Worksheet 1.6 Signed Numbers

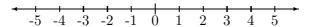
Section 1 Introduction to Signed Numbers

Signed numbers are positive and negative numbers. So far in the worksheets we have mainly talked about the counting numbers, which are all positive. In maths we call the set of these natural numbers \mathbb{N} . We have also talked about fractions. All fractions including whole numbers are in a set called the rationals and represented by the symbol \mathbb{Q} . If we include the negative numbers to the counting numbers we end up with the set of integers represented by \mathbb{Z} . The set of real numbers \mathbb{R} includes positive and negative integers, fractions and irrational numbers.

If my cheque account was in credit \$30 I could say that the balance was +\$30. If it was overdrawn \$30 the balance would be -\$30. Temperatures are measured in degrees. In Sydney the average summer temperature is about $+27^{\circ}$. In Berlin the average winter temperature is about -10° . In some way, we consider the negative of a number to be its opposite. If you were in debit \$30 and you made a deposit of \$30, i.e. you added \$30 to your account, you would have a zero balance. Since we consider -b to be the opposite of b, what is -(-b)? It is the opposite of -b which is b. This statement can be written as a rule:

Two negatives make a positive. Only in maths, of course.

When we want to picture the ordinary number system we can often think about it as a line which is infinitely long in both directions. We choose a point to be zero and the numbers to the right of zero are positive. The numbers to the left of zero are negative. The number line looks like this:



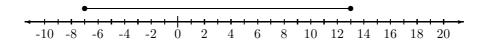
Fractions and other numbers can be fitted in on the number line. For instance $1\frac{1}{2}$ is half-way between 1 and 2. And $-1\frac{1}{2}$ is half-way between -1 and -2. If b is any point on the number line then -b is the same distance from zero but on the opposite side of the number line.

Section 2 Operations involving signed numbers

Example 1: My cheque account is overdrawn \$7, and I make a deposit of \$20. What is my new balance? It is \$13. This can be written as

$$-7 + 20 = 13$$

When we add a number, we go to the right on the number line:



Example 2: The temperature outside in the morning is -15° . During the day the temperature rises by 10° . At the end of the day it is -5° . This can be written as

$$-15 + 10 = -5$$

In example 1 we have

$$-7 + 20 = 20 - 7 = 13$$

In example 2 we have

$$-15 + 10 = -5 = -(15 - 10)$$

These two examples illustrate the general rules:

$$-a+b = b-a$$
$$-a+b = -(a-b)$$

<u>Note:</u> These are just two ways of writing the same thing. Because -(-a) = a we can write a positive sign or a plus sign as two minuses. Also recall that a + b = b + a, so we can change the order of an addition without affecting it. These rules will become familiar with practice.

Example 3: My cheque account balance is \$13 and I write a cheque for \$25. My account becomes \$12 overdrawn. This can be written as

$$13 - 25 = -12$$

It is not long before I receive a letter from the bank informing me of a \$30 dishonour fee. Then the amount in arrears is

$$-12 - 30 = -42$$

Example 4: The temperature in downtown London is -10° during the day. At night it drops a further 5° . This brings the temperature down to -15° . We can write this as

$$-10 - 5 = -15$$

In example 3 we have

$$13 - 25 = -12 = -(25 - 13)$$

In example 4 we have

$$-10 - 5 = -15 = -(10 + 5)$$

These examples illustrate the following rules

$$b-a = -(a-b)$$
$$-a-b = -(a+b)$$

These rules will also become familiar with practice.

The rearranging of these formulae to make things easier to calculate uses some basic rules of arithmetic. One mentioned earlier is that a + b = b + a. Another is that subtracting a positive number is the same as adding a negative number, so that

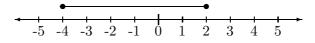
$$a - b = a + (-b)$$

And a final one is that a minus sign in front of a bracket means you should change the sign of everything inside the brackets when you remove them. So

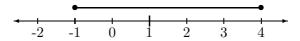
$$-(a-b) = -a+b$$
$$-(-a+b) = a-b$$

Example 5: Calculate the following using a number line:

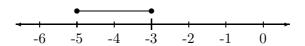
(a) -4+6 (Go to -4 on the number line, then go 6 places to the right).



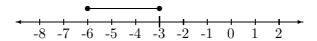
(b) 4-5 (Go to 4 on the number line, then go 5 places to the left).



(c) -3-2 (Go to -3 on the number line, then go 2 places to the left).



(d) -6 - (-3) = -6 + 3 (Go to -6 on the number line, then go 3 places to the right).



Example 6:

- (a) 2 (-4) = 2 + 4 = 6
- (b) 2 (-4 1) = 2 (-5) = 2 + 5 = 7
- (c) -3+8=8-3=5
- (d) -3 12 = -(3 + 12) = -15
- (e) -7+4=-(7-4)=-3
- (f) $\frac{5}{2} \frac{6}{2} = \frac{5-6}{2} = \frac{-(6-5)}{2} = -\frac{1}{2}$

Exercises:

- 1. Using any of the methods given in this worksheet, but without the use of a calculator, work out the following:
 - (a) -3 + 5
- (b) 15 20
- (c) -8-2
- (d) -10 + 14
- (e) -3 (-2)
- (f) -8+7
- (g) -4+4
- (h) -40-9
- (i) 6 14
- (j) 2-12

Exercises 1.6 Signed Numbers

- 1. Evaluate the following:
 - (a) 12 7
 - (b) -12 + 7
 - (c) -12-7
 - (d) -4 + 12
 - (e) -2 (-1)
 - (f) -(-3)+7
 - (g) 10-2-3-5
 - (h) -6-2-12
 - (i) -6 3 + (-2)
- 2. Evaluate the following:
 - (a) 26.3 13.2
 - (b) -14.02 + 11.7
 - (c) -210.1 306.2
 - (d) -27.5 (-1.3)
 - (e) 79.6 + (-0.05)
 - (f) 61.03 (9.1)
- 3. (a) A bank account has an account balance of -\$600. That is, it is \$600 overdrawn. If there is a debit of \$250, what is the new balance? If there has been a bank error, and the debit of \$250 is cancelled, what is the balance now?
 - (b) In a quiz show, you win a point for each correct answer, and lose a point for each incorrect answer. Dale gets 15 right and 20 wrong: what is her score? If the rules change, and it is decided that incorrect answers are ignored, what would Dale's score be now?
 - (c) If the temperature is -4 degrees one morning, and 11 degrees the next, what is the temperature difference?
 - (d) Joan went on a three-month diet. She lost 2 kg in the first month, 1.1 kg in the second, and 3.2 kg in the third. How much weight did she lose altogether? Write an expression, using negative numbers, to show this loss.
 - (e) Peter starts at point A and walks 6 km south, then 17km north, then 2km south. How far is he from his starting point? Write an expression using negative and positive numbers; let north be positive and south be negative.

Answers 1.6

Section 2

1. (a) 2

(c) -10

(e) -1

(g) 0

(i) -8

(b) -5

(d) 4

(f) -1

(h) -49

(j) -10

Exercises 1.6

1. (a) 5

(b) -5

(c) -19

2. (a) 13.1

(b) -2.32

3. (a) -\$850, -\$600

(b) -5, 15

(d) 8

(e) -1

(f) 10

(c) -516.3

(d) -26.2

(c) 15 degrees

(d) -6.3 kg

(g) 0

(h) -20

(i) -11

(e) 79.55

(f) 51.93

(e) 9 km north