

# RATIO & PROPORTION

A **RATIO** is a comparison between two quantities.

We use ratios everyday; one Pepsi costs 50 cents describes a ratio. On a map, the legend might tell us one inch is equivalent to 50 miles or we might notice one hand has five fingers. Those are all examples of comparisons – ratios.

A ratio can be written three different ways. If we wanted to show the comparison of one inch representing 50 miles on a map, we could write that as;

	1 to 50	or
using a colon	1:50	or
Using a fraction	$\frac{1}{50}$	

Because we are going to learn to solve problems, it's easier to write the ratios using fractional notation. If we looked at the ratio of one inch representing 50 miles,  $1/50$ , we might determine 2 inches represents 100 miles, 3 inches represents 150 miles by using equivalent fractions.

That just seems to make sense. Look at that from a mathematical standpoint, it appears that we might also be able to reduce  $3/150$  to  $1/50$ .

Does  $3/150$  represent the same comparison as  $1/50$ ?

The answer is yes – and if we looked at other ratios, we would see that reducing ratios does not effect those comparisons.

Now, we have some good news. We not only discovered how to write ratios, we also learned they can be reduced. Neato!

We noticed that  $3/150$ , 3 inches represents 150 miles, could be reduced to  $1/50$  meaning 1 inch represents 50 miles.

Mathematically, by setting the ratios equal, we could write

$$\frac{1}{50} = \frac{3}{150}$$

That leads us to a new definition.

A **PROPORTION** is a statement of equality between 2 ratios.

Looking at a proportion like  $\frac{1}{2} = \frac{3}{6}$ , we might see some relationships that exist if we take time and manipulate the numbers.

For instance, what would happen if we tipped both ratios **up-side down**?

$$\frac{2}{1} \text{ and } \frac{6}{3}, \text{ notice they are also equal, so } \frac{2}{1} = \frac{6}{3}$$

How about writing the original proportion **sideways**, will we get another equality?

$$\frac{1}{3} \text{ and } \frac{2}{6}, \text{ notice they are equal also, so } \frac{1}{3} = \frac{2}{6}$$

If we continued looking at the original proportion, we might also notice we **could cross multiply** and retain an equality. In other words  $1 \times 6 = 2 \times 3$ . This idea of manipulating numbers is pretty interesting stuff, don't you think.

Makes you wonder whether tipping ratios up-side down, writing them sideways or cross multiplying only works for our original proportion?

Well, to make that determination, we would have to play with some more proportions. Try some, if our observation holds up, we'll be able to generalize what we saw.

Let's try these observations with the proportion  $\frac{2}{3} = \frac{4}{6}$  Can I tip them upside down and

still retain an equality? In other words, does  $\frac{3}{2} = \frac{6}{4}$ ?

How about writing them sideways, does  $\frac{2}{4} = \frac{3}{6}$ ?

How about cross multiplying in the original proportion, does  $2 \times 6 = 3 \times 4$ ?

The answer to all three questions is yes.

Since everything seems to be working, we will generalize our observations using letters instead of numbers.

If  $\frac{a}{b} = \frac{c}{d}$ , then

1.  $\frac{b}{a} = \frac{d}{c}$

2.  $\frac{a}{c} = \frac{b}{d}$

3.  $ad = bc$

.....

Those 3 observations are referred to as **Properties of Proportions**. Those properties can be used to help us solve problems.

To solve problems, most people use either equivalent fractions or cross multiplying to solve proportions.

**Example** If a turtle travels 5 inches every 10 seconds, how far will it travel in 50 seconds?

What we are going to do is set up a proportion. How surprising? The way we'll do this is to identify the comparison we are making. In this case we are saying 5 inches every 10 seconds. Therefore, and this is very important, we are going to set up our proportion by saying inches is to seconds.

On one side we have  $\frac{5}{10}$  describing inches to seconds. On the other side we have to again use the same comparison, inches to seconds. We don't know the inches, so we'll call it "n". Where will the 50 go in the ratio, top or bottom? Bottom, because it describes seconds – good deal. So now we have.

$$\frac{5}{10} = \frac{n}{50}$$

Now, we can find n by equivalent fractions or we could use property 3 and cross multiply.

$$\frac{5}{10} = \frac{n}{50}$$

$$10n = 5 \times 50$$

$$10n = 250$$

$$n = 25 \quad \text{The turtle will travel 25 inches in 50 seconds}$$

It is very important to write the same comparisons on both sides of the equal signs. In other words, if we had a ratio on one side comparing inches to seconds, then we must write inches to seconds on the other side.

If we compared the number of boys to girls on one side, we would have to write the same comparison on the other side, boys to girls. We could also write it as girls to boys on one side as long as we wrote girls to boys on the other side. The first Property of Proportion, tipping the ratios upside down, permits this to happen.

Isn't this great how all this seems to come together? I know, you are saying; I love math, math is my life?

I also know what you're thinking, you want to do some of the problems on your own, right?

Solve these problems by setting up a proportion.

1. If there were 7 males for every 12 females at the dance, how many females were there if there were 21 males at the dance?

Ask yourself, is there a ratio, a comparison in that problem? What's being compared?

2. David read 40 pages of a book in 5 minutes. How many pages will he read in 80 minutes if he reads at a constant rate?
3. On a map, one inch represents 150 miles. If Las Vegas and Reno are five inches apart on the map, what is the actual distance between them?
4. Bob had 21 problems correct on a math test that had a total of 25 questions, what percent grade did he earn? (In other words, how many questions would we expect him to get correct if there were 100 questions on the test?)
5. If there should be three calculators for every 4 students in an elementary school, how many calculators should be in a classroom that has 44 students? If a new school is scheduled to open with 600 students, how many calculators should be ordered?
6. If your car can go 350 miles on 20 gallons of gas, at that rate, how much gas would you have to purchase to take a cross country trip that was 3000 miles long?

The ratio and proportions problems we have done up to this point have expressed a ratio, then given you more information in terms of the ratio previously expressed.

In other words, if the ratio expressed was male to female, then more information was given to you in terms of males or females and you set up the proportion. Piece of cake, right?

Well, what happens if you were given a ratio, like males to females, but then the additional information you received was not given in the terms of the original ratio? Maybe the additional information told you how many students were in a class altogether?

You wouldn't be able to set up a proportion based on what we know now.

But, don't you love it when somebody says but? It normally means more is to come. So hold on to your chair to contain the excitement, we get to learn more by seeing patterns develop.

Up to this point, we know if 2 fractions are equal, then I can tip them upside down, write them sideways and cross multiply and we will continue to have an equality.

If  $\frac{a}{b} = \frac{c}{d}$ , then

1.  $\frac{b}{a} = \frac{d}{c}$

2.  $\frac{a}{c} = \frac{b}{d}$  and

3.  $ad = bc$

Remember, the way we developed those properties was by looking at equal fractions and looking for patterns. If we continued to look at equal fractions, we might come up with even more patterns.

Let's look, we originally said that  $\frac{1}{3} = \frac{2}{6}$ . Now if we continued to play with these fractions, then looked to see if the same things we noticed with these held up for other equal fractions, then we might be able to make some generalizations.

For instance, if I were to keep the numerator 1 on the left side, and add the numerator and denominator together to make a new denominator on the left side, would I still have an equality if I did the same thing to the right side. Let's peek.

We have  $\frac{1}{3} = \frac{2}{6}$ , keeping the same numerators, then adding the numerator and denominator together for a new denominator, we get

$$\frac{1}{1+3} \text{ and } \frac{2}{2+6}. \quad \text{Are they equal? Does } \frac{1}{4} = \frac{2}{8}?$$

Oh boy, the answer is yes. Don't you wonder who stays up at night to play with patterns like this? If we looked at other equal fractions, we would find this seems to be true. So what do we do, we generalize this.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a}{a+b} = \frac{c}{c+d}$$

Other patterns we might see by looking at the fraction  $\frac{1}{3} = \frac{2}{6}$  include adding the numerators and writing those over the sum of the denominators, that would be equal to either of the original fractions.

$$\text{In other words, } \frac{1+2}{3+6} = \frac{1}{3} = \frac{2}{6}$$

And of course, since this also seems to work with a number of different equal fractions, we again make a generalization.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a+c}{b+d} = \frac{a}{b} = \frac{c}{d}$$

So now we have discovered two more patterns for a total of 5 Properties of Proportion.

Now, going back to the problem we were describing earlier that gave a ratio, then gave additional information that was not in terms of the ratio, we'll be able to manipulate the properties of proportion to solve additional problems. Say yes to math, this is really great stuff.

**EXAMPLE** If there are 3 boys for every 7 girls at school, how many boys attend the school if the total student enrollment consists of 440 students.

The first comparison given is boys to girls. Knowing this, we would like to set up a proportion that looks like this:

$$\frac{\text{boys}}{\text{girls}} = \frac{\text{boys}}{\text{girls}} \quad \text{just as we have done before.}$$

The problem that we encounter is while we can put the 3 and 7 on the left side to represent boys and girls, we have to ask ourselves, where does 440 go? It does not represent just boys or just girls, so we can't put it in either position on the right side. 440 represents the total number of boys and girls.

Remembering what we just did, see there was a reason for looking for more patterns, we noticed if we have a proportion like

$$\frac{b}{g} = \frac{b}{g}, \text{ then } \frac{b}{b+g} = \frac{b}{b+g} \quad \text{would be true.}$$

From our problem, we now can see  $b+g$  would be the total of the boys and girls.

Filling in this proportion, we have

$$\frac{3}{3+7} = \frac{b}{440} \quad - \quad \frac{3}{10} = \frac{b}{440}$$

$$\text{Solving; } 10b = 3 \times 440$$

$$10b = 1320$$

$$b = 132$$

There would be 132 boys, to find the number of girls we could subtract 132 from 440.

The point being, if you were given a problem being described by a ratio, then additional information was given to you not using the descriptors in the original ratio, you could manipulate the information using one of the properties of proportion.

To be quite frank, that is not the way I would usually attack that sort of problem. What I would prefer doing is setting up the problem algebraically.

Let's step back and see how ratios work. Remember, we said you could reduce ratios. In other words, if I had the ratio of 3/150, I could reduce it to 1/50.

Visually, the way you reduce is by dividing out a common factor. To reduce  $\frac{3}{150}$ , we could rewrite it as  $\frac{1 \times 3}{50 \times 3}$ . Dividing out the 3's, we would have  $\frac{1}{50}$ .

Now going back to the previous example, we had 3 boys for every 7 girls, with a total enrollment of 440. Doing this algebraically, we still have the same ratio, boys to girls. But, again, we realize the additional information is not given in terms of boys or girls. We just did this problem by playing with the properties of proportion.

Using algebra, the ratio of boys to girls,  $\frac{b}{g}$  is  $\frac{3}{7}$ .

Does that mean we have exactly three boys and seven girls? No, that ratio comes from reducing the actual number in the boys to girls ratio. Since we don't know the common factor we would have divided out a common factor, we'll call it X. Unbelievable concept.

Using the ratio;  $\frac{b}{g}$  or  $\frac{3}{7}$ , we now know we have 3X boys and 7X girls. So the ratio of  $\frac{b}{g}$

Looks like this;  $\frac{3X}{7X}$ .

But notice the sum of 3X and 7X would be the total number of students

The total number of students is 440, therefore we have

$$\begin{aligned} \text{boys} + \text{girls} &= 440 \\ 3X + 7X &= 440 \\ 10X &= 440 \\ X &= 44 \end{aligned}$$

The ratio of boys to girls  $\frac{3X}{7X}$ , that means there are  $3X$  boys or 3 (44) which is 132 boys.

That manipulation allowed us to solve a proportion problem where more information was not given in terms of the first ratio.

From my standpoint, there are two types of **Ratio & Proportion** problems.

**Type 1** A ratio is given, then more information is given in terms of the descriptors of the first ratio. For this type of problem, you set the two ratios equal and solve the resulting equation.

**Type II** A ratio is given, then more information is given to you that is not using the descriptors in the original ratio. This type of problem should be done algebraically as we did in the last example.

## The 5 Properties of Proportion

are if  $\frac{a}{b} = \frac{c}{d}$ , then

1.  $\frac{b}{a} = \frac{d}{c}$  (upside down)

2.  $\frac{a}{c} = \frac{b}{d}$  (sideways)

3.  $ad = bc$  (cross multiply)

4.  $\frac{a}{a+b} = \frac{c}{c+d}$  (numerator over sum of num and den)

5.  $\frac{a+c}{b+d} = \frac{a}{b} = \frac{c}{d}$  (add num, add den)

As you try some of these problems, first determine if they are Type I or Type II, then use the appropriate problem solving strategy.

## RATIO AND PROPORTION

- Express each ratio as a fraction.
  - 3 to 4
  - 8 to 5
  - 9:13
  - 15:7
- Express each ratio in simplest form.
  - 12 to 10
  - 24:36
  - $\frac{15}{18}$
- A certain math test has 50 questions. The first 10 are true – false and the rest are matching. Find:
  - The ratio of true-false questions to matching questions.
  - The ratio of true-false questions to the total number of questions.
  - The ration of the total number of questions to matching questions.
- A 30 pound moonling weighs 180 pounds on the earth. How much does a 300 pound Earthling weigh on the moon?
- The ratio of the sides of a certain triangle is 2:7:8. If the longest side of the triangle is 40 cm, how long are the other two sides?
- If a quarterback completes 20 out of 45 passes in his first game, how many passes do you expect him to complete in his second game if he only throws 18 passes?
- On a trip across the country Joe used 20 gallons of gas to go 300 miles. At this rate, how much gas must he use to go 3500 miles?
- On a map 3 inches represents 10 miles. How many miles do 16 inches represent?
- If our class is representative of the university and there are 2 males for every 12 females. How many men attend the university if the female population totals 15,000?
- The ratio of length to width of a rectangle is 8:3. Find the dimensions of the rectangle if the perimeter is 88dm.

## RATIO & PROPORTION

Students should be able to:

- Identify, write, and simplify ratios
- Set up a proportion from a word problem
- Solve problems using proportions
- Solve problems using unit pricing

### Problem set

1. Of 300 students in the cafeteria, 140 had lunch. Write the ratio of the students in the cafeteria to the students that had lunch.
2. A math test has 50 questions. The first ten are True-False and the rest are matching, find the ratio of True-False questions to matching questions. The ratio of True-False to the total number of questions.
3. Write a proportion to represent the conditional if there are 5 boys for every 7 girls in math class, how many boys are there if there are 35 girls?
4. A baseball team won 8 games and lost 3. What is the ratio of wins to games played?

5. Use the chart: 

Grade Distribution	
A	3
B	5
C	7
D	4
F	1

A	3
B	5
C	7
D	4
F	1

What is the ratio of A's to the class?

What is the ratio of B's to D's?

6. A 30 pound moonling weighs 180 pounds on earth. How much does a 300 pound Earthling weigh on the moon?
7. A quarterback completes 12 out of 20 passes in his first game, how many passes would you expect him to complete in his second game if he only throws 15 passes?
8. On a trip across country, Andy used 20 gallons of gas to travel 300 miles. At this rate, how much gas must he use to go 2000 miles?
9. On a map, if one inch represents 25 miles, how many miles does 7 inches represent?
10. At Intercede Inc., 7 out of 10 people received raises. If they employ 350 people, how many got raises?

11. Snow is falling at a rate of one inch every 3.5 hours. At this rate how long will it take for 6.5 inches of snow to fall?
12. The ratio of width to length in a rectangle is 3:8. If the length of the rectangle is 20 inches, what is the width?
13. There were 5 boys for every 3 girls in the class. Out of the 56 students in the class, how many were girls?
14. Fred paid \$72 last week in gas for his car. If he used 56 gallons of gas, how much did he pay per gallon?
15. A store sells bars of soap at 4 for \$3.56 or 3 for \$2.97. What is the unit price of the soap that is the better buy?
16. Bob drove 320 miles in 5 hours. At this rate, how far could he travel in three hours?
17. 1\_ cups of sugar will serve 6 people, how many people will 2\_ cups serve?
18. A 15 pound ham is enough to serve 20 people. How much ham do you need to serve 50 people?
19. The ratio length to width of a rectangle is 8:3. Find the dimensions of the rectangle if the perimeter is 88 inches.
20. The ratio of men to women in a class is 2:7. Of the 36 students in the class, how many were women?

## MORE PROBLEMS

- 1) On last week's math test Carol had 21 correct out of 25 problems. What percent grade did she receive.
- 2) Bob received an 85% on his history exam. If there were 20 questions, how many did he have correct?
- 3) John receives a 5% commission on his sales. If he received \$30. How much did he sell?
- 4) Bob received an 84% on his history exam. If there were 50 questions, how many did he have wrong?
- 5) If you receive 20% off on a pair of pants that cost \$25, how much would you pay?
- 6) Jessie earns \$25 per week. If 8% is deducted for social security, what is the amount Jessie receives?
- 7) A radio costs \$20, Harold buys it for \$16. What percent off did he receive?

- 8) A quarterback completed 18 of 25 passes in a football game. What percent of the passes did he complete?
- 9) Mrs. Freeze bought a watch for \$45, she was charged 10% federal tax and 4% sales tax. How much did she have to pay altogether?

#### SOLVING PROBLEMS

- 1) Jack sold his driver for \$17.85, making a profit of \$5.95. The profit is what percent of the selling price?
- 2) Twenty-one percent of Maria's salary goes to her car payment. If she earns \$400, how much does she have to pay for her car?
- 3) Miranda's ball club won 5 of 6 games. What percent of the games did his team lose?
- 4) At 4<sup>th</sup> Avenue there are 630 students. If 70 were absent Monday, what percent is this?
- 5) Last year I earned \$5000, this year I earned \$5500. What percent increase is this?
- 6) At 4<sup>th</sup> Avenue 60 students received a 1 in math. This was 20% of the students taking math in the 8<sup>th</sup> grade. How many math students in the 8<sup>th</sup> grade?
- 7) In the Falcon's Nest ice cream cost 20¢. If the school pays 15¢, what percent mark-up is it?
8. The pep club was decreased from 15 members to 12 members. What was the percent of decrease?
- 9) \$15 is what percent more than \$10?