x?
   $x = 4$
5. How can you find the value of y? What is the value of y?
   Sample answer: Substitute 4 for x in the equation from Exercise 2; $y = 2$.
6. How many weekdays and weekend days did Jamal park?
   4 weekdays, 2 weekend days

On the Back! **14 large notebooks, 10 small notebooks**

7. Krysta bought 24 notebooks and spent \$104. The large notebooks cost \$6 each and the small notebooks cost \$2 each. How many of each type of notebook did she buy?

6-2 Reteaching

Solving Systems Using Substitution

You can solve a system of equations by substituting an equivalent expression for one variable.

Problem

Solve and check the following system:

$$x + 2y = 4$$
$$2x - y = 3$$

Solution $x + 2y = 4$	The first equation is easiest to solve in terms of one variable.
$x = 4 - 2y$	Get x to one side by subtracting $2y$.
$2(4 - 2y) - y = 3$	Substitute $4 - 2y$ for x in the second equation.
$8 - 4y - y = 3$	Distribute.
$8 - 5y = 3$	Simplify.
$8 - 8 - 5y = 3 - 8$	Subtract 8 from both sides.
$-5y = -5$	Divide both sides by -5.
$y = 1$	You have the solution for y. Solve for x.
$x + 2(1) = 4$	Substitute in 1 for y in the first equation.
$x + 2 - 2 = 4 - 2$	Subtract 2 from both sides.
$x = 2$	The solution is (2, 1).

Check Substitute your solution into either of the given linear equations.

$x + 2y = 4$	
$2 + 2(1) \stackrel{?}{=} 4$	Substitute (2, 1) into the first equation.
$4 = 4$ ✓	You check the second equation.

Exercises

Solve each system using substitution. Check your answer.

1. $x + y = 3$ **(1, 2)**
$2x - y = 0$

2. $x - 3y = -14$ **(4, 6)**
$x - y = -2$

3. $2x - 2y = 10$ **infinitely many solutions**
$x - y = 5$

4. $4x + y = 8$ $\left(\frac{11}{7}, \frac{12}{7}\right)$
$x + 2y = 5$

6-2 Reteaching (continued)

Solving Systems Using Substitution

Problem

Solve and check the following system:

$\frac{x}{2} - 3y = 10$

$3x + 4y = -6$

Solve	$\frac{x}{2} - 3y = 10$	First, isolate x in the first equation.
	$\frac{x}{2} = 10 + 3y$	Add $3y$ to both sides and simplify.
	$x = 20 + 6y$	Multiply by 2 on both sides.
	$3x + 4y = -6$	Substitute $20 + 6y$ for x in second equation.
	$3(20 + 6y) + 4y = -6$	Simplify.
	$60 + 22y = -6$	Subtract 60 from both sides.
	$22y = -66, y = -3$	Divide by 22 to solve for y.
	$\frac{x}{2} - 3(-3) = 10$	Substitute -3 in the first equation.
	$\frac{x}{2} + 9 = 10$	Simplify.
	$x = 2$	Solve for x.

The solution is (2, −3).

Check $3(2) + 4(-3) \stackrel{?}{=} -6$

$-6 = -6$ ✓

Now you check the first equation.

Exercises

Solve each system using substitution. Check your answer.

5. $-2x + y = 8$
$3x + y = -2$ **(−2, 4)**

6. $3x - 4y = 8$
$2x + y = 9$ **(4, 1)**

7. $3x + 2y = 25$
$2x + 3y = -6$ **($17\frac{2}{5}$, $-13\frac{3}{5}$)**

8. $6x - 5y = 3$
$x - 9y = 25$ **(−2, −3)**

6-3 Additional Vocabulary Support

Solving Systems Using Elimination

Tony is trying to find the solution of the system using elimination.

$2x - 4y = 12$ $3x + 4y = 48$

He wrote these steps to solve the problem on note cards, but they got mixed up.

Solve for x.

Eliminate one variable. Since the sum of the coefficients of y is 0, add the equations to eliminate y.

Simplify.

Substitute 12 for x to solve for the eliminated variable.

Solve for y.

Since $x = 12$ and $y = 3$, the solution is (12, 3).

Use the note cards to complete the steps below.

1. First,
   eliminate one variable. Since the sum of the coefficients of y is 0, add the equations to eliminate y.
2. Second,
   solve for x.
3. Third,
   substitute 12 for x to solve for the eliminated variable.
4. Then,
   simplify.
5. Next,
   solve for y.
6. Finally,
   since $x = 12$ and $y = 3$, the solution is (12, 3).

Name ______________________________

Build Mathematical Literacy
5-4

Read the problem below. Then answer the questions to help you understand the problem.

Lily exercises for a total of 150 minutes each week. She does weight training and cardio workouts. She spends more time doing cardio workouts. The difference between the time she spends on cardio workouts and the time she spends weight training is 72 minutes. How much time does Lily spend on each type of activity?

1. Highlight the question that you need to answer.
   Check students' work.

2. You can write a system of equations to represent this problem. What quantities should you represent with variables?
   The number of minutes Lily spends on cardio workouts and the number of minutes Lily spends on weight training

3. Circle the information in the problem that is represented by the equation $x + y = 150$ where x represents cardio workouts and y represents weight training.
   Check students' work.

4. Underline the information in the problem that you will use to write another equation.
   Check students' work.

5. Circle the equation that can represent the information you underlined in Exercise 4.

   $x + y = 72$ $\quad$ $y = 72x$ $\quad$ $x - y = 72$ $\quad$ $y = 72 - x$

6. Describe the correct answer to this problem, including the units.
   Sample answer: The correct answer will be two lengths of time in minutes, the amount of time Lily spends on cardio workouts and the amount of time Lily spends weight training.

6-3 Think About a Plan

Solving Systems Using Elimination

Nutrition Half a pepperoni pizza plus three fourths of a ham-and-pineapple pizza contains 765 Calories. One fourth of a pepperoni pizza plus a whole ham-and-pineapple pizza contains 745 Calories. How many Calories are in a whole pepperoni pizza? How many Calories are in a whole ham-and-pineapple pizza?

Know

1. What equation will represent the 765 Calories combination of pizza? $\frac{1}{2}x + \frac{3}{4}y = 765$

2. What equation will represent the 745 Calories combination of pizza? $\frac{1}{4}x + y = 745$

Need

3. What possible methods can you use to solve the system of equations?

   You can solve the system by graphing, by substitution, or by elimination.

Plan

4. How can you solve the system of equations by elimination?

   You can multiply one equation by a constant and then add the revised equation to the other original equation. If the constant is chosen carefully, one variable will cancel out and you can solve the resulting equation for the other variable. Then you can substitute that value into one of the original equations to solve for the other variable.

5. How can you eliminate one of the variables to solve the system of equations?

   You can eliminate the variable x by multiplying the second equation by -2 and adding the two equations together.

6. Solve the system of equations.

   660; 580

7. What is the solution of the system?

   (660, 580)

8. How many Calories are in each kind of pizza?

   pepperoni: 660 calories; ham-and-pineapple: 580 calories

6-3 Reteaching

Solving Systems Using Elimination

Elimination is one way to solve a system of equations. Think about what the word "eliminate" means. You can eliminate either variable, whichever is easiest.

Problem

Solve and check the following system of linear equations. $\begin{aligned} 4x - 3y &= -4 \\ 2x + 3y &= 34 \end{aligned}$

Solution The equations are already arranged so that like terms are in columns.

Notice how the coefficients of the y-variables have the opposite sign and the same value.

$$\begin{aligned} 4x - 3y &= -4 \\ \underline{2x + 3y} &\underline{= 34} \\ 6x &= 30 \\ x &= 5 \end{aligned}$$

Add the equations to eliminate y.

Divide both sides by 6 to solve for x.

$$\begin{aligned} 4(5) - 3y &= -4 \\ 20 - 3y &= -4 \\ -3y &= -24 \\ y &= 8 \end{aligned}$$

Substitute 5 for x in one of the original equations and solve for y.

The solution is (5, 8).

Check

$$\begin{aligned} 4x - 3y &= -4 \\ 4(5) - 3(8) &\stackrel{?}{=} -4 \\ 20 - 24 &\stackrel{?}{=} -4 \\ -4 &= -4 \checkmark \end{aligned}$$

Substitute your solution into both of the original equations to check.

You can check the other equaton.

Exercises

Solve and check each system.

1. $3x + y = 3$ **(0, 3)**
$-3x + y = 3$

2. $6x - 3y = -14$ $\left(\frac{2}{3}, 6\right)$
$6x - y = -2$

3. $3x - 2y = 10$ **(2, −2)**
$x - 2y = 6$

4. $4x + y = 8$ **(1, 4)**
$x + y = 5$

6-3 Reteaching (continued)

Solving Systems Using Elimination

If none of the variables has the same coefficient, you have to multiply before you eliminate.

Problem

Solve the following system of linear equations. $\begin{aligned} -2x + 3y &= -1 \\ 5x + 4y &= 6 \end{aligned}$

Solution

$5(-2x - 3y) = (-1)5$ $2(5x + 4y) = (6)2$	Multiply the first equation by 5 (all terms, both sides) and the second equation by 2. You can eliminate the *x* variable when you add the equations together.
$-10x - 15y = -5$ $\underline{10x + 8y = 12}$ $-7y = 7$	Distribute, simplify and add.
$y = -1$	Divide both sides by 7.
$5x + 4(-1) = 6$	Substitute –1 in for y in the second equation to find the value of *x*.
$5x - 4 = 6$	Simplify.
$5x = 10$	Add 4 to both sides.
$x = 2$	Divide by 5 to solve for *x*.

The solution is $(2, -1)$.

Check $-2x + 3y = -1$	Substitute your solution into both original equations.
$-2(2) - 3(-1) \stackrel{?}{=} -1$	
$-1 = -1$ ✓	You can check the other equation.

Exercises

Solve and check each system.

5. $\begin{aligned} x - 3y &= -3 \\ -2x + 7y &= 10 \end{aligned}$ **(9, 4)**

6. $\begin{aligned} -2x - 6y &= 0 \\ 3x + 11y &= 4 \end{aligned}$ **(−6, 2)**

7. $\begin{aligned} 3x + 10y &= 5 \\ 7x + 20y &= 11 \end{aligned}$ $\left(1, \frac{1}{5}\right)$

8. $\begin{aligned} 4x + y &= 8 \\ x + y &= 5 \end{aligned}$ **(1, 4)**

6-4 Additional Vocabulary Support

Applications of Linear Systems

Use the list below to complete the diagram.

use when you want a visual display of the equations	use when it is easy to solve for one of the variables	use when you want an estimation of the solution
use when one equation is already solved for one of the variables	use when the coefficients of one variable are the same or opposites	use when it is not convenient to use graphing or substitution

Choosing a Method for Solving Linear Systems

Graphing	Substitution	Elimination
use when you want a visual display of the equations	use when one equation is already solved for one of the variables	use when the coefficients of one variable are the same or opposites
use when you want an estimation	use when it is easy to solve for one of the variables	use when it is not convenient to use graphing or substitution

6-4 Reteaching (continued)

Applications of Linear Systems

Exercises

1. You have a coin bank that has 275 dimes and quarters that total \$51.50. How many of each type of coin do you have in the bank?
   115 dimes; 160 quarters

2. **Open-Ended** Write a break-even problem and use a system of linear equations to solve it.
   Check students' work.

3. You earn a fixed salary working as a sales clerk making \$11 per hour. You get a weekly bonus of \$100. Your expenses are \$60 per week for groceries and \$200 per week for rent and utilities. How many hours do you have to work in order to break even?
   about 14.5 h

4. **Reasoning** Find A and B so that the system below has the solution $(1, -1)$.
   $Ax + 2By = 0$
   $2Ax - 4By = 16$
   $A = 4$; $B = 2$

5. You own an ice cream shop. Your total cost for 12 double cones is \$24 and you sell them for \$2.50 each. How many cones do you have to sell to break even?
   10 ice cream cones

6. **Multi-Step** A skin care cream is made with vitamin C. How many ounces of a 30% vitamin C solution should be mixed with a 10% vitamin C solution to make 50 ounces of a 25% vitamin C solution?
   - Define the variables.
   - Make a table or drawing to help organize the information.

   37.5 oz of 30% solution; 12.5 oz of 10% solution

7. Your hot-air balloon is rising at the rate of 4 feet per second. Another aircraft nearby is at 7452 feet and is losing altitude at the rate of 30 feet per second. In how many seconds will your hot-air balloon be at the same altitude as the other aircraft?
   about 219 s